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Approximate solution of an advanced reciprocal-radical functional equation

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Abstract. The significant role of square root mapping in probability, statistics, physics, architecture and engineering motivates us to emphasize on a new radical functional equation arising from a square root and reciprocal square root mappings. The interesting attribute of this equation is that it has both a square root mapping and a reciprocal square root mapping as solutions. We establish that the hyperstabilities of this equation exist using a fixed point alternative theorem. It is also demonstrated with an example that the stability may fail in special cases.

§1 Introduction

The role of functional equations has become imperative in many fields such as communication engineering, computer graphics, computer networks, digital image processing and decision theory. Due to its numerous applications, solving stability problems of mathematical equations has a substantial influence in many branches of mathematics such as algebra, geometry, probability, stochastic process, statistics and in other domains.

The stability problem pertinent to mathematical equations emerged through a famous query during the Mathematical Colloquium held at the University of Wisconsin in [25]. An exceptional partial response was provided in [11]. In due course, the stability result proved in [11] had a great impact in the research field of analysis and induced many academicians to find the solution of stability problems through different versions in [9, 15, 16].

The foremost result regarding hyperstability concerning homomorphims in groups was discussed in [2]. Moreover, a class of additive functional equations was considered to obtain its hyperstability in [14]. There are several hyperstability results of different equations (see [1, 4, 5, 6, 10, 13, 20]).

The stabilities of a multidimensional square root functional equation of the following form

$$p\left(\sum_{\ell=1}^{k} a_\ell x_\ell + 2\sum_{\ell=1}^{k-1}\sum_{m=\ell+1}^{k}\sqrt{a_\ell a_m x_\ell x_m}\right) = \sum_{\ell=1}^{k}\sqrt{a_\ell}\,p(x_\ell) \tag{1}$$

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were obtained in [18], where $a_\ell \neq 0$, for $\ell = 1, 2, \ldots, k$. It was proved that the equation (1) has a radical (square root) mapping $p(x) = \sqrt{x}$, $x \in \mathbb{R}^+$ as its solution.

The following radical functional equations were considered to obtain their Ulam stability under 2-normed spaces in [12], that is,

$$f(\sqrt{ax^2 + by^2}) = af(x) + b(f(y)$$

and

$$f(\sqrt{ax^2 + by^2}) + f(\sqrt{|ax^2 - by^2|}) = a^2 f(x) + b^2 f(y)$$

where $a, b > 0$ are fixed real numbers and $x, y \in \mathbb{R}$. A radical quartic functional equation of the form

$$f\left(\sqrt[4]{x^4 + y^4}\right) = f(x) + f(y)$$

was also dealt in [8] to find its stability in the setting of 2-Banach spaces.

On the contrary, the reciprocal functional equation

$$r(\mu + \nu) = \frac{r(\mu)r(\nu)}{r(\mu) + r(\nu)} \tag{2}$$

was dealt to study its stability in [19]. It was proved that the people reciprocal mapping $r(\mu) = \frac{1}{\mu}$ is a solution of equation (2). The stability results of (2) inspired to consider several forms of reciprocal functional equations (see [7, 17, 21, 22, 23, 24]).

An equation $f(x) = 0$ is a reciprocal equation if the multiplicative inverse of any solution of equation is also the solution of it. A reciprocal equation remains unchanged by replacing x by $\frac{1}{x}$. This concept stands behind us to frame a functional equation emerging from square root and reciprocal square root functions in this study.

So far in the literature, stability problems were solved pertaining to many mixed type functional equations. Such mixed type functional equations have linear, quadratic, cubic or quartic mappings as solutions. Also, there are different radical functional equations with radical functions only as solutions. But our intention is to focus on a new functional equation which has radical and reciprocal-radical functions as solutions.

The interesting theory and applications of square root functional equation (1) and reciprocal functional equation (2) influenced us to introduce a new equation of the following form

$$p(\alpha + \beta + 2\sqrt{\alpha\beta}) + p\left(\frac{\alpha\beta}{\alpha + \beta + 2\sqrt{\alpha\beta}}\right) = p(\alpha) + p(\beta) + \frac{p(\alpha)p(\beta)}{p(\alpha) + p(\beta)} \tag{3}$$

One can easily check that the functions $p(\alpha) = \sqrt{\alpha}$ and $p(\alpha) = \frac{1}{\sqrt{\alpha}}$ are solutions of equation (3). We discuss some fundamental results pertinent to equation (3). We also establish the hyperstabilities of (3) considering the domain as positive real numbers.

In the entire investigation of this paper, we will denote $\mathbb{N}$ as the set of all natural numbers, $\mathbb{N}_0$ as the set of all nonnegative integers, $\mathbb{N}_{m_0}$ as the set of all integers greater than or equal to m_0, $\mathbb{R}$ as the set of all real numbers and $\mathbb{R}^+$ as the set of all positive real numbers. As the solutions of equation (3) involve radicals and to avoid imaginary values for the functional value of p, we restrict the domain to be the set of positive reals.

§2 Prerequisites in hyperstability and fixed point theory

Here we refer to a few noteworthy ideas associated with fixed point theorem and hyperstability [3] which are important to obtain the major outcomes of this study. We utilize the following important statements to obtain the stabilities of equation (3).

(S1) Assume that $\mathcal{X}$ is a non-empty set and $\mathcal{Y}$ is a complete normed space (Banach space). Let $p_1, p_2, \ldots, p_k : \mathcal{X} \longrightarrow \mathcal{X}$ and $G_1, G_2, \ldots, G_k : \mathcal{X} \longrightarrow \mathbb{R}_+$ be given mappings.

(S2) Assume that an operator $\mu : \mathcal{Y}^{\mathcal{X}} \longrightarrow \mathcal{Y}^{\mathcal{X}}$ satisfies the following inequality

$$\|\mu g(\alpha) - \mu h(\alpha)\| \leq \sum_{j=1}^{k} G_j(\alpha) \left\| g\left(p_j(\alpha)\right) - h\left(p_j(\alpha)\right) \right\| \tag{4}$$

for all $g, h \in \mathcal{Y}^{\mathcal{X}}$, $\alpha \in \mathcal{X}$.

(S3) Let $\Lambda : \mathbb{R}_+^{\mathcal{X}} \longrightarrow \mathbb{R}_+^{\mathcal{X}}$ be a mapping defined by

$$\Lambda\Delta(\alpha) = \sum_{j=1}^{k} G_j(\alpha) \Delta\left(p_j(\alpha)\right), \quad \Delta \in \mathbb{R}_+^{\mathcal{X}},\ \alpha \in \mathcal{X}. \tag{5}$$

We apply the following theorem to prove the existence of unique fixed point operator $\mu : \mathcal{Y}^{\mathcal{X}} \longrightarrow \mathcal{Y}^{\mathcal{X}}$.

Theorem 2.1 *Suppose the statements* (S1)–(S3) *hold. Let $\varphi : \mathcal{X} \longrightarrow \mathbb{R}_+$ be a function and $\chi : A \longrightarrow B$ be a mapping satisfying the following two inequalities*

$$\|\mu\varphi(\alpha) - \varphi(\alpha)\| \leq \chi(\alpha), \quad \alpha \in \mathcal{X}, \tag{6}$$

$$\chi^\star(\alpha) = \sum_{m=0}^{\infty} \Lambda^m \chi(\alpha) < \infty, \quad \alpha \in \mathcal{X}. \tag{7}$$

Then, there exists a unique fixed point ξ of μ such that

$$\|\chi(\alpha) - \xi(\alpha)\| \leq \chi^\star(\alpha), \quad \alpha \in \mathcal{X}. \tag{8}$$

Moreover,

$$\xi(\alpha) = \lim_{m\to\infty} \mu^m \chi(\alpha) \tag{9}$$

for all $\alpha \in \mathcal{X}$.

§3 Fundamental results pertinent to equation (3)

In the present section, we introduce a definition and prove a theorem which will be keys to obtain our main results.

Definition 3.1 *A single real-valued function $p : \mathbb{R}^+ \longrightarrow \mathbb{R}$ is called reciprocal-radical if it is a solution of the following equation*

$$p(4\alpha) + p\left(\frac{\alpha}{4}\right) = \frac{5}{2}p(\alpha) \tag{10}$$

for all $\alpha \in \mathbb{R}^+$.

In the following theorem, we prove that a reciprocal-radical function $p : \mathbb{R}^+ \longrightarrow \mathbb{R}$ also satisfies a general functional equation with p as a single real-valued function.

Theorem 3.1 *Assume a single real-valued function $p:\mathbb{R}^+\longrightarrow\mathbb{R}$ satisfies (3). Then it also satisfies the following general functional equation*

$$p\left(4^r\alpha\right)+p\left(\frac{\alpha}{4^r}\right)=\frac{4^r+1}{2^r}p(\alpha) \tag{11}$$

for all $\alpha\in\mathbb{R}^+$, where r is a positive integer.

Proof. We will prove this theorem by strong induction method. For any positive integer r, let $A(r)$ be such that

$$A(r):p\left(4^r\alpha\right)+p\left(\frac{\alpha}{4^r}\right)=\frac{4^r+1}{2^r}p(\alpha). \tag{12}$$

Let us first put $\beta=\alpha$ in (3) to get

$$p(4\alpha)+p\left(\frac{\alpha}{4}\right)=\frac{5}{2}p(\alpha) \tag{13}$$

for all $\alpha\in\mathbb{R}^+$, which is true for $r=1$ in $A(r)$. Now, assume $A(r)$ holds for $r=n-1$ and $r=n$. That is, assume that

$$p\left(4^{n-1}\alpha\right)+p\left(\frac{\alpha}{4^{n-1}}\right)=\frac{4^{n-1}+1}{2^{n-1}}p(\alpha) \tag{14}$$

and

$$p\left(4^n\alpha\right)+p\left(\frac{\alpha}{4^n}\right)=\frac{4^n+1}{2^n}p(\alpha) \tag{15}$$

holds for all $\alpha\in\mathbb{R}^+$. Now, plugging 4α into α in (15), we obtain

$$p(4^{n+1}\alpha)+p\left(\frac{\alpha}{4^{n-1}}\right)=\frac{4^n+1}{2^n}p(\alpha) \tag{16}$$

for all $\alpha\in\mathbb{R}^+$. Besides, substituting $\alpha=\frac{\alpha}{4}$ in (15), we get

$$p(4^{n-1}\alpha)+p\left(\frac{\alpha}{4^{n+1}}\right)=\frac{4^n+1}{2^n}p\left(\frac{\alpha}{4}\right) \tag{17}$$

for all $\alpha\in\mathbb{R}^+$. Now, summing the equations (16) and (17) and then simplifying by using (13), (14) and (15), we arrive at

$$p\left(4^{n+1}\alpha\right)+p\left(\frac{\alpha}{4^{n+1}}\right)=\frac{4^{n+1}+1}{2^{n+1}}p(\alpha)$$

for all $\alpha\in\mathbb{R}^+$. Therefore, $A(r)$ is true for $r=n+1$. Hence, the proof is completed. $\square$

§4 Approximate solution of equation (3)

In the present section, we determine the existence of unique approximate solution of (3) using hyperstability concept. Just for the purpose of convenience, we define $Dp(\alpha,\beta):\mathbb{R}^+\times\mathbb{R}^+\longrightarrow\mathbb{R}$ via

$$Dp(\alpha,\beta)=p(\alpha+\beta+2\sqrt{\alpha\beta})+p\left(\frac{\alpha\beta}{\alpha+\beta+2\sqrt{\alpha\beta}}\right)-p(\alpha)-p(\beta)-\frac{p(\alpha)p(\beta)}{p(\alpha)+p(\beta)}$$

for all $\alpha,\beta\in\mathbb{R}^+$.

Theorem 4.1 *Let $\lambda,a>0$ be fixed real numbers. Assume that there exists a positive integer m_0 with the condition $m\alpha\in\mathbb{R}^+$ for $\alpha\in\mathbb{R}^+$, $m\in\mathbb{N}_{m_0}$. Assume a single real-valued function $p:\mathbb{R}^+\longrightarrow\mathbb{R}$ satisfies the following approximation*

$$|Dp(\alpha,\beta)|\leq\lambda\left(\alpha^a+\beta^a\right) \tag{18}$$

for all $\alpha, \beta \in \mathbb{R}^+$. Then a unique single real-valued reciprocal-radical function $P : \mathbb{R}^+ \longrightarrow \mathbb{R}$ exists and satisfies (3) *with*

$$|P(\alpha) - p(\alpha)| \leq \frac{2^{a+1}\lambda}{5 \cdot 2^{a-1} - 4^a - 1}\alpha^a \tag{19}$$

for all $\alpha \in \mathbb{R}^\star$.

Proof. We begin the proof with letting $\beta = \alpha$ in (18) and then multiplying with $\frac{2}{5}$ to obtain

$$\left|\frac{2}{5}p(4\alpha) + \frac{2}{5}p\left(\frac{\alpha}{4}\right) - p(\alpha)\right| \leq \frac{4\lambda}{5}\alpha^a \tag{20}$$

for all $\alpha \in \mathbb{R}^+$. Let us define

$$\mu g(\alpha) = \frac{2}{5}g(4\alpha) + \frac{2}{5}g\left(\frac{\alpha}{4}\right), \quad \alpha \in \mathbb{R}^+, \quad g \in \mathbb{R}^{\mathbb{R}^+}, \tag{21}$$

$$\Upsilon(\alpha) = \frac{4\lambda}{5}\alpha^a, \quad \alpha \in \mathbb{R}^+, \tag{22}$$

with g as a single real-valued function. Now, let $\alpha, \beta \in \mathbb{R}^+$. Suppose $\alpha = \beta$. Then $g(4\alpha) = g(4\beta)$ and $g\left(\frac{\alpha}{4}\right) = g\left(\frac{\beta}{4}\right)$ implies

$$\frac{2}{5}g(4\alpha) + \frac{2}{5}g\left(\frac{\alpha}{4}\right) = \frac{2}{5}g(4\beta) + \frac{2}{5}g\left(\frac{\beta}{4}\right)$$

which produces $\mu g(\alpha) = \mu g(\beta)$. Hence, the function μg is well-defined. Clearly, the function Υ is also well-defined. Using the notations defined in (21) and (22), the inequality (20) becomes

$$|\mu p(\alpha) - p(\alpha)| \leq \Upsilon(\alpha), \quad \alpha \in \mathbb{R}^+. \tag{23}$$

Let us define the following function as

$$\Lambda\sigma(\alpha) = \frac{2}{5}\sigma(4\alpha) + \frac{2}{5}\sigma\left(\frac{\alpha}{4}\right), \quad \sigma \in \mathbb{R}_+^{\mathbb{R}^+}, \quad \alpha \in \mathbb{R}^+. \tag{24}$$

The above function is in the form of (S3) with $k = 2$ and $p_1(\alpha) = 4\alpha$, $p_2(\alpha) = \frac{\alpha}{4}$ and $G_1(\alpha) = G_2(\alpha) = \frac{2}{5}$ for $\alpha \in \mathbb{R}^+$. Also, for each $g, h \in \mathbb{R}^{\mathbb{R}^+}$, $\alpha \in \mathbb{R}^+$,

$$\begin{aligned}
&|\mu g(\alpha) - \mu h(\alpha)| \\
&\quad = \left|\frac{2}{5}g(4\alpha) + \frac{2}{5}g\left(\frac{\alpha}{4}\right) - \frac{2}{5}h(4\alpha) - \frac{2}{5}h\left(\frac{\alpha}{4}\right)\right| \\
&\quad \leq \frac{2}{5}\left|(g-h)(4\alpha)\right| + \frac{2}{5}\left|(g-h)\left(\frac{\alpha}{4}\right)\right| \\
&\quad \leq \sum_{j=1}^{2} G_j(\alpha)\left|(g-h)p_j(\alpha)\right|.
\end{aligned} \tag{25}$$

Since $\frac{2}{5}\left(\frac{4^a+1}{2^a}\right) < 1$, we have

$$\begin{aligned}
\Upsilon^\star(\alpha) = \sum_{m=0}^{\infty}\Lambda^m\Upsilon(\alpha) &= \sum_{m=0}^{\infty}\frac{4\lambda}{5}\left(\frac{2}{5}\left(\frac{4^a+1}{2^a}\right)\right)^m\alpha^a \\
&= \frac{2^{a+1}\lambda}{5 \cdot 2^{a-1} - 4^a - 1}\alpha^a.
\end{aligned} \tag{26}$$

Using the result of Theorem 2.1, one can notice that a unique solution $P : \mathbb{R}^+ \longrightarrow \mathbb{R}$ of the equation (3) exists and it is defined through

$$P(\alpha) = \frac{2}{5}p(4\alpha) + \frac{2}{5}p\left(\frac{\alpha}{4}\right) \tag{27}$$

such that the inequality (19) holds. Furthermore, we can define

$$P(\alpha)=\lim_{m\to\infty}\mu^m p(\alpha). \tag{28}$$

For the purpose of showing that P satisfies (3), we find

$$|\mu^m Dp(\alpha,\beta)|\le\lambda\left(\frac{2}{5}\right)^m\left(\frac{4^a+1}{2^a}\right)^m(\alpha^a+\beta^a) \tag{29}$$

for all $\alpha,\beta\in\mathbb{R}^+$, and $m\in\mathbb{N}_0$. Suppose that $m=0$, then (29) turns out to be (18). So, by taking $m\in\mathbb{N}_0$ and supposing that (29) holds for m and $\alpha,\beta\in\mathbb{R}^+$, then

$$\begin{aligned}
\left|\mu^{m+1}Dp(\alpha,\beta)\right| &= \Big|\frac{2}{5}\mu^m p\left(4\alpha+4\beta+8\sqrt{\alpha\beta}\right)+\frac{2}{5}\mu^m p\left(\frac{\alpha}{4}+\frac{\beta}{4}+2\sqrt{\frac{\alpha}{4}\frac{\beta}{4}}\right)\\
&+\frac{2}{5}\mu^m p\left(\frac{16\alpha\beta}{4\alpha+4\beta+2\sqrt{16\alpha\beta}}\right)+\frac{2}{5}\mu^m p\left(\frac{\frac{\alpha\beta}{16}}{\frac{\alpha}{4}+\frac{\beta}{4}+2\sqrt{\frac{\alpha}{4}\frac{\beta}{4}}}\right)\\
&-\frac{2}{5}\mu^m\left(p(4\alpha)+p(4\beta)\right)-\frac{2}{5}\mu^m\left(p\left(\frac{\alpha}{4}\right)+p\left(\frac{\beta}{4}\right)\right)\\
&-\frac{2}{5}\mu^m\left(\frac{p(4\alpha)p(\beta)}{p(4\alpha)+p(4\beta)+2\sqrt{p(4\alpha)p(4\beta)}}\right)-\frac{2}{5}\mu^m\left(\frac{p(\frac{\alpha}{4})p(\frac{\beta}{4})}{p(\frac{\alpha}{4})+p(\frac{\beta}{4})+2\sqrt{p(\frac{\alpha}{4})p(\frac{\beta}{4})}}\right)\Big|\\
&\le\lambda\left(\frac{2}{5}\right)^m\left(\frac{4^a+1}{2^a}\right)^m\left[\frac{2}{5}(\alpha^a+\beta^a)+\frac{2}{5}\left(\left(\frac{\alpha}{4}\right)^a+\left(\frac{\beta}{4}\right)^a\right)\right]\\
&\le\lambda\left(\frac{2}{5}\right)^{m+1}\left(\frac{4^a+1}{2^a}\right)^{m+1}(\alpha^a+\beta^a).
\end{aligned} \tag{30}$$

By mathematical induction, the preceeding inequality (30) shows that (29) holds for all $\alpha,\beta\in\mathbb{R}^+$. We observe that P satisfies (3) when n approaches ∞ in (29), which completes the proof. □

The following example proves that equation (3) is not stable for a special case.

Example 4.2 *Let $B=(0,1]$ and let $p:B\longrightarrow\mathbb{R}$ be defined by $p(\alpha)=\sqrt{\alpha}$, $\alpha\in B$. Then for $\alpha,\beta\in B$ such that $4\alpha+4\beta+8\sqrt{\alpha\beta},\frac{\alpha}{4}+\frac{\beta}{4}+2\sqrt{\frac{\alpha}{4}\frac{\beta}{4}},16\alpha\beta,\frac{\alpha\beta}{16}\in B$, then*

$$|Dp(\alpha,\beta)|\le\alpha^a+\beta^a$$

with $a>0$, but p does not satisfy (3).

We obtain the hyperstability of equation (3) by considering product of different powers of norms. Since the proof of the following theorem is achieved through akin arguments as in Theorem 4.1, we present only the statement.

Theorem 4.3 *Let $\lambda>0$ be a real number. Let $a_1,a_2\in\mathbb{R}$ such that $a=a_1+a_2>0$. Suppose the function $p:\mathbb{R}^+\longrightarrow\mathbb{R}$ satisfies the following inequality*

$$|Dp(\alpha,\beta)|\le\lambda\left(\alpha^{a_1}\beta^{a_2}\right)$$

for all $\alpha,\beta\in\mathbb{R}^+$. Then a unique reciprocal-radical function $P:\mathbb{R}^+\longrightarrow\mathbb{R}$ exists and satisfies equation (3) with

$$|P(\alpha)-p(\alpha)|\le\frac{2^a\lambda}{5\cdot 2^{a-1}-4^a-1}\alpha^a$$

for all $\alpha\in\mathbb{R}^+$.

§5 Conclusion

Different types of functional equations have been taken into consideration up to this point in order to determine their hyperstability by using the direct and fixed point methods. This is our foremost venture to study a functional equation emerging through square root function and reciprocal-square root function. An inspiring aspect of equation (3) is that it has two different solutions $p(\alpha) = \sqrt{\alpha}$ and $p(\alpha) = \frac{1}{\sqrt{\alpha}}$. Hence, we call equation (3) as reciprocal-radical functional equation. From the investigation carried out in this study, we find that the hyperstability of equation (3) is valid in the setting of real numbers. An example is also presented to disprove the stability of equation (3)

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Declarations

Conflict of interest The authors declare no conflict of interest.

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