Appl. Math. J. Chinese Univ.
2025, 40(1): 33-52

Breathers and multi wave solutions of three different space-time fractional nonlinear coupled waves dynamical models and their applications

Ambreen Sarwar[1] GANG Tao[1,*]
Muhammad Arshad[2,3,*] Iftikhar Ahmed[3]
LU Dian-chen[3]

Abstract. In this article, several kinds of novel exact waves solutions of three well-known different space-time fractional nonlinear coupled waves dynamical models are constructed with the aid of simpler and effective improved auxiliary equation method. Firstly we will investigate space-time fractional coupled Boussinesq-Burger dynamical model, which is used to model the propagation of water waves in shallow sea and harbor, and has many applications in ocean engineering. Secondly, we will investigate the space-time fractional coupled Drinfeld-Sokolov-Wilson equation which is used to characterize the nonlinear surface gravity waves propagation over horizontal seabed. Thirdly, we will investigate the space-time-space fractional coupled Whitham-Broer-Kaup equation which is used to model the shallow water waves in a porous medium near a dam. We obtained different solutions in terms of trigonometric, hyperbolic, exponential and Jacobi elliptic functions. Furthermore, graphics are plotted to explain the different novel structures of obtained solutions such as multi solitons interaction, periodic soliton, bright and dark solitons, Kink and anti-Kink solitons, breather-type waves and so on, which have applications in ocean engineering, fluid mechanics and other related fields. We hope that our results obtained in this article will be useful to understand many novel physical phenomena in applied sciences and other related fields.

§1 Introduction

Waves play a vital role in the aquatic environment and more broadly influence the climate of earth. Shallow water waves are one significant example of major aquatic waves [1-4]. These waves are generated where the depth of water is shorter relative to the wave propagation. These waves are also used in modeling of coastal regions, tidal flats, hydrodynamics of lakes and also

Received: 2020-04-10. Revised: 2021-10-06.
MR Subject Classification: 35Dxx, 35Q51, 35A24, 35C07, 35C08.
Keywords: conformable fractional derivative, solitons, breather waves, jacobi elliptic function solutions.
Digital Object Identifier(DOI): https://doi.org/10.1007/s11766-025-4099-y.
*Corresponding author.

to study the physical phenomena such as tidal fluctuation, storm surges and waves of tsunamis [5-8]. So due to their great importance shallow water waves dynamics has become an essential area of research [9-13]. With the achievement in the field of nonlinear science, a large number of mathematical models are constructed and evaluated for comprehensive analysis of dynamical behaviour of shallow water waves [14-18].

The coupled Drinfeld-Sokolov-Wilson (DSW) dynamical model is introduced as a water wave model [3,19]. The authors in [20] investigate waves solutions in solitary form and their structure. It was first introduced by Drinfeld and Sokolov, afterwards developed and modified by Wilson in 1982. The time-space fractional DSW equation is used to characterize the nonlinear surface gravity waves propagation over the horizontal seabed. Because of its imperative and vital applications in hydrodynamical and physical science, a better understanding of its solutions is also useful for interpretations behaviors of shallow water waves model in ocean engineering [21].

The authors in [22-24] studied the coupled Whitham-Broer-Kaup (cWBK) dynamical model of long dispersive waves in shallow water and obtained it via utilizing Boussinesq approximation. The parameters in this model represent different diffusion powers. The space-time fractional cWBK equation is an important model which elaborates the propagation of shallow water waves in a porous medium near a dam, it is also used to absorb wave energy and predict and prevent tsunami. Due to its great significant applications, a better insight of its solutions is very useful and beneficial for both engineers and physicists [25].

The time-space fractional Coupled-Boussinesq-Berger equation is a well-known model which illustrates the generation and propagation of shallow water waves in a hydrodynamic system. Due to the great significance of this shallow water waves equation model in ocean engineering, a good insight of its solutions is useful and beneficial for coastal and civil engineers for implementation of this shallow water wave model to harbor and coastal designs [26]. It is very helpful for engineers and coastal engineers to pay attention to nonlinear water wave models in different fields of science and engineering. Therefore, it is a basic curiosity of hydrodynamics to find different types of traveling wave solutions of coupled systems.

The fractional calculus has attracted many researchers in the last and present centuries. The impact of this fractional calculus in both pure and applied branches of science and engineering [27-30]. A remarkable improvement has done in the field of nonlinear fractional differential equations [31-34]. A large number of effective and convenient methods have been developed and improved for overall exact and numerical solutions of nonlinear fractional differential equations [35-39].

These nonlinear fractional differential equations are frequently used for precise modeling of nonlinear phenomena occurring in the execution of shallow water waves [40-43]. The comprehensive study of the literature revealed that these equations exhibit their progressive and beneficial use in the mathematical modeling of shallow water waves [44-46]. There are a huge number of models of shallow water waves equations that are represented with the help of fractional calculus [47-49]. So it become a curious topic for researchers. That's why the new definition of fractional derivatives called conformable fractional derivatives was proposed for a better explanation of nonlinear phenomena [50]. In this way, a new direction in fractional calculus was opened, which has shown to be interesting from a theoretical viewpoint and useful in the applications.

The conformable fractional derivative has two advantages over the classical fractional derivatives. First, the conformable fractional derivative definition is natural and it satisfies most of the properties which the classical integral derivative has such as linearity, product rule, quotient rule, power rule, chain rule, vanishing derivatives for constant functions, Rolles theorem, and mean value theorem. Second, the conformable derivative brings us a lot of conveniences when it is applied to modelling many physical problems, because the differential equations with conformable fractional derivatives are easier to solve numerically than those associated with the Riemann-Liouville or Caputo fractional derivative [36-40]. In fact, many researchers have already applied conformable fractional derivative to many fields and a lot of corresponding techniques were developed [46-50].

The aim of the article is to plan an improved auxiliary equation method to obtain of three well-known different space-time fractional nonlinear coupled waves dynamical models are constructed with the aid of conformable fractional derivatives. Furthermore graphics are plotted to explain the different novel structures of obtained solutions such as multi solitons interaction, periodic soliton, bright and dark solitons, Kink and anti-Kink solitons, breather type waves and so on, which have applications in ocean engineering, fluid mechanics and other related fields.

The organization of this article is as follows: in Section 2, the conformable fractional derivatives are described. In Section 3, the proposed improved auxiliary equation method is described and its application is given in Section 4. The results are discussed in Section 5, and the conclusion is given in Section 6.

§2 Conformable fractional derivative

Innovative progress in fractional calculus opens a new porthole for researchers to elucidate the physical phenomena in a new way. Lately, the authors [27-29,50] introduced a new simple and thought provoking definition of fractional derivative (FD), named conformable FD. This derivative is performed systematically and follows the Leibniz and chain rules as well. Here, we give preliminaries of the new derivative with some effective and advantageous properties [47,51].

Definition: The conformable fractional derivative of a function $h = h(t) : [0, \infty) \to \mathbb{R}$, of order δ, where $0 < \delta \leqslant 1$, is defined as

$$D_t^\eta h(t) = \lim_{\eta \to 0} \frac{h\left(\delta t^{1-\eta} + t\right) - h(t)}{\delta}.$$

The above conformable fractional derivative satisfied the following properties:

- $D_t^\eta h(t) = \frac{t^{1-\eta}(dh(t))}{dt}$.
- $D_t^\eta t^m = m t^{n-\eta}, \forall m \in \mathbb{R}$.
- $D_t^\eta c = 0, \forall$ constant functions $h(t) = c$.
- $D_t^\eta (p_1 * h(t) + p_2 * h_1(t)) = p_1 * D_t^\eta h(t) + p_2 * D_t^\eta h_1(t),\ \forall p_1, p_2 \in \mathbb{R}$.
- $D_t^\eta (h(t) h_1(t)) = h(t) D_t^\eta h_1(t) + h_1(t) D_t^\eta h(t)$.

- $\frac{D_t^\eta h(t)}{h_1(t)} = \frac{h_1(t)D_t^\eta h(t) - h(t)D_t^\eta h_1(t)}{h_1(t)^2}$.
- $D_t^\eta(h \circ h_1)(t) = t^{1-\eta}h_1'(t)h'(h_1(t))$.

§3 Proposed Improved auxiliary equation method

This section describes the algorithm of the improved extended auxiliary equation technique for nonlinear FPDEs. Consider a general nonlinear FPDE having conformable space-time fractional derivatives in the following form

$$R\left(U, U_x, D_x^a U, U_t, D_t^b U, U_{tx}, D_x^a(D_t^b U), ...\right) = 0, \quad 0 < a, b \le 1. \tag{1}$$

above R is a polynomial function of $U(x,t)$ corresponding to partial derivatives, nonlinear terms and highest order derivatives. The following stages of this technique are as

Step 1: Assume the fractional traveling wave transformation as

$$U(x,t) = u(\eta), \quad \eta = \kappa\frac{x^a}{a} - \lambda\frac{t^b}{b}. \tag{2}$$

where λ and κ are constants. By utilizing the transformation (2) on Eq.(1), The Eq.(1) reduces into ODE as

$$S(u', u'', u''', ...) = 0, \tag{3}$$

where S is the polynomial function of $u(\eta)$ and its derivatives.

Step 2: Assume the solution of Eq.(3) as

$$u(\eta) = \sum_{i=0}^{2N} c_i F^i(\eta), \tag{4}$$

where $a_i (i = 0, 1, 2, \dots, 2N)$ are arbitrary constants and N is a positive integer which is determined by applying the balance principle on Eq.(3). Let $F(\eta)$ satisfy the novel ansatz ODE

$$F'(\eta) = \sqrt{a_0 + a_2F(\eta)^2 + a_4F(\eta)^4 + a_6F(\eta)^6}, \tag{5}$$

where a_0, a_2, a_4, a_6 are arbitrary constants and Eq.(5) has the following solutions as

$$F(\eta) = \frac{1}{2}\sqrt{-\frac{a_4}{a_6}(1 \pm z(\eta))}. \tag{6}$$

It should be pointed out that $z(\eta)$, can be expressed in terms of Jacobi elliptic functions (JEFs) $sn(\eta, m)$, $cn(\eta, m)$, $dn(\eta, m)$, their inverse ratio and so on, m is modulus of JEFs, and its value is $0 < m < 1$. When it realizes the value of 0 or 1, the Jacobian elliptic function is transformed into a trigonometric function and hyperbolic function.

Step 3: Deputing Eqs. (4) and (5) into (3), collect and set the coefficient of $F^i(\eta)$ equal to zero, the system of equations in parameters a_0, a_2, a_4, a_6, c_i, ω and λ are attained. This system will solved by Mathematica software for the parameters values.

Step 4: By deputy the parameters values along with solutions of Eq.(6) into Eq.(4), we get the solutions of Eq.(1).

§4 Application of the described method

In this section, we use the improved auxiliary equation method to construct the solitary wave solutions of the space-time fractional wave equations.

4.1 Space-Time fractional coupled Boussinesq-Burger equation

The time-space fractional coupled Boussinesq-Burger equation, with a conformable fractional derivative can be written as

$$D_t^b U - \frac{1}{2} D_x^a V + 2U D_x^a U = 0,$$
$$D_t^b V - \frac{1}{2} D_x^{3a} U + 2D_x^a (UV) = 0, \quad 0 < a, b \leq 1. \tag{7}$$

Assume the traveling waves solutions of the above equations as

$$U(x,t) = u(\eta) = \sum_{i=0}^{2n} c_i F^i(\eta),$$
$$V(x,t) = v(\eta) = \sum_{j=0}^{2n'} c_j F^j(\eta), \quad \eta = \kappa \frac{x^a}{a} - \lambda \frac{t^b}{b}. \tag{8}$$

Utilizing fractional traveling wave transformation (8) on Eq.(7), it reduces into ODE as

$$-\lambda u' - \frac{1}{2}\kappa v' + 2\kappa u u' = 0, \tag{9}$$
$$-\lambda v' - \frac{1}{2}\kappa^3 u^{(3)} + 2\kappa (uv)' = 0. \tag{10}$$

Integrating Eq.(9) with respect to η, by taking an integration constant equal to zero and solving for the value of v, it yields

$$v = 2\left(u^2 - \frac{\lambda}{\kappa} u\right). \tag{11}$$

Substituting the Eq.(11) into Eq.(10), solving and integrating, it yields

$$\kappa^4 u'' - 8\kappa^2 u^3 + 12\kappa\lambda u^2 - 4\lambda^2 u = 0. \tag{12}$$

Utilizing the balancing principal on Eq.(12), got $N = 1$, and assume the solution as,

$$U(\eta) = c_0 + c_1 F(\eta) + c_2 F(\eta)^2. \tag{13}$$

Substituting Eq.(13) along Eq.(5) into Eq.(12), it yields a system of algebraic in parameters $a_0, a_2, a_4, a_6, c_0, c_1, c_2$, κ and λ. These algebraic equations are resolved for the parameters with the help of Mathematica, we have

$$c_0 = -\frac{\sqrt[4]{3\sqrt{a_2^2 - 2a_0a_4} - a_2}\sqrt{\lambda} + \sqrt{\lambda\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right)\sqrt{\frac{1}{3\sqrt{a_2^2-2a_0a_4}-a_2}}}}{2\sqrt[4]{2}}, c_1 = 0,$$
$$c_2 = \frac{\left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right)\sqrt{\frac{\lambda^2}{3\sqrt{a_2^2-2a_0a_4}-a_2}}}}{2\sqrt[4]{2}a_0},$$
$$a_6 = \frac{a_4\left(a_2 - \sqrt{a_2^2 - 2a_0a_4}\right)}{4a_0},$$
$$\kappa = -\frac{\sqrt[4]{2}\sqrt{\lambda}}{\sqrt[4]{3\sqrt{a_2^2 - 2a_0a_4} - a_2}}. \tag{14}$$

Substituting Eq.(14) into Eq.(6), it yields

$$F(\eta)=\sqrt{-\frac{a_0(1\pm z(\eta))}{a_2-\sqrt{a_2^2-2a_0a_4}}}. \tag{15}$$

Substituting the values $z(\eta)$ in terms of Jacobi elliptic functions and putting in Eq.(13), it yields the solutions as

Case 1: If $a_0=\frac{a_4^3(m^2-1)}{32a_6^2m^2}$, $a_2=\frac{a_4^2(5m^2-1)}{16a_6m^2}$, $a_6>0$ then

$$U_{11}(\eta)=\frac{\sqrt{\lambda}\left(-\sqrt[4]{3\sqrt{a_2^2-2a_0a_4}-a_2}\pm\sqrt{\frac{a_2+\sqrt{a_2^2-2a_0a_4}}{\sqrt{3\sqrt{a_2^2-2a_0a_4}-a_2}}\mathrm{sn}(\tau\eta)}\right)}{2\sqrt[4]{2}}, \tag{16}$$

$$V_{11}(\eta)=2\left(U_{11}^2-\frac{\lambda U_{11}}{\kappa}\right). \tag{17}$$

or

$$U_{12}(\eta)=\frac{\sqrt{\lambda}\left(-\sqrt[4]{3\sqrt{a_2^2-2a_0a_4}-a_2}\pm\frac{\sqrt{\frac{a_2+\sqrt{a_2^2-2a_0a_4}}{\sqrt{3\sqrt{a_2^2-2a_0a_4}-a_2}}}}{m\mathrm{sn}(\tau\eta)}\right)}{2\sqrt[4]{2}}, \tag{18}$$

$$V_{12}(\eta)=2\left(U_{12}^2-\frac{\lambda U_{12}}{\kappa}\right). \tag{19}$$

As $m\to 1$, Eq.(18) and Eq.(19) generate the following solutions,

$$U_{13}(\eta)=\frac{\sqrt{\lambda}\left(-\sqrt[4]{3\sqrt{a_2^2-2a_0a_4}-a_2}\pm\sqrt{\frac{a_2+\sqrt{a_2^2-2a_0a_4}}{\sqrt{3\sqrt{a_2^2-2a_0a_4}-a_2}}}\coth(\tau\eta)\right)}{2\sqrt[4]{2}}, \tag{20}$$

$$V_{13}(\eta)=2\left(U_{13}^2-\frac{\lambda U_{13}}{\kappa}\right). \tag{21}$$

Case 2: If $a_0=\frac{a_4^3(1-m^2)}{32a_6^2}$, $a_2=\frac{a_4^2(5-m^2)}{16a_6}$ and $a_6>0$, then

$$U_{21}(\eta)=\frac{\sqrt{\lambda}\left(-\sqrt[4]{3\sqrt{a_2^2-2a_0a_4}-a_2}\pm\sqrt{\frac{a_2+\sqrt{a_2^2-2a_0a_4}}{\sqrt{3\sqrt{a_2^2-2a_0a_4}-a_2}}m\mathrm{sn}(\tau\eta)}\right)}{2\sqrt[4]{2}}, \tag{22}$$

$$V_{21}(\eta)=2\left(U_{21}^2-\frac{\lambda U_{21}}{\kappa}\right). \tag{23}$$

or

$$U_{22}(\eta)=\frac{\sqrt{\lambda}\left(-\sqrt[4]{3\sqrt{a_2^2-2a_0a_4}-a_2}\pm\frac{\sqrt{\frac{a_2+\sqrt{a_2^2-2a_0a_4}}{\sqrt{3\sqrt{a_2^2-2a_0a_4}-a_2}}}}{\mathrm{sn}(\tau\eta)}\right)}{2\sqrt[4]{2}}, \tag{24}$$

$$V_{22}(\eta)=2\left(U_{22}^2-\frac{\lambda U_{22}}{\kappa}\right). \tag{25}$$

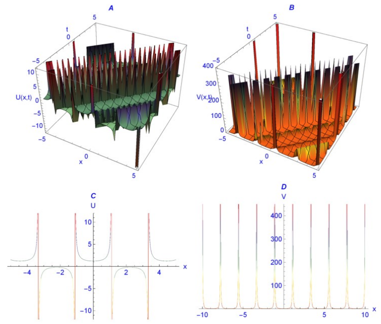

Figure 1. The graph of solutions (35) and (36) are depicted at different values of parameters and we obtained: (A) Multi solitons interaction, (B) periodic ten solitons, (C) 2D plot of U and (D) 2D plot of V.

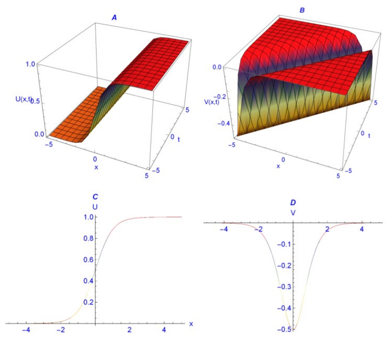

Figure 2. The graph of solutions (38) and (39) are depicted at different values of parameters and we obtained: (A) Kink-type wave, (B) dark soliton, (C) 2D plot of U and (D) 2D plot of V.

As $m \to 1$, solutions (22) and (23) generate the following solutions,

$$U_{23}(\eta) = \frac{\sqrt{\lambda}\left(-\sqrt[4]{3\sqrt{a_2^2-2a_0a_4}-a_2} \pm \sqrt{\frac{a_2+\sqrt{a_2^2-2a_0a_4}}{\sqrt{3\sqrt{a_2^2-2a_0a_4}-a_2}}}\tanh(\tau\eta)\right)}{2\sqrt[4]{2}}, \tag{26}$$

$$V_{23}(\eta) = 2\left(U_{23}^2 - \frac{\lambda U_{23}}{\kappa}\right). \tag{27}$$

Case 3: If $a_0 = \frac{a_4^3}{32a_6^2(1-m^2)}$, $a_2 = \frac{a_4^2(4m^2-5)}{16a_6(m^2-1)}$, $a_6 > 0$,

$$U_{31}(\eta) = \frac{\sqrt{\lambda}\left(-\sqrt[4]{3\sqrt{a_2^2-2a_0a_4}-a_2} \pm \frac{\sqrt{\frac{a_2+\sqrt{a_2^2-2a_0a_4}}{\sqrt{3\sqrt{a_2^2-2a_0a_4}-a_2}}}}{\mathrm{cn}(\tau\eta)}\right)}{2\sqrt[4]{2}}, \tag{28}$$

$$V_{31}(\eta) = 2\left(U_{31}^2 - \frac{\lambda U_{31}}{\kappa}\right). \tag{29}$$

or

$$U_{32}(\eta) = \frac{\sqrt{\lambda}\left(-\sqrt[4]{3\sqrt{a_2^2-2a_0a_4}-a_2} \pm \sqrt{\frac{fraca_2+\sqrt{a_2^2-2a_0a_4}\sqrt{3\sqrt{a_2^2-2a_0a_4}-a_2}\,\mathrm{dn}(\tau\eta)}{\sqrt{1-m^2}\,\mathrm{sn}(\tau\eta)}}\right)}{2\sqrt[4]{2}}, \tag{30}$$

$$V_{32}(\eta) = 2\left(U_{32}^2 - \frac{\lambda U_{32}}{\kappa}\right). \tag{31}$$

As $m \to 0$,Eq.(32) and Eq.(33) generate the following solutions,

$$U_{33}(\eta) = \frac{\sqrt{\lambda}\left(-\sqrt[4]{3\sqrt{a_2^2-2a_0a_4}-a_2} \pm \sqrt{\frac{a_2+\sqrt{a_2^2-2a_0a_4}}{\sqrt{3\sqrt{a_2^2-2a_0a_4}-a_2}}}\,\mathrm{csc}(\tau\eta)\right)}{2\sqrt[4]{2}}, \tag{32}$$

$$V_{33}(\eta) = 2\left(U_{33}^2 - \frac{\lambda U_{33}}{\kappa}\right). \tag{33}$$

Special solutions: It should be noted that the Eq.(5) has some special solutions other than Jacobi elliptic function solutions, which are as follows:

Case IV: If $a_0 = 0, a_6 = 0$ and $a_2 < 0, a_4 > 0$ then, applying the conditions on parametric equations system, we obtained the values of parameters as

$$\kappa = \frac{\lambda}{2c_0}, a_2 = -\frac{32c_0^4}{\lambda^2}, a_4 = \frac{16c_0^2c_1^2}{\lambda^2}, c_2 = 0. \tag{34}$$

The following solution of equation (7) from solution set (34) is obtained as

$$U_{41}(\eta) = \frac{\lambda}{\sqrt{2}\kappa}\left(\frac{1}{\sqrt{2}} + \sec\left(\frac{\sqrt{2}\lambda\eta}{\kappa^2}\right)\right), \tag{35}$$

$$V_{41}(\eta) = 2\left(U_{41}^2 - \frac{\lambda U_{41}}{\kappa}\right). \tag{36}$$

Case V: If $a_0 = \frac{a_2^2}{4a_4}, a_6 = 0$ and $a_2 < 0, a_4 > 0$ then, applying the conditions on the parametric equations system, we obtained the values of parameters as

$$\lambda = 2c_0\kappa, a_2 = -\frac{8c_0^2}{\kappa^2}, a_4 = \frac{4c_1^2}{\kappa^2}, c_2 = 0. \tag{37}$$

The following solution of equation (7) from the solution set (37) is obtained as

$$U_{51}(\eta) = \frac{\lambda}{2\kappa}\left(1 \pm \tanh\left(\frac{\lambda\eta}{\kappa^2}\right)\right), \tag{38}$$

$$V_{51}(\eta) = 2\left(U_{51}^2 - \frac{\lambda U_{51}}{\kappa}\right). \tag{39}$$

4.2 Space-Time fractional coupled Drinfeld-Sokolov-Wilson equation

The space-time fractional coupled DSW equation is another distinguished nonlinear shallow water wave model that describes the significant characteristics of shallow water waves. The space-time fractional Drinfeld-Sokolov-Wilson equation with conformable fractional derivative can be written as

$$
\begin{aligned}
&D_t^b U + \delta V D_x^a V = 0,\\
&D_t^b V + \alpha D_x^{3a} V + \beta U D_x^a V + \gamma V D_x^a U = 0, \quad 0 < a, b \le 1.
\end{aligned} \tag{40}
$$

Assume the traveling waves solutions of the above equations as

$$
\begin{aligned}
&U(x,t) = U(\eta) = \sum_{i=0}^{2n_1} c_i F^i(\eta),\\
&V(x,t) = V(\eta) = \sum_{j=0}^{2n_1'} c_j F^j(\eta), \quad \eta = \kappa \frac{x^a}{a} - \lambda \frac{t^b}{b}.
\end{aligned} \tag{41}
$$

Applying fractional traveling wave transformation (41) on Eq.(40), it reduces to ODE as

$$-\lambda U' + \delta\kappa V V' = 0, \tag{42}$$

$$-\lambda V' + \alpha\kappa^3 V^{(3)} + \beta\omega U V' + \gamma\omega V U' = 0. \tag{43}$$

Integrating Eq.(42) with respect to η, by taking an integration constant equal to zero and solving for the value of U, it yields

$$U = \frac{\delta\kappa}{2\lambda} V^2. \tag{44}$$

Substituting the Eq.(44) into Eq.(43), solving and integrating, it yields

$$2\alpha\lambda\kappa^3 V'' - 2\lambda^2 V + \frac{\delta\kappa^2}{3}(\beta + 2\gamma)V^3 = 0. \tag{45}$$

Utilizing the balancing principal on Eq.(45), got $n_1 = 1$, and assume the solution in the form as,

$$V(\eta) = c_0 + c_1 F(\eta) + c_2 F(\eta)^2. \tag{46}$$

Substituting Eq.(46) along Eq.(5) into Eq.(45), it yields a system of algebraic equations in parameters $a_0, a_2, a_4, a_6, c_0, c_1, c_2, \alpha, \beta, \gamma, \delta, \kappa$ and λ. This algebraic system of equations is resolved with the help of Mathematica, we have

$$
\begin{aligned}
&\alpha = \frac{\lambda}{\left(a_2 + 3\sqrt{a_2^2 - 2a_0a_4}\right)\kappa^3},\\
&\beta = -\frac{2\left(3\sqrt{a_2^2 - 2a_0a_4}\left(\gamma c_0^2\delta\kappa^2 - \lambda^2\right) + a_2\left(\gamma c_0^2\delta\kappa^2 + 3\lambda^2\right)\right)}{c_0^2\kappa^2\left(a_2\delta + 3\sqrt{a_2^2 - 2a_0a_4}\,\delta\right)},\\
&c_1 = 0, c_2 = \frac{a_2c_0 - \sqrt{\left(a_2^2 - 2a_0a_4\right)c_0^2}}{a_0},\\
&a_6 = \frac{a_4\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right)}{4a_0}.
\end{aligned} \tag{47}
$$

Substituting Eq.(47) into Eq.(6),it yields

$$F(\eta) = \sqrt{-\frac{a_0(1 \pm z(\eta))}{a_2 + \sqrt{a_2^2 - 2a_0a_4}}}. \tag{48}$$

By deputing substituting the values $z(\eta)$ in terms of Jacobi elliptic functions and putting in Eq.(46), it yields the following results as

Case 1: If $a_0 = \frac{a_4^3\left(m^2-1\right)}{32a_6^2m^2}$, $a_2 = \frac{a_4^2\left(5m^2-1\right)}{16a_6m^2}$, $a_6 > 0$ then

$$V_{11}(\eta) = \left(1 - \frac{\left(a_2 - \sqrt{a_2^2 - 2a_0a_4}\right)(1 \pm \mathrm{sn}(\tau\eta))}{a_2 + \sqrt{a_2^2 - 2a_0a_4}}\right) c_0, \tag{49}$$

$$U_{11}(\eta) = \frac{\delta\kappa}{2\lambda} V_{11}^2. \tag{50}$$

or

$$V_{12}(\eta) = \left(1 - \frac{\left(a_2 - \sqrt{a_2^2 - 2a_0a_4}\right)\left(1 \pm \frac{1}{m\mathrm{sn}(\tau\eta)}\right)}{a_2 + \sqrt{a_2^2 - 2a_0a_4}}\right) c_0, \tag{51}$$

$$U_{12}(\eta) = \frac{\delta\kappa}{2\lambda} V_{12}^2. \tag{52}$$

As $m \to 1$, Eq.(51) and Eq.(52) generate the following solutions,

$$V_{13}(\eta) = \left(1 - \frac{\left(a_2 - \sqrt{a_2^2 - 2a_0a_4}\right)(1 \pm \coth(\tau\eta))}{a_2 + \sqrt{a_2^2 - 2a_0a_4}}\right) c_0, \tag{53}$$

$$U_{13}(\eta) = \frac{\delta\kappa}{2\lambda} V_{13}^2. \tag{54}$$

Case 2: If $a_0 = \frac{a_4^3\left(1-m^2\right)}{32a_6^2}$, $a_2 = \frac{a_4^2\left(5-m^2\right)}{16a_6}$ and $a_6 > 0$, then

$$V_{21}(\eta) = \left(1 - \frac{\left(a_2 - \sqrt{a_2^2 - 2a_0a_4}\right)(1 \pm m\mathrm{sn}(\tau\eta))}{a_2 + \sqrt{a_2^2 - 2a_0a_4}}\right) c_0, \tag{55}$$

$$U_{21}(\eta) = \frac{\delta\kappa}{2\lambda} V_{21}^2. \tag{56}$$

or

$$V_{22}(\eta) = \left(1 - \frac{\left(a_2 - \sqrt{a_2^2 - 2a_0a_4}\right)\left(1 \pm \frac{1}{\mathrm{sn}(\tau\eta)}\right)}{a_2 + \sqrt{a_2^2 - 2a_0a_4}}\right) c_0, \tag{57}$$

$$U_{22}(\eta) = \frac{\delta\kappa}{2\lambda} V_{22}^2. \tag{58}$$

As $m \to 1$, solutions (55) and (56) generate the following solutions

$$V_{23}(\eta) = \left(1 - \frac{\left(a_2 - \sqrt{a_2^2 - 2a_0a_4}\right)(1 \pm \tanh(\tau\eta))}{a_2 + \sqrt{a_2^2 - 2a_0a_4}}\right) c_0, \tag{59}$$

$$U_{23}(\eta) = \frac{\delta\kappa}{2\lambda} V_{23}^2. \tag{60}$$

Case 3: If $a_0 = \frac{a_4^3}{32a_6^2(1-m^2)}$, $a_2 = \frac{a_4^2\left(4m^2-5\right)}{16a_6(m^2-1)}$, $a_6 > 0$,

$$V_{31}(\eta) = \left(1 - \frac{\left(a_2 - \sqrt{a_2^2 - 2a_0a_4}\right)\left(1 \pm \frac{1}{\mathrm{cn}(\tau\eta)}\right)}{a_2 + \sqrt{a_2^2 - 2a_0a_4}}\right) c_0, \tag{61}$$

$$U_{31}(\eta) = \frac{\delta\kappa}{2\lambda} V_{31}^2. \tag{62}$$

or

$$V_{32}(\eta) = \left(1 - \frac{\left(a_2 - \sqrt{a_2^2 - 2a_0a_4}\right)\left(1 \pm \frac{\mathrm{dn}(\tau\eta)}{\sqrt{1-m^2}\mathrm{sn}(\tau\eta)}\right)}{a_2 + \sqrt{a_2^2 - 2a_0a_4}}\right) c_0, \tag{63}$$

$$U_{32}(\eta) = \frac{\delta\kappa}{2\lambda} V_{32}^2. \tag{64}$$

As $m \to 0$, solutions (63) and (64) generate the following solutions

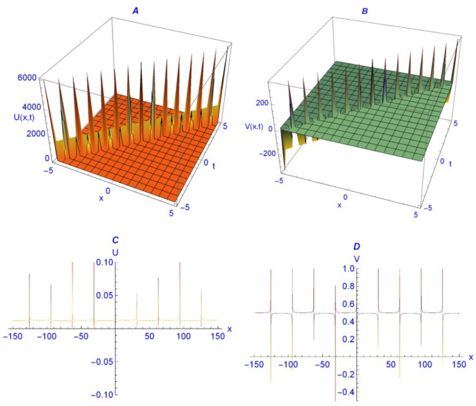

Figure 3. The graph of solutions (66) and (65) are depicted evaluate at different values of parameters: (A) Multi-peak solitons of higher amplitude 6000 units, (B) Multi peak solitons of shorter amplitude of 200 units, (C) 2D plot of U and (D) 2D plot of V.

$$V_{33}(\eta) = (1 - \frac{\left(a_2 - \sqrt{a_2^2 - 2a_0a_4}\right)(1 \pm \csc(\tau\eta))}{a_2 + \sqrt{a_2^2 - 2a_0a_4}})c_0, \tag{65}$$

$$U_{33}(\eta) = \frac{\delta\kappa}{2\lambda} V_{33}^2. \tag{66}$$

Special solutions: It should be noted that the Eq.(5) has some special solutions other than Jacob elliptic function solutions, which are as:

Case IV: If $a_0 = 0, a_6 = 0$ and $a_2 > 0, a_4 < 0$ then, applying the conditions on the parametric equations system, we obtained the values of parameters as

$$a_2 = \frac{\lambda}{\alpha\kappa^3}, a_4 = -\frac{c_1^2\delta(\beta + 2\gamma)}{12\alpha\kappa\lambda}, c_0 = 0, c_2 = 0. \tag{67}$$

The following solution of equation (40) from solution set (67) is obtained as

$$V_{41}(\eta) = \frac{2\lambda}{\kappa}\sqrt{\frac{3}{\delta(\beta + 2\gamma)}}\mathrm{sech}\left(\sqrt{\frac{\lambda}{\alpha\kappa^3}}\eta\right), \tag{68}$$

$$U_{41}(\eta) = \frac{V_{41}^2(\delta\kappa)}{2\lambda}. \tag{69}$$

Case V: If $a_0 = \frac{a_2^2}{4a_4}, a_6 = 0$ and $a_2 < 0, a_4 > 0$ then, applying the conditions on parametric equations system, we obtained the values of parameters as

$$\alpha = -\frac{c_1^2\delta(\beta + 2\gamma)}{12a_4\kappa\lambda}, a_2 = -\frac{12a_4\lambda^2}{c_1^2\delta\kappa^2(\beta + 2\gamma)}, c_0 = 0, c_2 = 0. \tag{70}$$

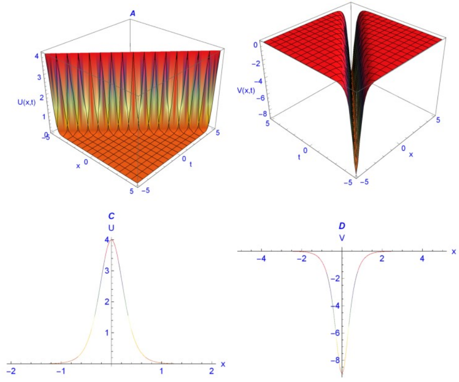

Figure 4. The The graph of wave solutions (69) and (68) are depicted at different values of parameters: (A) bright soliton of solution U, (B) dark soliton of solution V, (C) 2D plot of U and (D) 2D plot of V.

The following solution of equation (40) from the solution set (70) is obtained as

$$V_{51}(\eta) = \pm\frac{\lambda}{\kappa}\sqrt{\frac{6}{\delta(\beta+2\gamma)}}\tanh\left(\frac{\lambda\sqrt{\frac{6a_4}{\delta(\beta+2\gamma)}}}{\kappa}c_1\eta\right), \tag{71}$$

$$U_{51}(\eta) = \frac{V_{51}^2(\delta\kappa)}{2\lambda}. \tag{72}$$

4.3 Space-Time fractional coupled Whitham-Broer-Kaup equation

The space-time fractional Whitham-Broer-Kaup equation, with conformable fractional derivative can be written as,

$$\begin{aligned} &D_t^b U + U D_x^a U + D_x^a V + \epsilon D_x^{2a} U = 0,\\ &D_t^b V + D_x^a(UV) - \epsilon D_x^{2a} V + \rho D_x^{3a} U = 0, \quad 0 < a, b \le 1. \end{aligned} \tag{73}$$

Assume the traveling waves solutions of the above equations as

$$\begin{aligned} U(x,t) &= U(\eta) = \sum_{i=0}^{2n_2} c_i F^i(\eta),\\ V(x,t) &= V(\eta) = \sum_{j=0}^{2n_2'} c_j F^j(\eta), \quad \eta = \kappa\frac{x^a}{a} - \lambda\frac{t^b}{b}. \end{aligned} \tag{74}$$

Applying fractional traveling wave transformation (74) on Eq.(73), it reduces to ODE as

$$-\lambda U' + \kappa U U' + \kappa V' + \kappa^2 \epsilon U'' = 0, \tag{75}$$

$$-\lambda V' + \kappa(UV)' - \kappa^2 \epsilon V'' + \rho\kappa^3 U''' = 0. \tag{76}$$

Integrating Eq.(75) with respect to η, by taking an integration constant equal to zero and solving for value of U,it yields

$$V = \frac{\lambda U}{\kappa} - \frac{U^2}{2} - \kappa\epsilon U'. \tag{77}$$

Substituting the Eq.(77) into Eq.(76),solving and integrating ,it yields

$$(\rho+\epsilon^2)\kappa^3 U'' - \frac{\lambda^2 U}{\kappa} + \frac{3}{2}\lambda U^2 - \frac{\kappa \mathrm{U}^3}{2} = 0. \tag{78}$$

Utilizing the balancing principal on Eq.(78), got $n_2 = 1$, and assume the solution as

$$U(\eta) = c_0 + c_1 F(\eta) + c_2 F(\eta)^2. \tag{79}$$

Substituting Eq.(79) along Eq.(5) into Eq.(78), it yields a system of algebraic equations in parameters $a_0, a_2, a_4, a_6,\ c_0, c_1, c_2, \epsilon, \rho, \kappa$ and λ . This algebraic system of equations is resolved with the help of Mathematica, we obtained the parameters values as

$$\begin{aligned}
\rho &= \frac{1}{4}\left(\frac{\left(a_2 + 3\sqrt{a_2^2 - 2a_0a_4}\right)\lambda^2}{\left(4a_2^2 - 9a_0a_4\right)\kappa^4} - 4\epsilon^2\right),\\
c_0 &= \frac{\lambda}{\kappa}\left(1 - \frac{a_2 + \sqrt{a_2^2 - 2a_0a_4}}{\sqrt{2\left(\sqrt{a_2^2 - 2a_0a_4}\,a_2 + (a_2^2 - 3a_0a_4)\right)}}\right),\\
c_1 &= 0, c_2 = \frac{\sqrt{2}a_4\lambda}{\sqrt{\sqrt{a_2^2 - 2a_0a_4}\,a_2 + (a_2^2 - 3a_0a_4)\kappa}},\\
a_6 &= \frac{a_4\left(a_2 - \sqrt{a_2^2 - 2a_0a_4}\right)}{4a_0}.
\end{aligned} \tag{80}$$

Substituting Eq.(80) into Eq.(6),it yields

$$F(\eta) = \sqrt{-\frac{a_0(1 \pm z(\eta))}{a_2 - \sqrt{a_2^2 - 2a_0a_4}}}. \tag{81}$$

Deputing the values $z(\eta)$ in terms of JEFs, it yields the following solutions as

Case 1: If $a_0 = \frac{a_4^3(m^2-1)}{32a_6^2m^2}$, $a_2 = \frac{a_4^2(5m^2-1)}{16a_6m^2}$, $a_6 > 0$ then

$$U_{11}(\eta) = \frac{\lambda\left(\sqrt{2}a_0a_4(2 \pm \mathrm{sn}(\tau\eta)) + \left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\right)}{\left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4\kappa}}, \tag{82}$$

$$V_{11}(\eta) = \frac{\lambda U_{11}}{\kappa} - \kappa\epsilon U'_{11} - \frac{U_{11}^2}{2}. \tag{83}$$

or

$$U_{12}(\eta) = \frac{\lambda\left(\sqrt{2}a_0a_4\left(2 \pm \frac{1}{m\mathrm{sn}(\tau\eta)}\right) + \left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\right)}{\left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4\kappa}}, \tag{84}$$

$$V_{12}(\eta) = \frac{\lambda U_{12}}{\kappa} - \kappa\epsilon U'_{12} - \frac{U_{12}^2}{2}. \tag{85}$$

As $m \to 1$, solutions (84) and (85) generate the following solutions

$$U_{13}(\eta) = \frac{\lambda\left(\sqrt{2}a_0a_4(2 \pm \coth(\tau\eta)) + \left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\right)}{\left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\,\kappa}, \tag{86}$$

$$V_{13}(\eta) = \frac{\lambda U_{13}}{\kappa} - \kappa\epsilon U_{13}' - \frac{U_{13}^2}{2}. \tag{87}$$

Case 2: If $a_0 = \frac{a_4^3\left(1-m^2\right)}{32a_6^2}$, $a_2 = \frac{a_4^2\left(5-m^2\right)}{16a_6}$ and $a_6 > 0$, then

$$U_{21}(\eta) = \frac{\lambda\left(\sqrt{2}a_0a_4(2 \pm m\mathrm{sn}(\tau\eta)) + \left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\right)}{\left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\,\kappa}, \tag{88}$$

$$V_{21}(\eta) = \frac{\lambda U_{21}}{\kappa} - \kappa\epsilon U_{21}' - \frac{U_{21}^2}{2}. \tag{89}$$

or

$$U_{22}(\eta) = \frac{\lambda\left(\sqrt{2}a_0a_4\left(2 \pm \frac{1}{\mathrm{sn}(\tau\eta)}\right) + \left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\right)}{\left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\,\kappa}, \tag{90}$$

$$V_{22}(\eta) = \frac{\lambda U_{22}}{\kappa} - \kappa\epsilon U_{22}' - \frac{U_{22}^2}{2}. \tag{91}$$

As $m \to 1$, solutions (88) and (89) generate the following solutions,

$$U_{23}(\eta) = \frac{\lambda\left(\sqrt{2}a_0a_4(2 \pm \tanh(\tau\eta)) + \left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\right)}{\left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\,\kappa}, \tag{92}$$

$$V_{23}(\eta) = \frac{\lambda U_{23}}{\kappa} - \kappa\epsilon U_{23}' - \frac{U_{23}^2}{2}. \tag{93}$$

Case 3: If $a_0 = \frac{a_4^3}{32a_6^2(1-m^2)}$, $a_2 = \frac{a_4^2\left(4m^2-5\right)}{16a_6(m^2-1)}$, $a_6 > 0$,

$$U_{31}(\eta) = \frac{\lambda\left(\sqrt{2}a_0a_4\left(2 \pm \frac{1}{\mathrm{cn}(\tau\eta)}\right) + \left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\right)}{\left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\,\kappa}, \tag{94}$$

$$V_{31}(\eta) = \frac{\lambda U_{31}}{\kappa} - \kappa\epsilon U_{31}' - \frac{U_{31}^2}{2}. \tag{95}$$

or

$$U_{32}(\eta) = \frac{\lambda\left(\sqrt{2}a_0a_4\left(2 \pm \frac{\mathrm{dn}(\tau\eta)}{\sqrt{1-m^2}\mathrm{sn}(\tau\eta)}\right) + \left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\right)}{\left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\,\kappa}, \tag{96}$$

$$V_{32}(\eta) = \frac{\lambda U_{32}}{\kappa} - \kappa\epsilon U_{32}' - \frac{U_{32}^2}{2}. \tag{97}$$

As $m \to 0$, solutions (96) and (97) generate the following solutions

$$U_{33}(\eta) = \frac{\lambda\left(\sqrt{2}a_0a_4(2 \pm \csc(\tau\eta)) + \left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4}\right)}{\left(\sqrt{a_2^2 - 2a_0a_4} - a_2\right)\sqrt{a_2\left(a_2 + \sqrt{a_2^2 - 2a_0a_4}\right) - 3a_0a_4\kappa}}, \tag{98}$$

$$V_{33}(\eta) = \frac{\lambda U_{33}}{\kappa} - \kappa\epsilon U_{33}' - \frac{U_{33}^2}{2}. \tag{99}$$

Special solutions: It should be noted that the Eq.(5) has some special solutions other than Jacobi elliptic function solutions, which are as:

Case IV: If $a_0 = 0, a_6 = 0$ and $a_2 < 0, a_4 > 0$, then applying the conditions on the parametric equations system, we obtained the values of parameters as

$$\kappa = \frac{\lambda}{c_0}, a_2 = -\frac{c_0^4}{2\lambda^2(\rho + \epsilon^2)}, a_4 = \frac{c_0^2c_1^2}{4\lambda^2(\rho + \epsilon^2)}, c_2 = 0. \tag{100}$$

The following solution of equation (73) from solution set (100) is obtained as

$$U_{41}(\eta) = \frac{\lambda}{\kappa}(1 + \sqrt{2}\operatorname{sech}\left(\frac{\lambda\sqrt{-\frac{1}{2(\rho+\epsilon^2)}}\eta}{\kappa^2}\right)), \tag{101}$$

$$V_{41}(\eta) = \frac{\lambda U_{41}}{\kappa} - \kappa\epsilon U_{41}' - \frac{U_{41}^2}{2}. \tag{102}$$

Case V: If $a_0 = \frac{a_2^2}{4a_4}, a_6 = 0$ and $a_2 < 0, a_4 > 0$ then, applying the conditions on parametric equations system, we obtained the values of parameters as

$$\epsilon = \frac{\sqrt{c_1^2 - 4a_4\kappa^2\rho}}{2\sqrt{a_4}\kappa}, \lambda = c_0\kappa, a_2 = -\frac{2a_4c_0^2}{c_1^2}, c_2 = 0. \tag{103}$$

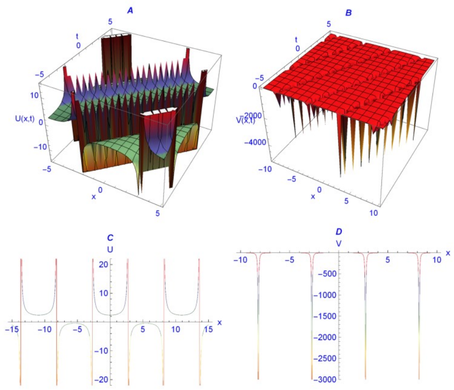

Figure 5. The graph of waves solutions (101) and (102) are depicted at different values of parameters: (A) Breather wave of strange structure, (B) Dark type breather waves of different amplitude, (C) 2D plot of U and (D) 2D plot of V.

The following solution of equation (73) from solution set (103) is obtained as

$$U_{51}(\eta)=\frac{\lambda}{\kappa}(1\pm\tanh\left(\frac{\eta\left(\sqrt{a_4}c_1\lambda\right)}{\kappa}\right)), \tag{104}$$

$$V_{51}(\eta)=\frac{\lambda U_{51}}{\kappa}-\kappa\epsilon U_{51}'-\frac{U_{51}^2}{2}. \tag{105}$$

§5 Discussion of results and Physical Interpretation of results

It has been observed that the exact waves solutions of space-time fractional coupled dynamical equations constructed via the current method are novel and in more general form. For the first coupled wave model which is space-time fractional coupled Boussinesq-Burger dynamical equation, on comparing the solutions to the generalized kudryashov method [26], it has been seen that our solutions are more simple, general and novel. Similarly, for the second coupled wave model which is space-time fractional coupled Drinfeld-Sokolov-Wilson dynamical equation, on comparing the solutions to Sine-Gordon expansion method [21], it has been seen some of our solutions are similar by taking different values of parameters and the remaining solutions are novel. For the third coupled wave model which is a space-time fractional coupled Whitham-Broer-Kaup equation, on comparing the solutions to modified exp-function method [25], it has been observed that our solutions are in a more general and simpler form. It indicates that our method is more effective, simple and has easy implementations. The advantages of this technique are as

- The attained results concerned with some unknown parameters which are used to obtain the different kinds of structures of solutions.
- Our results are more general and comprehensive.
- The current technique gives many types of novel soliton solutions to explore many physical phenomena in nonlinear physical sciences and other related field.
- Our techniques are very useful and easy to study, when the balance number is high.
- This technique is more suitable for exploring exact soliton solutions than these of the other schemes.

Figures 1 and 2 elaborate the wave solutions in various shapes of space-time fractional coupled Boussinesq-Burger equation. The Figures 3 and 4, elaborate the solitons wave in various shapes of space-time fractional coupled Drinfeld-Sokolov-Wilson equation. In the Figure 5, elaborates the analytical wave solutions in various shapes of space-time fractional coupled Whitham-Broer-Kaup dynamical equation. We use different values of parameters $a_0, a_2, a_4, a, b, \alpha, \beta, \gamma, \epsilon, \rho$, κ, λ and τ, for obtaining a novel graphical representation of solutions which is helpful for researcher to understand the physical phenomena of these fractional models.

§6 Conclusions

In this work, the proposed improved auxiliary equation method has been successfully used to obtain analytical wave solutions of three well-known different space-time fractional nonlinear coupled waves dynamical models, namely, space-time fractional coupled Boussinesq-Burger dynamical model, space-time fractional coupled Drinfeld-Sokolov-Wilson equation and space-time fractional coupled Whitham-Broer-Kaup equation. We obtained different solutions in terms of trigonometric, hyperbolic, exponential and Jacobi elliptic functions. Furthermore graphics are plotted to explain the different novel structures of obtained solutions such as multi solitons interaction, periodic soliton, bright and dark solitons, multi peak solitons, Kink and anti-Kink solitons, breather type waves and so on, which have applications in ocean engineering, fluid mechanics and other related fields. We hope that our results obtained in this article will be useful to understand many novel physical phenomena in physical sciences and other related fields. Our future work would be intense towards investigating the new wave solutions of other fractional models of higher order and dual-mode models by using this scheme. Furthermore, the executed techniques can be employed for further studies to explain the realistic phenomena arising in fluid dynamics correlated with any physical and engineering problems.

Declarations

Conflict of interest The authors declare no conflict of interest.

References

[1] M J Ablowitz, P A Clarkson. *Solitons, Nonlinear Evolution Equations and Inverse Scattering*, CUP, New York, 1991.

[2] R Hirota. *Exact Solutions of the Korteweg-de Vries Equation for Multiple Collisions of Solitons*, Phys Rev Lett, 1971, 27(2): 1192-1194.

[3] Y Kai, Z Yin. *On the Gaussian traveling wave solution to a special kind of Schrödinger equation with logarithmic nonlinearity*, Mod Phys Letters B, 2022, 36(2): 2150543.

[4] X Hu, M Arshad, Lu Xiao, et al. *Bright-Dark and Multi wave Novel Solitons Structures of Kaup-Newell Schrodinger Equations and their Applications*, Alex Eng J, 2021, 60(4): 3621-3630.

[5] X Y Wen. *Construction of new exact rational form non-travelling wave solutions to the (2+1)-dimensional generalized Broer-Kaup system*, Appl Math Comput, 2010, 217(4): 1367-1375.

[6] G B Whitham. *Variational methods and applications to water waves*, Proc R Soc Lond A, 1967, 299(1456): 6-25.

[7] W Sun, Y Jin, G Lu. *Genuine multipartite entanglement from a thermodynamic perspective*, Physical Review A, 2024, 109(4): 042422.

[8] S J Chen, X Lü, W X Ma. *Bäcklund transformation, exact solutions and interaction behaviour of the (3+1)-dimensional Hirota-Satsuma-Ito-like equation*, Commun Nonlinear Sci Numer Simul, 2020, 83(4): 105135.

[9] M Arshad, A R Seadawy, D Lu, et al. *Travelling wave solutions of Drinfeld-Sokolov-Wilson, Whitham-Broer-Kaup and (2+1)-dimensional Broer-Kaup-Kupershmit equations and their applications*, Chin J Phys, 2017, 55(3): 780-797.

[10] A Kurt. *New periodic wave solutions of a time fractional integrable shallow water equation*, Appl Ocean Res, 2019, 85: 128-135.

[11] X Lü. *Madelung fluid description on a generalized mixed nonlinear Schrödinger equation*, Nonlinear Dyn, 2015, 81(2): 239-247.

[12] A R Seadawy, M Arshad, D Lu. *Stability analysis of new exact traveling-wave solutions of new coupled KdV and new coupled Zakharov-Kuznetsov systems*, Euro Phys J Plus, 2017, 132(4): 162.

[13] X An, L Du, F Jiang, et al. *A few-shot identification method for stochastic dynamical systems based on residual multipeaks adaptive sampling*, Chaos, 2024, 34(7): 073118.

[14] J Xie, Z Xie, H Xu, et al. *Resonance and attraction domain analysis of asymmetric duffing systems with fractional damping in two degrees of freedom*, Chaos Solitons Fractals, 2024, 187: 115440.

[15] D Lu, B Hong. *New exact solutions for the (2 + 1)-dimensional generalized Broer-Kaup system*, Appl Math Comput, 2008, 199(2): 572-580.

[16] M Arshad, D Lu, M Ur Rehman, et al. *Optical Solitary Wave and Elliptic function Solutions of Fokas-Lenells Equation in Presence of Perturbation Terms and its Modulation Instability*, Phys Scr, 2019, 94(10): 105202.

[17] J W Xia, Y W Zhao, X Lü. *Predictability, fast calculation and simulation for the interaction solutions to the cylindrical Kadomtsev-Petviashvili equation*, Commun Nonlinear Sc Numer Simul, 2020, 90(11): 105260.

[18] X Lü, S J Chen. *Interaction solutions to nonlinear partial differential equations via Hirota bilinear forms: one-lump-multi-stripe and one-lump-multi-soliton types*, Nonlinear Dyn, 2021, 103(1): 947-977.

[19] X Zeng, D S Wang. *A generalized extended rational expansion method and its application to (1+1)-dimensional dispersive long wave equation*, Appl Math Comput, 2009, 212(2): 296-304.

[20] X Y Yao, Z B Li. *New exact solutions for three nonlinear evolution equations*, Phys Lett A, 2001, 297(1): 196-204.

[21] A Kurt, O Ozkan. *New solutions of fractional Drinfeld-Sokolov-Wilson system in shallow water waves*, J Ocean Eng, 2018, 161(1): 62-68.

[22] X J Yang, J A T Machado, D Baleanu. *Exact traveling-wave solution for local fractional Boussinesq equation in fractal domain*, Fractals, 2017, 25(4): 1740006.

[23] D J Kaup. *A higher-order water wave equation and method for solving it*, Prog Theor Phys, 1975, 54(1): 396-408.

[24] G Xu, Z Li. *Exact travelling wave solutions of the whitham-broer-kaup and broer-kaup-kupershmidt equations*, Chaos Solitons Fractals, 2005, 24(2): 549-566.

[25] Y Wang, Y F Zhang, Y Wang, et al. *A fractional whitham-broer-kaup equation and its possible application to tsunami prevention*, Thermal Science, 2017, 21(4): 1847-1855.

[26] M M A Khater, D Kumar. *New exact solutions for the time fractional coupled Boussinesq-Burger equation and approximate long water wave equation in shallow water*, J Ocean Eng Sci, 2017, 2(3): 223-228.

[27] A A Kilbas, H M Srivastava, J J Trujillon. *Theory and applications of fractional differential equations*, Elsevier Science Amsterdam, 2006.

[28] I Petras. *Fractional order nonlinear systems modeling, analysis and simulation*, Beijing Higher Education Press, 2011.

[29] A Tozar, A Kurt, O Tasbozan. *New wave solutions of an integrable dispersive wave equation with a fractional time derivative arising in ocean engineering models*, Kuwait J Sci, 2020, 47(2): 22-33.

[30] K S Miller. *An Introduction to Fractional Calculus and Fractional Differential Equations*, J Wiley & Sons, New York, 1993.

[31] S J Chen, X Lü, X F Tang. *Novel evolutionary behaviors of the mixed solutions to a generalized Burgers equation with variable coefficients*, Commun Nonlinear Sci Numer Simul, 2021, 95: 105628.

[32] M Arshad, D Lu, J Wang. *(N+1)-dimensional fractional reduced differential transform method for fractional order partial differential equations*, Commun Nonlinear Sci Numer Simul, 2017, 48(1): 509-519.

[33] A Kurt. *New Analytical and Numerical Results For Fractional Bogoyavlensky-Konopelchenko Equation Arising in Fluid Dynamics*, Appl Math J Chinese Univ, 2020, 35(1): 101-112.

[34] Y He, Y Kai. *Wave structures, modulation instability analysis and chaotic behaviors to Kudryashov's equation with third-order dispersion*, Nonlinear Dyn, 2024, 112(12): 10355-10371.

[35] S Guo, S Wang. *Twisted relative Rota-Baxter operators on Leibniz conformal algebras*, Communications in Algebra, 2024, 52(9): 3946-3959.

[36] M Arshad, D Lu, J Wang, et al. *Exact Traveling Wave Solutions of a Fractional Sawada-Kotera Equation*, East Asian J Appl Math, 2018, 8(2): 211-223.

[37] K A Gepreel. *Exact solutions for nonlinear integral member of Kadomtsev-Petviashvili hierarchy differential equations using the modified (G / G)-expansion method*, Comput Math Appl, 2016, 72(9): 2072-2083.

[38] S S Ray, C S Wise. *A novel method for travelling wave solutions of fractional Whitham-Broer-Kaup, fractional modified Boussinesq and fractional approximate long wave equations in shallow water*, Math Methods Appl Sci, 2015, 38(7): 1352-1368.

[39] A M Wazwaz. *The Cole-Hopf transformation and multiple soliton solutions for the integrable sixth-order Drinfeld-Sokolov-Satsuma-Hirota equation*, Appl Math Comput, 2009, 207(1): 248-255.

[40] O Tasbozan. *New Analytical Solutions for Time Fractional Benjamin-Ono Equation Arising Internal Waves in Deep Water*, China Ocean Eng, 2019, 33(5): 593-600.

[41] D Lu, C Yue, M Arshad. *Traveling Wave Solutions of Space-Time Fractional Generalized Fifth-Order KdV Equation*, Adv Math Phys, 2017, 17(1): 1-7.

[42] J E Macías-Díaz. *Numerical study of the process of nonlinear supratransmission in Riesz space-fractional sine-Gordon equations*, Commun Nonlinear Sci Numer Simul, 2017, 46: 89-102.

[43] M D Hossain, M K Alam, M A Akbar. *Abundant wave solutions of the Boussinesq equation and the (2+1)-dimensional extended shallow water wave equation*, Ocean Eng, 2018, 165(1): 69-76.

[44] Y Yang, H Li. *Neural Ordinary Differential Equations for Robust Parameter Estimation in Dynamic Systems with Physical Priors*, Applied Soft Computing, 2024, 169: 112649.

[45] T S El-danaf, A R Hadhoud. *Parametric spline functions for the solution of the one time fractional Burgers equation*, Appl Math Model, 2012, 36(10): 4557-4564.

[46] K K Ali, R I Nuruddeen. *New hyperbolic structures for the conformable time-fractional variant bussinesq equations*, Opt Quantum Electron, 2018, 50(2): 1-10.

[47] Z Zheng, W Zhao, H Dai. *A new definition of fractional derivative*, Int J Non Linear Mech, 2019, 108(1): 1-6.

[48] J Zhang, F Liu, Z Lin, V Anh. *Analytical and numerical solutions of a multi-term time-fractional Burgers fluid model*, Appl Math Comput, 2019, 356(1): 1-12.

[49] C Li, Q Guo, M Zhao. *New solitary wave solutions of (2+1)-dimensional space-time fractional Burgers equation and Korteweg-de Vries equation*, Comput Math Appl, 2019, 77(8): 2255-2262.

[50] R Khalil, M Al Horani, A Yousef, et al. *A new definition of fractional derivative*, J Comput Appl Math, 2014, 264: 65-70.

[51] W S Chung. *Fractional Newton mechanics with conformable fractional derivative*, J Comput Appl Math, 2015, 209(1): 150-158.

[1]School of Energy and Power Engineering, Nanjing University of Science and Technology, Nanjing 210094, China.

Email: taogang@mail.njust.edu.cn

[2]Department of Mathematics & Statistics, University of Agriculture, Faisalabad, Pakistan.

Email: muhammad.arshad18@yahoo.com

[3]Faculty of Science, Jiangsu University, Zhenjiang 212013, China.

Email: dclu@ujs.edu.cn