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# Modeling deforestation due to population growth and wood industrialization

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**Abstract.** More than 30% of the earth's land surface is covered by the forest. Increase in population undergoes activities like construction, grazing, agriculture activities, and industrialization causing permanent clearing of land to make room for something besides the forest, which is called deforestation. Considering this scenario, the mathematical model is framed for studying the dynamics with using four compartments such as deforestation of the dense forest, deforestation of the urban forest, population growth and wood industrialization. Using the dynamical phenomenon, the boundedness of the system is proposed. The proposed model has five equilibria. Behaviour of the system around all feasible equilibria is scrutinized through local stability theory of differential equations. The 3d phase portrait gives the chaotic behavior of each compartment. Basic reproduction number value assists the bifurcation and the sensitivity analysis. Bifurcation analysis gives the ideal value, then the comparison of threshold and ideal value suggests the permissible situation of the compartment. For these findings, analytics results are verified through numerically validated data.

## §1 Introduction

Forests spread over the globe are the dominant terrestrial ecosystem of Earth (Wikipedia) (KusumLata (2017)). Forest maintains the air quality, the earth's biodiversity, prevents soil and water quality, global greenhouse gases, and provides valuable medicines, etc. (Qureshi (2019), Repo et al. (2015), Hendrey (1999)). The increase in the human population poses a big menace for the forests. Land for the development of the needed infrastructure of the increased population mainly comes from clearing forests. All over the world the dense forests are cleared to utilize the growing woods or the forest area (Nath (2012), Otu (2011)). For many years, around 13 million hectares of forest area were lost through logging and burning the dense forests (Brad-

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ford (2018)). To expand the forest area and eco-tourism, the Forest and Wildlife Department of different countries regularly conduct plantation programs by encouraging students, farmers, women, and localities in the rainy season (SA (2016)).

Numerous pieces of research have been conducted relating to mathematical modeling that describes the understanding of deforestation due to population growth (Schneider (2011), Bologna (2020), Lawson (2020)). Qureshi and Yusuf have shown the impacts of deforestation on wildlife species using a nonlinear fractional-order model (Qureshi (2019)). To study the depletion of forestry resources caused by population and population pressure augmented industrialization, Misra et. al. (2014) has proposed a nonlinear mathematical model including the control of population pressure, using economic efforts.The epidemic model is used widely by researchers to study the dynamics COVID-19 outbreak. (Khan et al. (2021), Baba et al. (2021)).

Teru and Koya (2020) have projected a nonlinear mathematical model of deforestation due to population density and population pressure and validated local as well as global stability of the defined model. Chaudhary and Dhar (2013) have proposed a compartmental model to study the effect of maturation delay on forestry biomass with a wood based and synthetic industries. By exploring the hopf bifurcation analysis Chaudhary et al. (2013) constructed a model of synthetic based industries causing the depletion of forestry. A model of deforestation due to human population and its effect on farm fields has been developed by Sebastian and Victor (2018).

Literature gives the idea about the formulation of model. The above study of deforestation using mathematical modeling carried out is related to stability and parametric behavior to the model. Motivation of this current study comes up with the compartmental model by observing the fact that the wood-based industries are increasing to fulfill the requirements of the growing human population, which ultimately results in deforestation. The model has more focus on threshold value and its critical value as it is ideal value for securing environment. Moreover, the urban forests play an influential role to maintain the environmental and the economic conditions of respective cities. In the present model, the impact of increased population on dense forest and the urban forest is considered separately to relate a detailed study of deforestation.

Keeping mentioned above in mind, the formulation of a mathematical model with a system of nonlinear differential equations has been done in section 2. It also includes the data collection, and boundedness of equilibria which are solutions of the system. Section 3 produces the basic reproduction number and bifurcation respectively. Bifurcation gives significant output of this dynamical model. Local stability analysis works out in section 4. Section 5 consists of the sensitivity analysis of each parameter. Section 6 is numerical simulation part of interpreted results. Finally concluding the model is in section 7.

## §2 Formulation of model

It is fact that forest helps in forming the home for various wildlife. It gives other aids for human beings like recreation area, source of fuel, raw material for industries and many more. Nowadays it is very well observed that, growing population is the reason for the depletion of

Table 1. Model parameters and their interpretation.

| Notation | Parameter Description | Parametric values |
|---|---|---|
| $B_1$ | Growth rate of dense forest biomass | 0.7 |
| $B_2$ | Growth rate of urban forest biomass | 0.5 |
| $B_3$ | Intrinsic growth rate of human population | 0.6 |
| $B_4$ | Constant rate of raw material available for wood industries | 0.01 |
| $\beta$ | Depletion rate of urban forest due to population growth | 0.03 |
| $\gamma_1$ | Depletion rate of dense forest due to wood industries | 0.8 |
| $\gamma_2$ | Depletion rate of urban forest due to wood industries | 0.8 |
| $\epsilon$ | New plantation rate for dense forest | 0.012 |
| $\mu_{i=1,2,3,4}$ | Natural depletion rate for respective compartment | 0.01,0.02,0.1,0.24 |
| $\eta_1$ | Carrying capacity of dense forest biomass | 0.8 |
| $\eta_2$ | Carrying capacity of urban forest biomass | 0.6 |
| $\eta_3$ | Carrying capacity of human population | 0.2 |

natural resources. This leads to develop a dynamical model having four compartments density of dense forest ($D_F$), density of urban forest ($U_F$), density of population growth ($P_g$) and density of wood industrialization ($W_d$).

$$\begin{aligned}
\frac{dD_F}{dt} &= B_1 D_F \left(1 - \frac{D_F}{\eta_1}\right) - \gamma_1 D_F W_d + \varepsilon D_F - \mu_1 D_F, \\
\frac{dU_F}{dt} &= B_2 U_F \left(1 - \frac{U_F}{\eta_2}\right) - \gamma_2 U_F W_d - \beta P_g U_F - \mu_2 U_F, \\
\frac{dP_g}{dt} &= B_3 P_g \left(1 - \frac{P_g}{\eta_3}\right) + \beta P_g U_F - \mu_3 P_g, \\
\frac{dW_d}{dt} &= B_4 + \gamma_1 D_F W_d + \gamma_2 U_F W_d - \mu_4 W_d.
\end{aligned} \tag{1}$$

Each compartment is connected with some rate. Description and parametric values are given in Table 1. The data has been taken historically between years 2013-2020. Parametric values of $B_1$, $B_3$, $\beta$, $\eta_1$, $\eta_3$ and $\varepsilon$ are taken from Teru and Koya (2020). Values of $\gamma_1$ and $\mu_4$ are observed from Chaudhary et al. (2020). From Sebastian (2018) value of $\mu_1$ is taken. $\mu_3$ is from Qureshi (2019). Except other parameters are taken hypothetically.

## 2.1 Equilibrium points

After solving the system (1), the proposed model has five equilibrium points:
(i) First is urban deforestation-free equilibrium point $E_1\left(\frac{\eta_1(-\gamma_1 r_1+B_1-\mu_1+\varepsilon_1)}{B_1}, 0, \frac{\eta_1(B_3-\mu_3)}{B_3}, r_1\right)$;
(ii) Second equilibrium point is dense deforestation-free due to wood industries $E_2\left(0, \frac{\eta_2(-\gamma_2 r_2+B_2-\mu_2)}{B_2}, 0, r_2\right)$;
(iii) Third is population growth-free equilibrium point

$E_3\left(\frac{\eta_1(-\gamma_1 r_3+B_1-\mu_1+\varepsilon_1)}{B_1}, \frac{\eta_2(-\gamma_2 r_3+B_2-\mu_2)}{B_2}, 0, r_3\right)$;

(iv) Fourth is dense deforestation-free equilibrium point

$E_4\left(0, \frac{B_3(r_4-1)+\mu_3}{\beta}, r_4\eta_3, -\frac{r_4\left(\beta^2\eta_2\eta_3+B_2B_3\right)-\beta\eta_2(B_2-\mu_2)-B_2(B_3-\mu_3)}{\beta\eta_2\gamma_2}\right)$;

(v) Fifth is optimum issue point

$$E^*\Big(\frac{\eta_1\left(\gamma_1 r_5\left(\beta^2\eta_2\eta_3+B_2B_3\right)+\beta\eta_2\left(\gamma_2\left(B_1-\mu_1\right)-\gamma_1\left(B_2-\mu_2\right)+\gamma_2\varepsilon_1\right)-B_2\gamma_1\left(B_3-\mu_3\right)\right)}{B_1\beta\eta_2\gamma_2},$$
$$\frac{B_3\left(r_5-1\right)+\mu_3}{\beta}, r_5\eta_3, \frac{r_5\left(\beta^2\eta_2\eta_3+B_2B_3\right)-\beta\eta_2\left(B_2-\mu_2\right)-B_2\left(B_3-\mu_3\right)}{\beta\eta_2\gamma_2}\Big),$$

where $r_i$, $(i=1,2,...,5)$ are defined in Appendix A.

## 2.2 Boundedness of solutions

In this section, the boundedness of the solutions is proved with detailed computation showing the validation of mathematical models.

**Lemma:** The set

$\Omega=\left\{(D_F,U_F,P_g,W_d): 0\le D_F\le L_1, 0\le U_F\le L_2, 0\le P_g\le L_3, 0\le W_d le\frac{B_4}{k}\right\}$

is a feasible region for all solutions of system (1).

**Proof:** Consider the first equation of system (1),

$\frac{dD_F}{dt}=B_1D_F\left(1-\frac{D_F}{\eta_1}\right)-\gamma_1D_FW_d+\varepsilon_1D_F-\mu_1D_F$,

then $\frac{dD_F}{dt}\le B_1D_F\left(1-\frac{D_F}{\eta_1}\right)+\varepsilon_1D_F$,

and $0\le B_1D_F\left(\frac{\eta_1-D_F}{\eta_1}\right)+\varepsilon_1D_F\Rightarrow D_F\left(t\right)\le\frac{\eta_1}{B_1}\left(B_1+\varepsilon\right)=L_1$.

Next equation, $\frac{dU_F}{dt}\le B_2U_F\left(1-\frac{U_F}{\eta_2}\right)\Rightarrow U_F\left(t\right)\le\eta_2=L_2$.

Similarly, $\frac{dP_g}{dt}\le B_3P_g\left(1-\frac{P_g}{\eta_3}\right)+\beta P_gU_F\le B_3P_g-\frac{B_3P_g^2}{\eta_3}+\beta P_g\eta_2$,

then $P_g\left(t\right)\le\frac{\eta_3}{B_3}\left(B_3+\beta\eta_2\right)=L_3$,

and $\frac{dW_d}{dt}\le B_4+\left(\gamma_1D_F+\gamma_2U_F\right)W_d\Rightarrow\frac{dW_d}{dt}-\left(\gamma_1L_1+\gamma_2L_2\right)W_d\le B_4$,

Let $\gamma_1L_1+\gamma_2L_2=k$, then

$W_d(t)\le\frac{B_4}{k}e^{-kt}$. As $t\to\infty$, $W_d(t)\le\frac{B_4}{k}$.

Hence, the solutions are bounded and the model is well-defined. $\Omega$ expresses the set of all the solutions.

# §3 Basic reproduction number

Basic reproduction number, also called a threshold value is calculated by using the next generation matrix method (Diekmann (1990)). It suggests the spread of deforestation of dense forest and urban forest with respect to each equilibrium point due to both causes. With reference to this threshold value, analysis of backward bifurcation is carried out for each equilibrium point in this section. After applying bifurcation, ideal threshold value ($R_C$) has been evaluated and compared with basic reproduction number in Table 2. Values admits the fact that evalu-

Table 2. Comparison of threshold and ideal value.

| Equilibrium Point | Threshold Value ($R_0$) | Ideal Value ($R_C$) |
|---|---|---|
| $E_1$ | 0.8190 | 0.72 |
| $E_2$ | 0.1750 | 0.60 |
| $E_3$ | 3.0690 | 0.27 |
| $E_4$ | 0.0195 | 0.02 |
| $E^*$ | 3.2700 | 0.10 |

ated values are permissible compartmental growth values.

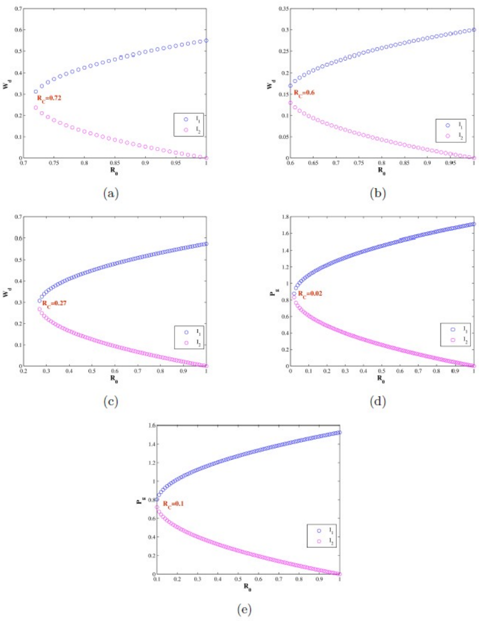


Figure 1. Backward Bifurcation Analysis.

Recall the equilibrium points with its interpretation, the description of threshold and bifurcation analysis has been expressed as shown in Figure 1(a-e).

Table 2 is the comparison of threshold value and ideal value at each equilibrium point. At $E_1$, due to wood industrialization and population growth, deforestation of dense forest is 81.9% while ideal value 72% is permissible for keeping the environment safe. From $E_2$ one can say that, only wood industrialization affects 17.5% deforestation of urban forest while observation suggests that 60% deforestation of urban forest may not be affected by surroundings. Point $E_3$ proposes that, wood industrialization causes deforestation of urban forest and dense forest is 306% and ideal value advice that both causes average 27% deforestation is admissible. In the same manner, $E_4$ suggests for population growth as 1.95% while critical point 2%. At $E^*$ that is endemic point, deforestation of both forests is 327% which is not recommended as the percentage seems very high while the ideal value is only 10%.

## §4 Local stability

Stability expresses the behaviour of a model around its equilibrium points. Here, the local stability is analysed by determining the sign of eigenvalues of the corresponding jacobian matrix $(J)$ of the system (1)

$$J = \begin{bmatrix} J_{11} & 0 & 0 & -\gamma_1 D_F \\ 0 & J_{22} & -\beta U_F & -\gamma_2 U_F \\ 0 & \beta P_g & J_{33} & 0 \\ \gamma_1 W_d & \gamma_2 W_d & 0 & J_{44} \end{bmatrix},$$

where

$J_{11} = B_1\left(1 - \frac{D_F}{\eta_1}\right) - \frac{B_1 D_F}{\eta_1} - \gamma_1 W_d + \varepsilon_1 - \mu_1,$

$J_{22} = B_2\left(1 - \frac{U_F}{\eta_2}\right) - \frac{B_2 U_F}{\eta_2} - \gamma_2 W_d + \beta P_g - \mu_2,$

$J_{33} = B_3\left(1 - \frac{P_g}{\eta_3}\right) - \frac{B_3 P_g}{\eta_3} - \beta U_F - \mu_3,$

$J_{44} = D_F\gamma_1 + U_F\gamma_2 - \mu_4.$

After evaluating all the eigenvalues of the jacobian matrix at each equilibrium point, the following theorems have been obtained, coming up with necessary conditions.

**Theorem 4.1:** Equilibrium point $E_1$ is locally asymptotically stable if,

(i) $\frac{\beta\eta_3\mu_3}{B_3} + B_2 < \beta\eta_3 + \gamma_2 r_1 + \mu_2;$

(ii) $\eta_1 (B_1 + \varepsilon_1) < \frac{B_1\mu_4}{\gamma_1} + \eta_1 (\gamma_1 r_1 + \mu_1)$ or $2\gamma_1 r_1 + \mu_1 < B_1 + \varepsilon_1.$

**Theorem 4.2:** Equilibrium point $E_2$ is locally asymptotically stable if,

(i) $B_1 + \varepsilon_1 < \gamma_1 r_2 + \mu_1$

(ii) $max\left(\frac{B_2(\beta\eta_2 + B_3) - B_3\mu_3}{\beta\eta_2}, \frac{\gamma_2\eta_2 B_2 - B_2\mu_4}{\eta_2\gamma_2}\right) < \gamma_2 r_2 + \mu_2.$

**Theorem 4.3:** Equilibrium point $E_3$ is locally asymptotically stable if,

(i) $\gamma_1 r_3 + \mu_1 < B_1 + \varepsilon_1;$

(ii) $B_2 (1 + 2\gamma_1 r_3 + 2\mu_1) < 2 (B_1 + \varepsilon_1) + \gamma_2 r_3 + \mu_2;$

(iii) $\frac{\beta\eta_2}{B_2} (B_1 + \varepsilon_1) + B_3 < \mu_3 + \frac{\beta\eta_2}{B_2} (\gamma_1 r_3 + \mu_1)$

(iv) $(B_1 + \varepsilon_1) > (\gamma_1 r_3 + \mu_1).$

**Theorem 4.4:** Equilibrium point $E_4$ is locally asymptotically stable if,

(i) $\beta\eta_2\gamma_1 (\eta_2\eta_3 r_4 + \mu_2) + B_2\gamma_1 (B_3 r_4 + \mu_3) + B_1 + \varepsilon_1 < \beta B_2\eta_2\gamma_1 (1 + \beta\eta_2) + \mu_1;$

(ii) (iii) $(B_3 r_4 + \mu_3)\gamma_2 < \beta (\mu_4 + B_3\gamma_2).$

**Theorem 4.5:** Equilibrium point $E^*$ is locally asymptotically stable if,

(i) $B_1 + \varepsilon_1 < \frac{2B_1 D_F}{\eta_1} + \gamma_1 W_d + \mu_1;$

(ii) $B_2 < \frac{2B_2 U_F}{\eta_2} + \gamma_2 W_d + \beta P_g + \mu_2;$

(iii) $B_3 + \beta U_F < \frac{2B_3 P_g}{\eta_3} + \mu_3;$

(iv) $\gamma_1 D_F + \gamma_2 U_F < \mu_4.$

These conditions are to be followed to have stable behaviour of the model around its equilibrium.

## §5 Sensitivity analysis

In this section, sensitivity analysis determines how parametric values affect each equilibrium point. Sensitivity analysis has been worked out with reference to basic reproduction number.

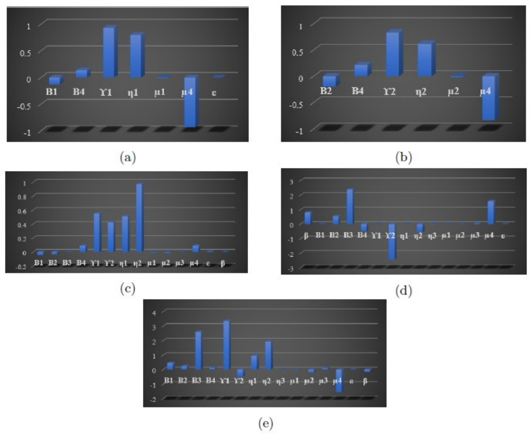

Figure 2. Sensitivity Analysis of Parameters on Threshold Value.

Figure 2 shows values of basic reproduction number ($R_0$) vary positively, negatively with respect to the model parameters. Moreover, it is observed that ($R_0$) is unaffected by some parameters. Figures 2(a), (c) and (e) illustrate that depletion rate ($\gamma_1$) and carrying capacity of dense forest ($\eta_1$) is a common parameter which is more effective causing deforestation in the first, third and endemic equilibrium points. Carrying capacity of urban forest ($\eta_2$) is much affected by second, third and endemic equilibrium points. Analysis suggests that the growth rate of population rate ($B_3$) and growth rate of woods for industries ($B_4$) affected deforestation. Other parameters are less effective.

## §6 Numerical simulation

In this section, the analytical results are numerically simulated in this section using MATLAB by taking parametric values given in Table (1). Figure 3(a) and Figure 3(b) illustrate the periodic diagram of dense forest and urban forest with variation in increasing population growth and wood industries respectively. Figure 3(c) represents urban forest with population growth and wood industries and Figure 3(d) is for dense forest. Each figure suggests that as population growth and the number of wood industries increase, deforestation of both forests increases. As time goes, it turns into chaos which advocates periodicity in nature. Comparing all the figures it is seen that the wood-based industries are more dependent on dense forests compared to urban forests as intensity of deforestation for urban forests is less.

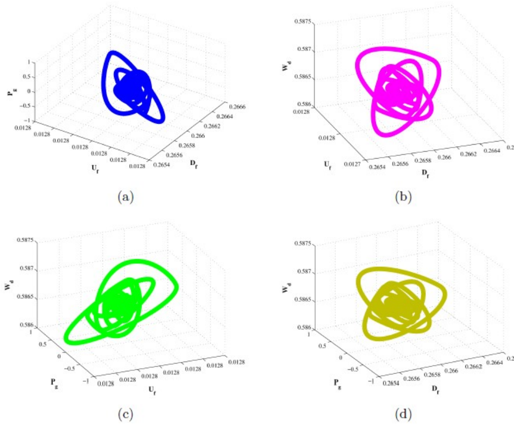

Figure 3. 3d Phase Potrait.

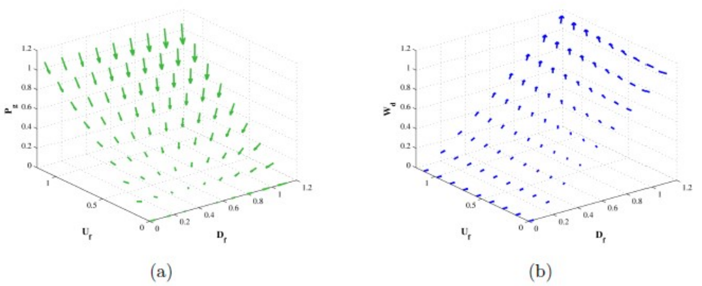

Figure 4. 3d Directed graph.

Figure 4(a) and Figure 4(b) show relation of deforestation of both the forests (dense and urban) with population growth and wood-based industries respectively. The direction of arrows in Figure 4(a) shows that, as population growth is directly proportional to the deforestation of urban forest and dense forest. Intensity of arrows in Figure 4(b) interprets that, as wood supply in industrialization increases, deforestation of urban forest and dense forest increases. It is advisable to control growth of population and supplying woods to industries.

## §7 Conclusion

In this paper, a mathematical model of deforestation is formulated with wood industrialization and population growth having important parameters by taking historical data. The feasible region of solution advocates the well-defined model. Local stability of equilibrium points shows the dynamical behaviour of the system with necessary conditions. More focus has been carried out related to the threshold value. Threshold value of each equilibrium has been analysed using next generation matrix method. Supporting the threshold value, backward bifurcation has been articulated. The ideal value in bifurcation suggests the maximum percentage allowance of deforestation due to population growth and supply for wood industrialization in accordance with each respective equilibrium point. Ideal value is a critical value advocates permissible percentage value of deforestation for caring environment.

Sensitivity analysis is executed to show the role of each parameter on the specified system. Simulation displays the numerically interpreted results in graphical manner. Periodic nature leads to a fact that cumulative rate of deforestation of urban forest and dense forest will go on increase if there is no control on growth of population and delivering woods to industries.

# Appendix

$$r_1 = \eta_1\gamma_1^2\_Z^2 + (\eta_1\gamma_1(-B_1+\mu_1-\varepsilon_1)+B_1\mu_4)\_Z - B_4B_1;$$

$$r_2 = \eta_2\gamma_2^2\_Z^2 + (\eta_2\gamma_2(-B_2+\mu_2)+\mu_4)\_Z - B_4B_2;$$

$$\begin{aligned} r_3 = {} & \left(B_1\eta_2\gamma_2^2 + B_2\eta_1\gamma_1^2\right)\_Z^2 \\ & + (B_1(-B_2(\eta_1\gamma_1+\eta_2\gamma_2)+\eta_2\gamma_2\mu_2+B_2\mu_4)+\eta_1\gamma_1B_2(\mu_1-\varepsilon_1))\_Z - B_4B_2B_1; \end{aligned}$$

$$\begin{aligned} r_4 = {} & B_3\gamma_2\left(\beta^2\eta_2\eta_3 + B_2B_3\right)\_Z^2 \\ & - \left(\left(\beta^2\eta_2\eta_3+B_2B_3\right)(\gamma_2(B_3-\mu_3)+\beta\mu_4) - B_3\gamma_2(B_2(B_3-\mu_3)+\beta\eta_2(B_2-\mu_2))\right)\_Z \\ & + (B_3-\mu_3)(\beta\eta_2\gamma_2(B_2-\mu_2)+B_2\gamma_2(B_3-\mu_3)+B_2\beta\mu_4) \\ & + \beta^2\eta_2(\mu_4(B_2-\mu_2)-\gamma_2B_4). \end{aligned}$$

## Declarations

**Conflict of interest** The authors declare no conflict of interest.

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