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COVID-19 emergency decision-making using q-rung linear diophantine fuzzy set, differential evolutionary and evidential reasoning techniques

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Abstract. In this paper, a robust and consistent COVID-19 emergency decision-making approach is proposed based on q-rung linear diophantine fuzzy set (q-RLDFS), differential evolutionary (DE) optimization principles, and evidential reasoning (ER) methodology. The proposed approach uses q-RLDFS in order to represent the evaluating values of the alternatives corresponding to the attributes. DE optimization is used to obtain the optimal weights of the attributes, and ER methodology is used to compute the aggregated q-rung linear diophantine fuzzy values (q-RLDFVs) of each alternative. Then the score values of alternatives are computed based on the aggregated q-RLDFVs. An alternative with the maximum score value is selected as a better one. The applicability of the proposed approach has been illustrated in COVID-19 emergency decision-making system and sustainable energy planning management. Moreover, we have validated the proposed approach with a numerical example. Finally, a comparative study is provided with the existing models, where the proposed approach is found to be robust to perform better and consistent in uncertain environments.

§1 Introduction

For the past one and half years, the whole world has been facing the pandemic caused by the novel coronavirus (COVID-19), and it has affected nearly 200 countries. The first human infected with the new coronavirus was reportedly identified in Wuhan [1]. As COVID-19 has affected the entire world and made human life arduous in these circumstances, WHO [4] issued a health emergency and declared COVID-19 as a pandemic in March, 2020. It has done severe damage to all the countries in different aspects like economy, education, health, and daily life routine. It has become very difficult for people to live a normal life because of the needed restrictions and imposition of the lockdown. In order to curtail the damage, the world health organization (WHO), scientists, doctors, and researchers are trying their best to eradicate the virus. For the prevention and control of the spreading of coronavirus, we need to imply suitable decision-making approaches for the safety of people around the world. Some significant and

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relevant research works on COVID-19 are summarized below. Ashraf and Abdullah [2] presented some novel decision-making methods for COVID-19 emergencies using spherical fuzzy sets based on Einstein aggregation operators. Ashraf et al. [3] introduced another novel decision- making method to diagnose COVID-19 as an emergency service based on a spherical fuzzy set, TOPSIS and COPRAS methods. Chan et al. [5] studied the clinical and laboratory findings of five patients in a particular family and analyzed the genomic sequences of these patients using phylogenetic analysis. Yu et al. [6] developed a few methods for diagnosing and treating pneumonia caused by the novel Coronavirus. Melin [19] presented a new hybrid prediction model for response integration that can consolidate the ensemble designs of fuzzy logic-based neural networks. To some extent, this model anticipates COVID-19 future trends and assists authorities in making necessary decisions to effectively manage the health care system. Fuzzy-based hybrid techniques for forecasting confirmed cases and deaths in countries based on time series. The proposed hybrid technique [21] with fuzzy logic incorporates the fractal dimension to enable COVID-19 time series forecasting. They used COVID-19 time series data to construct a hybrid technique for predicting COVID-19 data classification [22] by nation using time series and developing a plan to take appropriate action based on the countries' current status. Chander and Das [24] presented a similarity measure in the application of medical diagnosis using interval-valued pythagorean fuzzy set. Sun and Wang [20] collected COVID- 19 data and fitted it with the conventional differential condition model. They discovered that the affected visitors play an important role in the newly introduced cases of COVID-19 and that they can be rapidly expanded. Si et al. [23] proposed a decision-making approach using a picture fuzzy set, Dempster- Shafer theory of evidence and Grey relational analysis for preferable medicine for the treatment of COVID-19 patients and evaluated the preferences of the medicines based on the symptoms and signs of the COVID-19 patients. Castillo and Melin [26] proposed a fuzzy fractal control method for effectively controlling nonlinear dynamic systems and illustrated the proposed approach in achieving an efficient control of COVID-19 pandemic. In the other work, Melin and Castillo [27] proposed a neural network model with a self-organizing map for spatial data analysis and used the above proposed fuzzy fractal approach to represent the temporal trends of the time series of the countries such as Belgium, Italy, the United States, and Mexico with respect to COVID-19 cases. The above-mentioned studies assisted the decision makers in making ideal decisions in emergencies to avoid damage.

Some significant contributions and importance of various fuzzy sets to solve MADM problems are illustrated below. Zadeh [13] introduced fuzzy set (FS) theory in 1965, and it has been consistently employed in many decision-making procedures since then. To account for the importance of a non-membership degree, Atanassov [14] explored intuitionistic fuzzy set (IFS) that represents uncertainty using the grade of membership, non-membership, and indeterminacy as an extension to FS. To widen the structural space of membership and non-membership grades q-rung orthopair fuzzy set [15, 16] was introduced. The significance of the q^{th} reference parameter (q $\geq$ 1) in representing the q-rung orthopair fuzzy set (q-ROFS) allows experts or decision makers to issue grades flexibly. Riaz and Hashmi [9] proposed a new notion of linear diophantine fuzzy set (LDFS), which outperforms other fuzzy sets such as IFS, pythagorean fuzzy sets (PFS) and q-ROFS. The inclusion of reference parameters in the LDFS representation effectively enhances the structural space of membership and non-membership grades. For example, in some cases, the sum of membership (μ) and non-membership (ν) grades of an attribute of the alternative provided by a decision maker may be greater than 1. Consider

$\mu = 0.9$ and $\nu = 0.6$, hence, $\mu + \nu$, i.e., $0.9 + 0.6 > 1$. This kind of situation cannot be represented by IFS, since the summation of membership and non-membership grade is greater than 1, whereas PFS can represent it. However, when the summation of the squares of membership and non-membership grades are more than 1, i.e., $0.9^2 + 0.6^2 > 1$, PFS fails to express it. To manage such type of situations, q-ROFS are used. The condition $0 \leq \mu^q + \nu^q \leq 1$; $q > 1$ in q-ROFS overcomes the preceding membership and non-membership constraints. For bigger values of q, it can manage membership and non-membership grades. When both membership and non-membership grades are 1 ($\mu = \nu = 1$), however, we obtain ($1^q + 1^q > 1$), which violates the q-ROFS criterion. The drawback of the q-ROFS restriction can be avoided in LDFS by adding reference parameters (α, β) to the membership and non-membership grades with the conditions $0 \leq \alpha(\mu) + \beta(\nu) \leq 1$ and $0 \leq \alpha + \beta \leq 1$. But in some real-life cases, when the summation of the reference parameters (α, β) is more than one, i.e., ($\alpha + \beta > 1$), LDFS is not suitable to formulate those problems. Fuzzy set, IFS, PFS, q-ROFS, and LDFS are restricted in some cases to define the degree of membership and non-membership by the experts and decision makers, which affects choosing the right decision or right alternative. If the problem representation is bound to be limited in uncertain circumstances, then the handling of the problem will be approximate and cannot be concluded effectively with limited information. Almagrabi et al. [10] proposed a new methodology called the q-rung linear diophantine fuzzy set (q-RLDFS), which diminishes all restrictions and limitations in decision-making systems. A q-RLDFS handles more uncertainty compared to other above mentioned theories and eradicates all the restrictions and ambiguities in defining the membership and non-membership grades with the inclusion of reference parameters and q^{th} reference. The advantage of reference parameters and q^{th} reference allows the experts and decision makers to choose the membership and non-membership grades without any limits. These reference parameters can also be used as the physical sense in the categorization of the problem. For instance, the information of the problem is classified by the reference parameters with how much portion is still required to treat the patient and the grades of membership and non-membership decide the factor present in the medicine. This makes q-RLDFS more efficient in representing the problem information and enhances the wider applicability of structural space compared to the structural space in FS, IFS, PFS, q-ROFS, and LDFS. By using q-RLDFS, the integrity of the problem representation will not be disturbed or affected, which is the major advantage of q-RLDFS. It adapts to any kind of problem representation with the help of q^{th} reference parameters.

Optimization principles ensure the values reach global maxima or global minima, which is needed for choosing the optimal weight values of the attributes of the alternatives in the decision information, unlike the other methods, where optimal weight value calculation is based on experts assigned weights or other methods could be treated as static and specific attribute centric. The differential evolutionary algorithm [11] is a population-based stochastic optimization technique with the operations of mutation, crossover, and selection that generates an optimal weight vector of the attributes of the alternatives based on an objective function. The obtained optimal weight vector can ensure to be the global maxima or global minima. With uncertain and imprecise information on the path, aggregating decision problem information is critical in the domains such as medical sciences, planning and management, and a variety of other professions. Chander and Das [25] presented a differential evolutionary optimization based decision-making method using interval-valued pythagorean fuzzy set and compared DE with particle swarm optimization technique in the decision-making. The evidential reasoning (ER) approach [17] is different from other conventional decision-making models, which aggregates the decision information by employing a belief structure to represent as an assessment. It also enhances the problem-solving mechanism by assessing the belief degree structure for each attribute of all alternatives in the decision matrix. To rank the alternatives in a decision-making problem, Chen and Chiou [8] developed a MADM method using interval-valued intuitionistic fuzzy sets (IVIFS) based on the working principles of the evidential reasoning (ER) approach and particle swarm optimization (PSO) techniques. But in some cases, IVIFS of the form ([a, b], [c, d]) fails to describe the problem representation proficiently in making decisions by the grades of membership and non-membership with a condition $0 \leq$ b + d ≤ 1, where a, b are the lower and higher grades of membership, and c, d are the lower and higher grades of non-membership, respectively. Assume that [0.8, 0.9] and [0.2, 0.3] are the membership and non-membership interval grades respectively, hence 0.9 and 0.3 is not valid under IVIFS with the given condition and ensures restrictions to the decision maker, but it is valid under q-RLDFS for the same values with the qth reference parameters. Here q-RLDFS shows its wider applicability and removes the restrictions for the experts or decision makers. The novel q-RLDFS studied in [10] with reference parameters and a q^{th} reference parameter (q>1) widens the space of grades in membership and non-membership and outperforms other fuzzy sets. The authors demonstrated its applicability in the decision-making using q-RLDF aggregation operators and score functions.

Emergency decision-making has an important role in emergency services by ensuring quick and ideal decisions. Hence, the decision-makers ought to foster the decision-making approaches for humans to provide prominent ways to respond in emergencies. Few experiments have shown that the characteristics of human behavior, such as cognitive biases, insensitive disruption, etc., make the decisions uncertain and risky in some cases in emergency decision-making.

The motivation of this work is achieved from the structural space of bound in q-RLDFS, which is more than other fuzzy set theories, and the problem representation in uncertain environments can be well built by membership grades and non-membership grades without any restrictions given by the experts or decision makers. In some real-life problems, the sum of membership (μ) and non-membership (ν) grades for an attribute of an alternative provided by the decision maker can be greater than 1, which is outside the scope of fuzzy set and IFS. Although PFS, q-ROFS, and LDFS can handle such type of situations, but the structural space of bound is limited in these sets. However, q-RLDFS overcomes the restrictions mentioned above and successfully handles such scenarios. The advantage of q-RLDFS is the reference parameters and a q^{th} reference parameter which widens the structural space in grading membership and non-membership and outperforms other fuzzy sets. Any kind of problem representation can easily be adapted by q-RLDFS, which in turn allows the decision maker to freely grade the values without any restrictions. With the use of reference parameters and q^{th} parameter, q-RLDFS overcomes the information gap and can solve the limitations concerned with IFS, PFS, q-ROFS, and LDFS. In q-RLDFS, using the q^{th} power of reference parameter, the decision maker can grade membership and non-membership without any restriction. When the q value increases, the diophantine space also increases and thereby giving the space of bounds more scope to grade a wider range of information without restriction and can be best suited for complex and uncertain decision-making problems. Moreover, the ER methodology effectively handles the decision-making problems with uncertainties using degrees of belief structure. A belief degree structure is used to describe the assessment information of the attributes corresponding to the alternatives. The collective assessment information can be aggregated using ERM based on belief degree information, which can contribute to improving the decision-making process. In comparison to other optimization approaches, DE optimization is simple to implement and can consistently reach true optimal values. DE can help aggregate the belief degrees in ERM and contribute to the enhancement of MADM techniques by achieving optimal weight values for the attributes of the decision information. To manage the uncertainties related to COVID-19 emergency decision-making, benefit of high structural scope of bound in q-RLDFS, computation of the aggregated degree of belief in ERM, and the capabilities of obtaining the optimal weight values for the attributes in DE optimization may be combined to formulate a better decision-making framework. Motivated by the above facts and feeling the need for emergency decision-making during COVID-19, we aim to contribute an approach, which will help society for emergency decision-making during COVID-19.

The objective of this paper is to develop a multiattribute decision-making model built on methodologies that removes all the limitations and ambiguities for any kind of problem representations given by the experts or decision makers using q-RLDFS, DE optimization algorithm, and ER methodology. The proposed decision-making model can be more adaptable and appropriate in imprecise environments and establishes a strong relationship with MADM problems. In this paper, we propose a multi attribute decision-making model built on methodologies of q-RLDFS, DE optimization, and ER methodology. Initially, the decision information is provided as a decision matrix with a set of attributes for each alternative using q-RLDFS. By using the DE optimization technique, we obtain the optimal weight values of the attributes as the decision information. Then, the ER methodology aggregates the decision information by employing a belief degree structure for each of the attributes. Next, we calculate the score values of the alternatives by using the score functions of the q-RLDFS to rank the alternatives. The better alternative is chosen based on the larger score value of the alternatives. The proposed MADM approach has been illustrated with real-life case studies like COVID- 19 emergency decision-making and sustainable energy planning management followed by a numerical example. Finally, a comparative study with the existing methods has been demonstrated, where the proposed method is proved to be flexible and efficient.

The rest of the paper is organized as follows. Section 2 provides preliminaries comprising of some relevant ideas. Section 3 discusses the proposed MADM method. Section 4 provides the real-life case studies and numerical example followed by comparative study in Section 5. Section 6 concludes the work with a summary.

§2 Preliminaries

This section covers some preliminary information related to the q-RLDFS, such as fuzzy set, IFS, q-ROFS, LDFS, and q-LDFS.

Definition 2.1 [13]: If F is a fuzzy set on a non-empty discourse space $Z = \{\mathcal{K}_1, \mathcal{K}_2,.., \mathcal{K}_n\}$ then F is defined as

$$F = \{\mathcal{K}, \mu_F(\mathcal{K}) | \mathcal{K} \in Z\} \tag{1}$$

Here $\mu_F(\mathcal{K})$ represents the grade of membership of the entity $\mathcal{K}$ in the fuzzy set F, and it belongs to [0, 1].

Definition 2.2 [14]: If F be an IFS on a non-empty discourse space $Z = \{\mathcal{K}_1, \mathcal{K}_2, .., \mathcal{K}_n\}$, then

F is defined as

$$F = \{\mathcal{K}, \mu_F(\mathcal{K}), \nu_F(\mathcal{K}) | \mathcal{K} \in Z\}. \tag{2}$$

Here $\mu_F(\mathcal{K})$ and $\nu_F(\mathcal{K})$ ranges between [0, 1] respectively represent the grade of membership and non-membership of entity $\mathcal{K}$ in F with a condition $0 \leq \mu_F(\mathcal{K}) + \nu_F(\mathcal{K}) \leq 1$ for all $\mathcal{K} \in Z$. The grade of indeterminacy or hesitation π for the IFS F is $\pi_F = 1 - \mu_F(\mathcal{K}) - \nu_F(\mathcal{K})$. Figure 1 depicts IFS graphically.

Definition 2.3 [16]: If F is a q-ROFS on a non-empty discourse space $Z = \{\mathcal{K}_1, \mathcal{K}_2, .., \mathcal{K}_n\}$ then F is defined as

$$F = \{\mathcal{K}, \mu_F(\mathcal{K}), \nu_F(\mathcal{K}) | \mathcal{K} \in Z\}. \tag{3}$$

Here $\mu_F(\mathcal{K})$ and $\nu_F(\mathcal{K})$ ranges between [0, 1] respectively represent the grade of membership and non-membership of entity $\mathcal{K}$ in F with a condition $0 \leq (\mu_F(\mathcal{K}))^q + (\nu_F(\mathcal{K}))^q \leq 1$, q≥1 for all $\mathcal{K} \in Z$. The grade of indeterminacy π for the q-ROFS F is $\pi_F = \sqrt[q]{1 - (\mu_F(\mathcal{K}))^q - (\nu_F(\mathcal{K}))^q}$. Figure 2 depicts q-ROFS graphically.

Definition 2.4 [9]: If F is an LDFS on a non-empty discourse space $Z = \{\mathcal{K}_1, \mathcal{K}_2, ..., \mathcal{K}_n\}$ then F is distinguished as

$$F = \{\mathcal{K}, (\mu_F(\mathcal{K}), \nu_F(\mathcal{K})), (\alpha, \beta) | \mathcal{K} \in Z\}. \tag{4}$$

Here $\mu_F(\mathcal{K})$ and $\nu_F(\mathcal{K})$ respectively represent the grade of membership and non-membership of entity $\mathcal{K}$ in F, and α and β are the reference parameters, ranges between [0, 1], with a condition $0 \leq \alpha(\mu_F(\mathcal{K})) + \beta(\nu_F(\mathcal{K})) \leq 1$, and $0 \leq \alpha+\beta \leq 1$ for all $\mathcal{K} \in Z$. The grade of indeterminacy π for the LDFS F is $\pi_F = 1 - \alpha(\mu_F(\mathcal{K})) + \beta(\nu_F(\mathcal{K}))$. Figure 3 depicts LDFS graphically.

q-rung linear diophantine fuzzy set(q-RLDFS):

Definition 2.5 [10]: If F is a q-RLDFS on a non-empty discourse space $Z = \{\mathcal{K}_1, \mathcal{K}_2, .., \mathcal{K}_n\}$ then F is distinguished as

$$F = \{\mathcal{K}, (\mu_F(\mathcal{K}), \nu_F(\mathcal{K})), (\alpha, \beta) | \mathcal{K} \in Z\}, \tag{5}$$

Here $\mu_F(\mathcal{K})$, $\nu_F(\mathcal{K})$, α and β ranges between [0, 1] respectively represent the grade of membership, non-membership and the reference parameters of of entity $\mathcal{K}$ in F , where ($\mu_F(\mathcal{K})$, $\nu_F(\mathcal{K})$, α, β) is termed as q-rung linear diophantine fuzzy value (q-RLDFV) with a condition $0 \leq \alpha^q(\mu_F(\mathcal{K})) + \beta^q(\nu_F(\mathcal{K})) \leq 1$, and $0 \leq \alpha^q+\beta^q \leq 1$ for all $\mathcal{K} \in Z$. The grade of indeterminacy π for q-RLDFS F is $\pi_F = \sqrt[q]{1 - \alpha^q(\mu_F(\mathcal{K})) + \beta^q(\nu_F(\mathcal{K}))}$. Figure 4 depicts q-RLDFS graphically.

We note that,

(i) If$q = 1$ in Definition 2.5, the q-RLDFS reduces to LDFS.

(ii) If $q = 2$ in Definition 2.5, the q-RLDFS reduces to quadratic DFS.

(iii) If $q = 3$ in Definition 2.5, the q-RLDFS reduces to cubic DFS.

(iv) If $q = 4$ in Definition 2.5, the q-RLDFS reduces to bi-quadratic DFS and so on.

The advantage of the q^{th} parameter is that as we increase the q-rung value, the problem space increases accordingly. So, the fuzzy information can be broadly conveyed in the structure space. This structure removes the experts or decision makers limitations in establishing membership and non-membership grades. The physical perception of the system can also be altered by these reference parameters. One can overcome the systems limitations by increasing the grade space used in q-RLDFS. As a result, we may use q-RLDFS to grade a broader range of fuzzy information in problem representation. In other words, we can continue to tune the value of q to determine the information expression range and make it more appropriate for imprecise and ambivalent environments.

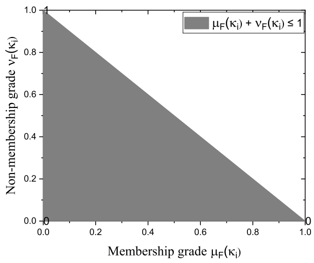

Figure 1. Intuitionistic fuzzy set.

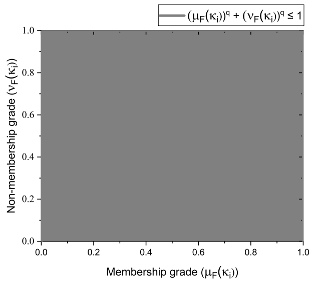

Figure 2. Q-rung orthopair fuzzy set.

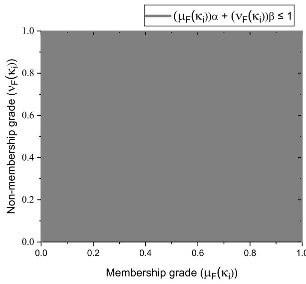

Figure 3. Linear diophantine fuzzy set.

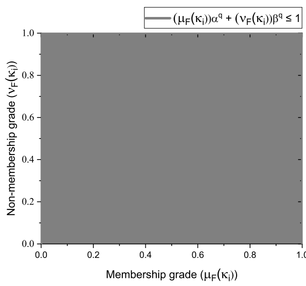

Figure 4. Q-rung linear diophantine fuzzy set.

Score functions and accuracy functions of q-RLDFS:
Let $F = ((\mu_F(\mathcal{K}),\nu_F(\mathcal{K})),\ (\alpha,\ \beta))$ be a q-RLDFV, where $\mu_F(\mathcal{K})$, $\nu_F(\mathcal{K})$, α, and β represent the grade of membership, non-membership, and the reference parameters respectively, with a condition $0 \le \alpha^q(\mu_F(\mathcal{K})) + \beta^q(\nu_F(\mathcal{K})) \le 1$ and $0 \le \alpha^q+\beta^q \le 1$. Almagrabi [10] presented three score functions such as score function, quadratic score function, and expectation score functions followed by respective accuracy functions, shown as follows:

Score functions (SF):
Assume $F = ((\mu_F, \nu_F), (\alpha, \beta))$ be a q-RLDFV, then score function (SF), accuracy function (AF), Quadratic Score function(QSF), qudratic accuracy function (QAF), expectation score function (ESF) are given by

$$\text{SF }(F) = [\tfrac{(\mu_F-\nu_F)+(\alpha^q-\beta^q)}{2}], \text{ where q} \ge 1, \text{ SF }(F) \in [-1, 1] .$$

$$\text{AF }(F) = [\tfrac{(\mu_F+\nu_F)}{4} + \tfrac{(\alpha^q+\beta^q)}{4}], \text{ where q} \ge 1, \text{ AF}(F) \in [0, 1].$$

$$\text{QSF }(F) = [\tfrac{((\mu_F)^2-(\nu_F)^2)+((\alpha^q)^2-(\beta^q)^2)}{2}], \text{ where q} \ge 1, \text{ QSF }(F) \in [-1, 1].$$

$$\text{QAF }(F) = [\tfrac{((\mu_F)^2+(\nu_F)^2)}{4} + \tfrac{((\alpha^q)^2+(\beta^q)^2)}{4}], \text{ where q} \ge 1, \text{ QAF }(F) \in [0, 1].$$

$$\text{ESF }(F) = [\tfrac{(\mu_F-\nu_F+1)}{4} + \tfrac{(\alpha^q-\beta^q+1)}{4}], \text{ q} \ge 1.$$

The score values of ESF score function lies between [0, 1] instead of [-1, 1] because it is generalized form of SF.

Differential evolutionary algorithm:

DEA [16] is considered the most widely used and effective population-based stochastic optimization technique. Individuals from the current generation are considered the target vectors in DEA, and the mutant vectors are determined using mutation operation from the target vector. Following that, the trial vectors are generated by combining the parameters of the mutant vector and the target vector using a crossover operation. The trial vector is chosen in the next generation based on the fitness value of the individuals and the greedy strategy (survival of the fittest). The DEA Pseudocode is given below in Algorithm 1.

Evidential reasoning methodology:

ER methodology was developed to enhance the handling of decision analysis problems based on the combination rule and evaluation framework, and is capable of handling imprecise information of both qualitative and quantitative data. It has been used to aggregate the assessments of the attributes of the alternatives with uncertainty in various decision-making problems, specifically in MADM environments [18]. It uses a belief decision matrix to improve decision-making and assesses attributes using evaluation grades. The degree of belief for each attribute of the alternatives is next assessed. A belief structure is given by an assumption to demonstrate a subjective assessment with uncertainty. To evaluate the performance of a bike, experts may find that 40 percent is good and 60 percent is excellent. Here good, excellent are the evaluation grades and the values in percentage are the degrees of belief [12]. So, the preceding statement can be represented as S(performance) = {(good, 0.4), (excellent, 0.6)}. To assess the bike on other attributes, other evaluation grades may also use the same grades such as good and excellent. For example, the assessment for attribute mileage of the bike, S(mileage) = {(good, 0.3), (excellent, 0.6)} is an incomplete assessment since $0.3 + 0.6 \le 1$ whereas, the assessment of the attribute performance is complete as $0.4 + 0.6 = 1$.

Algorithm 1 Pseudocode for DEA

Input: *Fitness function, lb, ub, Np, T, F, Pc*
Output: Optimal solution (weight vector $\mathcal{W}_j^*$) from the Np

Begin
Initialize population (P)
Compute the fitness (f) of $P(f)$ of P
for $i \leftarrow 1, T$ **do**
  for $i \leftarrow 1, N_p$ **do**
    Generate the donor vector (v_i) using mutation
    Perform crossover to generate offspring (U_i)
  end for
  for $i \leftarrow 1, N_p$ **do**
    Bound U_i
    Evaluate the fitness $(f(U_i))$ of U_i
    Perform greedy selection using $f(U_i)$ and f_i to update P
  end for
end for
End

Assume a MADM problem with $\mathcal{K}_i$ alternatives and C_j attributes, where $1 \leq i \leq m, 1 \leq j \leq n$, and the attributes have all the factors influencing the assessment of the general attribute E, where $E = \{C_1, C_2, .., C_j\}$. The weights of the attributes $\mathcal{W}_j = \{w_1, w_2, .., w_n\}$ are the relative weights, which play an important role in prioritizing and normalizing the attributes. Suppose p distinctive evaluation grades $H_1, H_2, .., H_p$ are given for assessing all attributes of the alternatives as, $H = \{H_1, H_2, .., H_p\}$, then the assessment of the j^{th} attribute of an alternative is defined as, $C_j(\mathcal{K}_i) = (H_q, \beta_{q,j}(\mathcal{K}_i))$, $q = 1, 2, .., p$ and $j = 1, 2, .., n$, where $\beta_{q,j}(\mathcal{K}_i) \geq 0$, $\sum_{q=1}^{p} \beta_{q,j}(\mathcal{K}_i)) \leq 1$ denotes the degree of belief. The above assessment says each attribute C_j is assessed with an evaluation grade (H_q) with a degree of belief.

Let $m_{q,j}$ be the basic probability mass for representing the degree of the j^{th} attribute assessed to the p evaluation grades, and $m_{H,j}$ be the remaining probability mass for the unassigned individual grades for the evaluation grades. They are calculated as,

$$m_{q,j}(\mathcal{K}_i) = \mathcal{W}_j^* \beta_{q,j}(\mathcal{K}_i),$$
$$m_{H,j}(\mathcal{K}_i) = 1 - \sum_{q=1}^{p} m_{q,j}(\mathcal{K}_i)$$

State $E_{I(y)}$ be the subset of the y attributes as $E_{I(y)} = \{C_1, C_2, .., C_j\}$, where $j = 1, 2, .., n$. Let $m_{q,I(y)}$ be the combined probability mass of the attribute y assessed to the grade Hp, and $m_{H,I(y)}$ be the combined remaining probability mass of the attributes unassigned to the individual grades of the attributes y, calculated as:

$$m_{q,I(y)}(\mathcal{K}_i) = R_{I(y)}(\mathcal{K}_i)(m_{q,I(y-1)}(\mathcal{K}_i) m_{q,y}(\mathcal{K}_i) +$$
$$m_{q,I(y-1)}(\mathcal{K}_i) m_{H,y)}(\mathcal{K}_i) + m_{H,I(y-1)}(\mathcal{K}_i) m_{q,y}(\mathcal{K}_i)),$$
$$m_{H,I(y)}(\mathcal{K}_i) = R_{I(y)}(\mathcal{K}_i) m_{H,I(y-1)}(\mathcal{K}_i) m_{H,y}(\mathcal{K}_i),$$

where $R_{I(y)}(\mathcal{K}_i) = (1 - \sum_{t=1}^{2} \sum_{g=1 g \neq t}^{2} m_{t,I(y-1)}(\mathcal{K}_i)\ m_{H,y}(\mathcal{K}_i))^{-1}$, $2 \leq y \leq n, 1 \leq i \leq m$ and $1 \leq q \leq 2$ is the normalized factor, and $m_{q,I(1)} = m_{q,1}$, and $m_{H,I(1)} = m_{H,1}$.

The combined degree of belief β_q is obtained by aggregating the evaluating values of the attributes of the alternatives. The aggregation problem generates β_q as,

Table 1. The decision matrix $(D)_{mxn}$.

	C_1	C_2		C_n
$\mathcal{K}_1$	$((\mu_{11},\nu_{11}),$ $(\alpha_{11},\beta_{11}))$	$((\mu_{12},\nu_{12}),$ $(\alpha_{12},\beta_{12}))$	$\cdots$	$((\mu_{1n},\nu_{1n}),$ $(\alpha_{1n},\beta_{1n}))$
$\mathcal{K}_2$	$((\mu_{21},\nu_{21}),$ $(\alpha_{21},\beta_{21}))$	$((\mu_{12},\nu_{22}),$ $(\alpha_{22},\beta_{22}))$	$\cdots$	$((\mu_{2n},\nu_{2n}),$ $(\alpha_{2n},\beta_{2n}))$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$
$\mathcal{K}_m$	$((\mu_{m1},\nu_{m1}),$ $(\alpha_{m1},\beta_{m1}))$	$((\mu_{m2},\nu_{m2}),$ $(\alpha_{m2},\beta_{m2}))$	$\cdots$	$((\mu_{mn},\nu_{mn}),$ $(\alpha_{mn},\beta_{mn}))$

$$\beta_q(\mathcal{K}_i) = 1 - \beta_H(\mathcal{K}_i)\frac{(m_{q,I(n)}(\mathcal{K}_i))}{1-(m_{H,I(n)}(\mathcal{K}_i))}$$

where $\beta_H(\mathcal{K}_i) = \sum_{j=1}^{n} \mathcal{W}_j(1-\sum_{q=1}^{2} \beta_{q,j}(\mathcal{K}_i))$ is the degree of belief unassigned to any individual evaluation grades.

§3 A Proposed approach based on q-RLDFS using DEA and ER methodology

In this section, we propose a MADM approach by taking advantage of q-rung LDFS to alleviate the problem space, differential evolutionary optimization and evidential reasoning methodologies to provide reliable decision-making in emergency environments.

Initially, in order to provide the decision information, we form a q-RLDFS based decision matrix q-RLDFSs $D_{mxn} = ((\mu_F(\mathcal{K}_i),\ \nu_F(\mathcal{K}_i)),(\alpha,\ \beta))\ q \geq 1$ with a set of m alternatives ($\mathcal{K}_1$, $\mathcal{K}_2$,..,$\mathcal{K}_m$) and n attributes (C_1, C_2,.., C_n). The decision matrix D_{mxn} is given below in Table 1. Here $((\mu_F(\mathcal{K}),\ \nu_F(\mathcal{K})),\ (\alpha,\ \beta))$ is called q-rung linear diophantine fuzzy value (q-RLDFV) and $0 \leq \alpha^q(\mu_F(\mathcal{K})) + \beta^q(\nu_F(\mathcal{K})) \leq 1$ and $0 \leq \alpha^q+\beta^q \leq 1$, q ≥ 1 for all $\mathcal{K} \in Z$. A q-RLDFV based weight matrix $\mathcal{W}_j$ for the corresponding attributes is given in the form $\mathcal{W}_j = \{\mathcal{W}_1, \mathcal{W}_2, .., \mathcal{W}_j\}$ where $j = 1, 2, .., n$.

Let $\mathcal{W}_j^* = \{\mathcal{W}_1^*, \mathcal{W}_2^*, .., \mathcal{W}_j^*\}$ be the optimal weights obtained for the corresponding attributes based on the DE working principles, where $0 \leq \mathcal{W}_j \leq 1, 1 \leq j \leq n$ and $\sum_{i=1}^{n} \mathcal{W}_j = 1$.

In the evidential reasoning methodology, we consider H_1, H_2, and H to be the assessment grades for assessing the attributes of alternatives, where H_1 and H_2 respectively represent satisfying and not satisfying, and H represents indeterminacy. Let $\beta_{1,j}(\mathcal{K}_i)$ and $\beta_{2,j}(\mathcal{K}_i)$ denote the respective belief degrees of the assessment grades H_1 and H_2 for the attribute C_j corresponding to the alternatives $\mathcal{K}_i$, where $0 \leq \beta_{1,j}(\mathcal{K}_i) \leq 1, 0 \leq \beta_{2,j}(\mathcal{K}_i) \leq 1, 0 \leq \beta_{1,j}(\mathcal{K}_i) + \beta_{2,j}(\mathcal{K}_i) \leq 1, 1 \leq i \leq m$, and $1 \leq j \leq n$. We present the proposed approach below in a stepwise manner.

Step 1: The initial decision matrix D is converted into a granular bounded decision matrix $\overline{D} = (\mu_{ij}, \nu_{ij})_{mxn}$ and levigated bounded decision matrix $\underline{D} = (\alpha_{ij}, \beta_{ij})_{mxn}$ as mentioned in Tables 2 and 3 respectively.

$$\overline{D} = (\mu_{ij}, \nu_{ij})_{mxn} \tag{6}$$

$$\underline{D} = (\alpha_{ij}, \beta_{ij})_{mxn} \tag{7}$$

Step 2: For the granular bounded decision matrix $\overline{D} = (\mu_{ij}, \nu_{ij})_{mxn}$, the degree of belief $(\beta_{1,j}(\mathcal{K}_i), \beta_{2,j}(\mathcal{K}_i))$ for the j^{th} attribute of the i^{th} alternative is given as

$$(\beta_{1,j}(\mathcal{K}_i), \beta_{2,j}(\mathcal{K}_i)) = (\mu_{ij}(\mathcal{K}_i) * \alpha_{ij}^q(\mathcal{K}_i)), (\nu_{ij}(\mathcal{K}_i) * \beta_{ij}^q(\mathcal{K}_i))), q \geq 1 \tag{8}$$

Table 2. Granular bounded decision matrix $\overline{D} = (\mu_{ij}, \nu_{ij})_{m\text{x}n}$.

	C_1	C_2		C_n
$\mathcal{K}_1$	(μ_{11}, ν_{11})	(μ_{12}, ν_{12})	$\ldots$	$(\mu_{1n}, \nu_{1n}$
$\mathcal{K}_2$	$((\mu_{21}, \nu_{21})$	$((\mu_{12}, \nu_{22})$	$\ldots$	$((\mu_{2n}, \nu_{2n})$
$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$
$\mathcal{K}_m$	$((\mu_{m1}, \nu_{m1})$	$((\mu_{m2}, \nu_{m2})$	$\ldots$	$((\mu_{mn}, \nu_{mn})$

Table 3. Levigated bounded decision matrix $\underline{D} = (\alpha_{ij}, \beta_{ij})_{m\text{x}n}$.

	C_1	C_2		C_n
$\mathcal{K}_1$	$(\alpha_{ij}, \beta_{ij})$	$(\alpha_{ij}, \beta_{ij})$	$\ldots$	$(\alpha_{ij}, \beta_{ij})$
$\mathcal{K}_1$	$(\alpha_{21}, \beta_{21})$	$(\alpha_{22}, \beta_{22})$	$\ldots$	$(\alpha_{2n}, \beta_{2n})$
$\vdots$	$\vdots$	$\vdots$	$\ldots$	$\vdots$
$\mathcal{K}_m$	$(\alpha_{m1}, \beta_{m1})$	$(\alpha_{m2}, \beta_{m2})$	$\ldots$	$(\alpha_{mn}, \beta_{mn})$

Step 2.1: Belief structure $\beta_{q,j}(\mathcal{K}_i)$ for the attribute C_j of alternative $\mathcal{K}_i)$ is transformed to basic probability mass $(m_{q,j}(\mathcal{K}_i))$ and remaining probability mass $(m_{H,j}(\mathcal{K}_i))$ using (9) and (10) as mentioned below.

$$m_{q,j}(\mathcal{K}_i) = w_j^* * \beta_{q,j}(\mathcal{K}_i) \tag{9}$$

$$m_{H,j}(\mathcal{K}_i) = 1 - \sum_{q=1}^{p} m_{q,j}(\mathcal{K}_i) \tag{10}$$

Step 2.2: Combined probability mass $m_{q,I(y)}$ and remaining combined probability mass $m_{H,I(y)}$ are computed using (11) and (12). Initially, it is considered $m_{q,I(1)} = m_{q,1}$, and $m_{H,I(1)} = m_{H,1}$ where $1 \le i \le m$ and $2 \le y \le n$.

$$\begin{aligned} m_{q,I(y)}(\mathcal{K}_i) = R_{I(y)}(\mathcal{K}_i)\big(&m_{q,I(y-1)}(\mathcal{K}_i)m_{q,y}(\mathcal{K}_i) + m_{q,I(y-1)}(\mathcal{K}_i)m_{H,y}(\mathcal{K}_i) + \\ &m_{H,I(y-1)}(\mathcal{K}_i)m_{q,y}(\mathcal{K}_i)\big) \end{aligned} \tag{11}$$

$$m_{H,I(y)}(\mathcal{K}_i) = R_{I(y)}(\mathcal{K}_i)m_{H,I(y-1)}(\mathcal{K}_i)m_{H,y}(\mathcal{K}_i) \tag{12}$$

where $R_{I(y)}(\mathcal{K}_i) = (1 - \sum_{t=1}^{2}\sum_{g=1 g\neq t}^{2} m_{t,I(y-1)}(\mathcal{K}_i)\ m_{H,y}(\mathcal{K}_i))^{-1}$, $2 \le y \le n, 1 \le i \le m$ and $1 \le q \le 2$ is the normalized factor, and $m_{q,I(1)} = m_{q,1}$, and $m_{H,I(1)} = m_{H,1}$.

Step 2.3: Belief degree $\beta_{q,j}(\mathcal{K}_i)$ is obtained by aggregating the evaluation values combined probability mass $m_{q,I(y)}(\mathcal{K}_i)$ and remaining combined probability mass $m_{H,I(y)}(\mathcal{K}_i)$ for the j^{th} attribute of the alternative $(\mathcal{K}_i)$ using (13).

$$\beta_q(\mathcal{K}_i) = 1 - \beta_H(\mathcal{K}_i)\frac{(m_{q,I(n)}(\mathcal{K}_i))}{1 - m_{(H,I(n))}(\mathcal{K}_i)} \tag{13}$$

Here $\beta_H(\mathcal{K}_i) = \sum_{j=1}^{n} w_j(1 - \sum_{q=1}^{2}\beta_{q,j}(\mathcal{K}_i))$, $1 \le i \le m$, and $1 \le j \le n$.
In order to represent the respective evaluation grades H_1 and H_2, belief degree $\beta_q(\mathcal{K}_i)$ is termed as $\overline{\beta_1}(\mathcal{K}_i)$ and $\overline{\beta_2}(\mathcal{K}_i)$.

step 3: For the levigated bounded decision matrix $\underline{D} = (\alpha_{ij}, \beta_{ij})_{m\text{x}n}$, the degree of belief structure $(\beta_{1,j}(\mathcal{K}_i), \beta_{2,j}(\mathcal{K}_i))$ for the j^{th} attribute of the i^{th} alternative is given as

$$(\beta_{1,j}(\mathcal{K}_i), \beta_{2,j}(\mathcal{K}_i)) = (\alpha_{ij}^q(\mathcal{K}_i), \beta_{ij}^q(\mathcal{K}_i)), q \ge 1 \tag{14}$$

Step 3.1: Belief structure $\beta_{q,j}(\mathcal{K}_i)$ for the attribute C_j of alternative $\mathcal{K}_i)$ is transformed to basic probability mass $(m_{q,j}(\mathcal{K}_i))$ and remaining probability mass $(m_{H,j}(\mathcal{K}_i))$ using (15) and (16) as mentioned below.

$$m_{q,j}(\mathcal{K}_i) = w_j^* \beta_{q,j}(\mathcal{K}_i) \quad (15)$$

$$m_{H,j}(\mathcal{K}_i) = 1 - \sum_{q=1}^{p} m_{q,j}(\mathcal{K}_i) \quad (16)$$

Step 3.2: Combined probability mass $m_{q,I(y)}$ and remaining combined probability mass $m_{H,I(y)}$ are computed using (17) and (18). Initially, it is considered $m_{q,I(1)} = m_{q,1}$, and $m_{H,I(1)} = m_{H,1}$ where $1 \leq i \leq m$ and $2 \leq y \leq n$.

$$m_{q,I(y)}(\mathcal{K}_i) = R_{I(y)}(\mathcal{K}_i)\big(m_{q,I(y-1)}(\mathcal{K}_i)m_{q,y}(\mathcal{K}_i) + m_{q,I(y-1)}(\mathcal{K}_i)m_{H,y}(\mathcal{K}_i) + m_{H,I(y-1)}(\mathcal{K}_i)m_{q,y}(\mathcal{K}_i)\big) \quad (17)$$

$$m_{H,I(y)}(\mathcal{K}_i) = R_{I(y)}(\mathcal{K}_i)m_{H,I(y-1)}(\mathcal{K}_i)m_{H,y}(\mathcal{K}_i) \quad (18)$$

where $R_{I(y)}(\mathcal{K}_i) = (1 - \sum_{t=1}^{2}\sum_{g=1 g\neq t}^{2} m_{t,I(y-1)}(\mathcal{K}_i)\ m_{H,y}(\mathcal{K}_i))^{-1}$, $2 \leq y \leq n, 1 \leq i \leq m$ and $1 \leq q \leq 2$ is the normalized factor, and $m_{q,I(1)} = m_{q,1}$, and $m_{H,I(1)} = m_{H,1}$.

Step 3.3: Belief degree $\beta_{q,j}(\mathcal{K}_i)$ is obtained by aggregating the evaluation values combined probability mass $m_{q,I(y)}(\mathcal{K}_i)$ and remaining combined probability mass $m_{H,I(y)}(\mathcal{K}_i)$ for the j^{th} attribute of the alternative $(\mathcal{K}_i)$ using (19).

$$\beta_q(\mathcal{K}_i) = 1 - \beta_H(\mathcal{K}_i)\frac{(m_{q,I(n)}(\mathcal{K}_i))}{1 - m_{(H,I(n))}(\mathcal{K}_i)} \quad (19)$$

Here $\beta_H(\mathcal{K}_i) = \sum_{j=1}^{n} w_j(1 - \sum_{q=1}^{2}\beta_{q,j}(\mathcal{K}_i))$, $0 \leq i \leq 1, 0 \leq j \leq 1$.
In order to represent the respective evaluation grades H_1 and H_2, belief degree $\beta_q(\mathcal{K}_i)$ is termed as $\underline{\beta_1}(\mathcal{K}_i)$ and $\underline{\beta_2}(\mathcal{K}_i)$.

Step 4: The belief degrees, $\overline{\beta_1}(\mathcal{K}_i), \overline{\beta_2}(\mathcal{K}_i)$ and $\underline{\beta_1}(\mathcal{K}_i)$, $\underline{\beta_2}(\mathcal{K}_i)$ obtained in Steps 2.3 and 3.3 mentioned above are combined to form aggregated q-RLDFV $((\overline{\beta_1}(\mathcal{K}_i), \overline{\beta_2}(\mathcal{K}_i)), (\underline{\beta_1}(\mathcal{K}_i), \underline{\beta_2}(\mathcal{K}_i)))$, where $0 \leq \overline{\beta_1}(\mathcal{K}_i) \leq 1, 0 \leq \overline{\beta_2}(\mathcal{K}_i) \leq 1, 0 \leq \underline{\beta_1}(\mathcal{K}_i) \leq 1, 0 \leq \underline{\beta_2}(\mathcal{K}_i) \leq 1, 1 \leq i \leq m.$

Step 5: Finally, the score values of the alternatives are computed based on the aggregated q-RLDFS $((\overline{\beta_1}(\mathcal{K}_i), \overline{\beta_2}(\mathcal{K}_i)), (\underline{\beta_1}(\mathcal{K}_i), \underline{\beta_2}(\mathcal{K}_i)))$ using one of the score functions such as expectation score function (ESF), quadratic score function (QSF), and score function (SF), referred in Eq. (20), (21), and (22), where ESF $(\mathcal{K}_i) \in$ [0, 1] and QSF and SF $(\mathcal{K}_i) \in$ [-1, 1].

$$ESF(\mathcal{K}_i) = [\frac{(\mu_F(\mathcal{K}_i) - \nu_F(\mathcal{K}_i) + 1)}{4} + \frac{(\alpha^q(\mathcal{K}_i) - \beta^q(\mathcal{K}_i) + 1)}{4}], \quad (20)$$

where q $\geq$ 1 and $1 \leq i \leq 2$

$$QSF(\mathcal{K}_i) = [\frac{((\mu_F(\mathcal{K}_i))^2 - (\nu_F(\mathcal{K}_i))^2) + ((\alpha^q(\mathcal{K}_i))^2 - (\beta^q(\mathcal{K}_i))^2)}{2}] \quad (21)$$

where q $\geq$ 1 and $1 \leq i \leq 2$.

$$SF(\mathcal{K}_i) = [\frac{(\mu_F(\mathcal{K}_i) - \nu_F(\mathcal{K}_i)) + (\alpha^q(\mathcal{K}_i) - \beta^q(\mathcal{K}_i))}{2}] \quad (22)$$

where q $\geq$ 1 and $1 \leq i \leq 2$.
The alternative with the higher score value is selected as better one.

§4 Case Study

This section illustrates the proposed approach using two real-life examples concerned with COVID-19 emergency decision-making system and sustainable energy planning followed by a numerical example.

4.1 COVID-19 emergency decision-making

This case study shows the implementation of the proposed approach for emergency decision-making in the public health care system. The proposed method with q-RLDFS works as an emergency decision support system, which suggests an optimal choice for the prevention of coronavirus and overcomes the spread of COVID-19 disease outbreaks. The problem information is represented as the input to the decision matrix D_1 using q-RLDFS with 4 alternatives and 5 criteria.

Let $\mathcal{K} = \{\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3, \mathcal{K}_4\}$={ medical-aid approach online, vaccination, governments mandate, medical support} be the set of four alternatives and C= $\{C_1, C_2, C_3, C_4, C_5\}$={Medical adherence, Awareness and protective gear, Expertise, Travel restrictions, Universal uncertainty} be the set of five attributes associated with each of the alternatives. A brief description of the alternatives and attributes are given below in Table 4 and Table 5 respectively.

Among these alternatives, the better alternative will be selected based on five attributes C=

Table 4. Description of the alternatives.

Symbol	Alternatives	Description
$\mathcal{K}_1$	medical-aid approach online	Sets people aware and inform them through online sources such as the WHO, the Higher Medical Institutes, the health ministry, and UNICEF in the context of COVID circumsta -nces.
$\mathcal{K}_2$	vaccination	COVID-19 vaccination is now accessible as an emergency ap -proval that enhances your protection against covetous treat -ment.
$\mathcal{K}_3$	Governments mandate	The directives given by the Govt. are vital, such as enforcing lockdowns, restricting crowd gatherings, providing quarantine centers and other social measures, etc. Following the orders can ensure no spread of the virus from person to person.
$\mathcal{K}_4$	medical support	Medical support provides safety to all the individuals with WHO recommended facial shields, gloves, PPEs, hand saniti -zers, and additional safeguards in these circumstances.

$\{C_1, C_2, C_3, C_4, C_5\}$ for precise and efficient decision-making by three decision makers $D_l = \{D_1, D_2, D_3\}$, where $1 \le i \le 4, 1 \le n \le 5$ and $1 \le l \le 3$. Opinions of the decision makers regarding the alternatives and attributes are represented using q-RLDFS which are shown using the decision matrices D_l $(l = 1, 2, 3)$ as shown below in Tables 6, 7, and 8 [10] respectively.

The weights assigned for the attributes are $\mathcal{W}_j = \{0.32, 0.27, 0.17, 0.14, 0.1\}$. Based on the input decision matrix and weights of the attributes, we obtain the optimal weights $\mathcal{W}_j^* = \{0.29, 0.23, 0.2, 0.19, 0.09\}$ based on the working principles of differential evolution optimization techniques.

Step 1: The initial decision matrix D_1 is converted into granular bounded decision matrix $\overline{D_1} = (\mu_{ij}, \nu_{ij})_{mxn}$ and levigated bounded decision matrix $\underline{D_1} = (\alpha_{ij}, \beta_{ij})_{mxn}$ as mentioned in

Table 5. Description of the attributes.

Symbol	Attributes	Description
C_1	Medical adherence	As the vaccines are being available to boost our immunity to fight against COVID-19, it is also necessary to undergo clinical treatme -nt in this challenging circumstances, particularly in the treatment of symptomatic patients with fever, cough, sore throat, respiratory issues, etc. for prevention or control measure.
C_2	Awareness and protective gear	To be safe, either the individual should have complete awareness about the disease by following an online nmedical-aid course and staying away from people who have COVID symptoms, wearing a mask, frequent hand wash and proper sanitization, and maintaining social distances.
C_3	Expertise	Doctors and scientists specializedin respective domains contributing towards the vaccination to prevent human from infections.
C_4	Travel restrictions	The viral transmission can occur through the transport system via the symptomatic or asymptomatic infections of people in one place. additional safeguards in these circumstances.
C_5	Universal uncertainty	COVID-19 already has a worldwide impact. It has affected most of the countries, notably healthcare, transport, trading, and tourism, and damaged their economic progress.

Table 6. Decision matrix D_1 $((\mu_F(\mathcal{K}_i), \nu_F(\mathcal{K}_i)),(\alpha, \beta))$, $q = 4$.

	C_1	C_2	C_3	C_4	C_5
$\mathcal{K}_1$	((.8,.9), (.85,.7))	((.9,.88), (.8,0.7))	((1,1), (.7,.8))	((.9,1), (.9,.6))	((.8,.9), (.7,.85))
$\mathcal{K}_2$	((1,1), (.7,.8))	((.98,.89), (.8,.7))	((.87,.95), (.8,.7))	((.82,1), (.8,.7))	((1,.9), (.6,.9))
$\mathcal{K}_3$	((.8,.7), (.6,.9))	((1,.8), (.7,.8))	((.94,.84), (.8,.7))	((.9,1), (.8,.7))	((.88,.96), (.8,.7))
$\mathcal{K}_4$	((.9,1), (.9,.6))	((1,.9), (.7,.85))	((.95,.88), (.8,.7))	((.9,.8), (.7,.85))	((..9,1), (.85,.7))

Table 7. Decision matrix D_2 $((\mu_F(\mathcal{K}_i), \nu_F(\mathcal{K}_i)),(\alpha, \beta))$, $q = 4$.

	C_1	C_2	C_3	C_4	C_5
$\mathcal{K}_1$	((1,1), (.7,.8))	((.9,.1), (.8,0.7))	((.95,.85), (.7,.8))	((.88,95), (.8,.7))	((.8,.95), (.7,.85))
$\mathcal{K}_2$	((.99,.88), (.7,.8))	((1,1), (.8,.7))	((.8,.1), (.8,.7))	((1,.9), (.7,.8))	((1,.9), (.6,.9))
$\mathcal{K}_3$	((.85,.88), (.6,.9))	((.8,.9), (.85,.7))	((.95,.85), (.7,.8))	((.9,1), (.8,.7))	((.88,.96), (.8,.7))
$\mathcal{K}_4$	((1,1), (.7,.8))	((.9,.1), (.8,.7))	((.95,.9), (.8,.7))	((.87,.95), (.87,.7))	((.9,1), (.9,.6))

Tables 9 and 10 respectively.

Table 8. Decision matrix D_3 $((\mu_F(\mathcal{K}_i), \nu_F(\mathcal{K}_i)),(\alpha, \beta))$, $q = 4$.

	C_1	C_2	C_3	C_4	C_5
$\mathcal{K}_1$	((.9,.1), (.8,.7))	((.88,.9), (.6,.9))	((.8,.9), (.7,.85))	((.85,.87), (.7,.85))	((.95,.85), (.7,.8))
$\mathcal{K}_2$	((1,.9), (.7,.8))	((.95,.78), (.8,.78))	((.85,1), (.7,.8))	((1,.8), (.7,.8))	((1,.9), (.6,.9))
$\mathcal{K}_3$	((.8,1), (.8,.7))	((1,1), (.7,.8))	((.8,.9), (.7,.85))	((.87,.95), (.8,.7))	((.88,.96), (.8,.7))
$\mathcal{K}_4$	((.9,.87), (.6,.9))	((1,.9), (.6,.9))	((1,1), (.8,.7))	((.98,.89), (.7,.8))	((1,.9), (.6,.9))

Table 9. Granular bounded decision matrix $\overline{D_1} = (\mu_{ij}, \nu_{ij})_{mxn}$.

	C_1	C_2	C_3	C_4	C_5
$\mathcal{K}_1$	(.8,.9)	(.9,.88)	(1,1)	(.9,1)	(.8,.9)
$\mathcal{K}_2$	(1,1)	(.98,.89)	(.87,.95)	(.82,1)	(1,.9)
$\mathcal{K}_3$	(.8,.7)	(1,.8)	(.94,.84)	(.9,1)	(.88,.96)
$\mathcal{K}_4$	(.9,1)	(1,.9)	(.95,.88)	(.9,.8)	(..9,1)

Step 2: For the granular bounded decision matrix $\overline{D_1} = (\mu_{ij}, \nu_{ij})_{mxn}$, the degree of belief $(\beta_{1,j}(\mathcal{K}_i), \beta_{2,j}(\mathcal{K}_i))$ for the j^{th} attribute of the i^{th} alternative is computed using (8) and shown in Table 11.

Table 10. Levigated bounded decision matrix $\underline{D_1} = (\alpha_{ij}, \beta_{ij})_{mxn}$.

	C_1	C_2	C_3	C_4	C_5
$\mathcal{K}_1$	(.85,.7)	(.8,0.7)	(.7,.8)	(.9,.6)	(.7,.85)
$\mathcal{K}_2$	(.7,.8)	(.8,.7)	(.8,.7)	(.8,.7)	((.6,.9)
$\mathcal{K}_3$	(.6,.9)	(.7,.8)	(.8,.7)	(.8,.7)	(.8,.7)
$\mathcal{K}_4$	(.9,.6)	(.7,.85)	(.8,.7)	(.7,.85)	(.85,.7)

Table 11. Degree of belief $(\beta_{1,j}(\mathcal{K}_i), \beta_{2,j}(\mathcal{K}_i))$ of $\overline{D_1}$.

	C_1	C_2	C_3	C_4	C_5
$\mathcal{K}_1$	(.41,.21)	(.36,.21)	(.24,.40)	(.59,.12)	(.19,.46)
$\mathcal{K}_2$	(.24,.40)	(.40,.21)	(.35,.22)	(.33,.24)	(.12,.59)
$\mathcal{K}_3$	(.10,.45)	(.24,.32)	(.38,.20)	(.36,.24)	(.36,.23)
$\mathcal{K}_4$	(.59,.12)	(.24,.46)	(.38,.21)	(.21,.41)	(.46,.24)

Based on $\beta_{q,j}(\mathcal{K}_i)$ and the obtained optimal weights by the programming model of DEA, we aggregate the evaluation values of the attributes of the alternatives($\mathcal{K}_i$) using (9) to (13) from steps 2.1 to 2.3 in order to obtain the belief degrees $\beta_q(\mathcal{K}_i) = (\overline{\beta_1}(\mathcal{K}_1), \overline{\beta_2}(\mathcal{K}_1)) =$ (0.73,0.79), $(\overline{\beta_1}(\mathcal{K}_2), \overline{\beta_2}(\mathcal{K}_2)) =$ (0.79,0.74), $(\overline{\beta_1}(\mathcal{K}_3), \overline{\beta_2}(\mathcal{K}_3)) =$ (0.75,0.70), $(\overline{\beta_1}(\mathcal{K}_4), \overline{\beta_2}(\mathcal{K}_4)) =$ (0.76,0.84).

Step 3: For the levigated bounded decision matrix $\underline{D_1} = (\alpha_{ij}, \beta_{ij})_{mxn}$, the degree of belief $(\beta_{1,j}(\mathcal{K}_i), \beta_{2,j}(\mathcal{K}_i))$ for the j^{th} attribute of the i^{th} alternative is computed using (14) and shown in Table 12.

Based on $\beta_{q,j}(\mathcal{K}_i)$ and the obtained optimal weights by the programming model of DEA,

Table 12. Degree of belief $(\beta_{1,j}(\mathcal{K}_i), \beta_{2,j}(\mathcal{K}_i))$ of $\underline{D_1}$.

	C_1	C_2	C_3	C_4	C_5
$\mathcal{K}_1$	(.52,.24)	(.40,.24)	(.24,.40)	(.65,.12)	(.24,.52)
$\mathcal{K}_2$	(.24,.40)	(.40,.24)	(.40,.24)	(.40,.24)	(.12,.65)
$\mathcal{K}_3$	(.12,.65)	(.24,.40)	(.40,.24)	(.40,.24)	(.40,.24)
$\mathcal{K}_4$	(.65,.12)	(.24,.52)	(.40,.24)	(.24,.52)	(.52,.24)

Table 13. Score values of the COVID-19 alternatives.

Score functions	$\mathcal{K}_1$	$\mathcal{K}_2$	$\mathcal{K}_3$	$\mathcal{K}_4$
ESF	0.491	0.470	0.483	0.501
SF	-0.016	-0.059	-0.0038	0.003
QSF	-0.029	-0.105	-0.058	0.006

we aggregate the evaluation values of the attributes of the alternatives($\mathcal{K}_i$) using (15) to (19) from steps 3.1 to 3.3 in order to obtain the belief degrees $\beta_q(\mathcal{K}_i) = (\underline{\beta_1}(\mathcal{K}_i), \underline{\beta_2}(\mathcal{K}_i)) =$ (0.73,0.79), $(\underline{\beta_1}(\mathcal{K}_i), \underline{\beta_2}(\mathcal{K}_i)) =$ (0.79,0.74), $(\underline{\beta_1}(\mathcal{K}_i), \underline{\beta_2}(\mathcal{K}_i)) =$ (0.75,0.70), and $(\underline{\beta_1}(\mathcal{K}_i), \underline{\beta_2}(\mathcal{K}_i))$ = (0.76,0.84).

Step 4: The belief degrees, $\overline{\beta_1}(\mathcal{K}_i), \overline{\beta_2}(\mathcal{K}_i)$ and $\underline{\beta_1}(\mathcal{K}_i), \underline{\beta_2}(\mathcal{K}_i)$ obtained in Steps 2.3 and 3.3 are combined to form aggregated q-RLDFV $((\overline{\beta_1}(\mathcal{K}_i), \overline{\beta_2}(\mathcal{K}_i)), (\underline{\beta_1}(\mathcal{K}_i), \underline{\beta_2}(\mathcal{K}_i)))$, we get

$$((\overline{\beta_1}(\mathcal{K}_1), \overline{\beta_2}(\mathcal{K}_1)),(\underline{\beta_1}(\mathcal{K}_1), \underline{\beta_2}(\mathcal{K}_1))) = ((0.73, 0.79), (0.79, 0.85)),$$
$$((\overline{\beta_1}(\mathcal{K}_2), \overline{\beta_2}(\mathcal{K}_2)),(\underline{\beta_1}(\mathcal{K}_2), \underline{\beta_2}(\mathcal{K}_2))) = ((0.79, 0.74), (0.82, 0.77)),$$
$$((\overline{\beta_1}(\mathcal{K}_3), \overline{\beta_2}(\mathcal{K}_3)),(\underline{\beta_1}(\mathcal{K}_3), \underline{\beta_2}(\mathcal{K}_3))) = ((0.75, 0.70), (0.84, 0.76)),$$
$$((\overline{\beta_1}(\mathcal{K}_4), \overline{\beta_2}(\mathcal{K}_4)),(\underline{\beta_1}(\mathcal{K}_4), \underline{\beta_2}(\mathcal{K}_4))) = ((0.76, 0.84), (0.81, 0.88)).$$

Similarly, the combined belief degrees for the decision matrices D_2 and D_3 in the form of aggregated q-RLDFVs are obtained by the same process respectively, we get

$$((\overline{\beta_1}(\mathcal{K}_1), \overline{\beta_2}(\mathcal{K}_1)),(\underline{\beta_1}(\mathcal{K}_1), \underline{\beta_2}(\mathcal{K}_1))) = ((0.84, 0.82), (0.81, 0.77)),$$
$$((\overline{\beta_1}(\mathcal{K}_2), \overline{\beta_2}(\mathcal{K}_2)),(\underline{\beta_1}(\mathcal{K}_2), \underline{\beta_2}(\mathcal{K}_2))) = ((0.86, 0.87), (0.82, 0.78)),$$
$$((\overline{\beta_1}(\mathcal{K}_3), \overline{\beta_2}(\mathcal{K}_3)),(\underline{\beta_1}(\mathcal{K}_3), \underline{\beta_2}(\mathcal{K}_3))) = ((0.86, 0.80), (0.81, 0.82)),$$
$$((\overline{\beta_1}(\mathcal{K}_4), \overline{\beta_2}(\mathcal{K}_4)),(\underline{\beta_1}(\mathcal{K}_4), \underline{\beta_2}(\mathcal{K}_4))) = ((0.81, 0.75), (0.86, 0.86)).$$

$$((\overline{\beta_1}(\mathcal{K}_1), \overline{\beta_2}(\mathcal{K}_1)),(\underline{\beta_1}(\mathcal{K}_1), \underline{\beta_2}(\mathcal{K}_1))) = ((0.85, 0.88), (0.69, 0.76)),$$
$$((\overline{\beta_1}(\mathcal{K}_2), \overline{\beta_2}(\mathcal{K}_2)),(\underline{\beta_1}(\mathcal{K}_2), \underline{\beta_2}(\mathcal{K}_2))) = ((0.83, 0.87), (0.71, 0.75)),$$
$$((\overline{\beta_1}(\mathcal{K}_3), \overline{\beta_2}(\mathcal{K}_3)),(\underline{\beta_1}(\mathcal{K}_3), \underline{\beta_2}(\mathcal{K}_3))) = ((0.76, 0.78), (0.74, 0.80)),$$
$$((\overline{\beta_1}(\mathcal{K}_4), \overline{\beta_2}(\mathcal{K}_4)),(\underline{\beta_1}(\mathcal{K}_4), \underline{\beta_2}(\mathcal{K}_4))) = ((0.90, 0.93), (0.68, 0.73)).$$

All the combined belief degrees obtained in the previous steps for D_1, D_2, and D_3 are aggregated. The optimal weight vector $\mathcal{W}_j^* = \{0.4,\ 0.352,\ 0.248\}$ is obtained by the working principles of DEA for the corresponding attributes of the alternatives for the combined belief degree matrix. The final evaluated q-RLDFV values of the alternatives are,

$$((\overline{\beta_1}(\mathcal{K}_1), \overline{\beta_2}(\mathcal{K}_1)),(\underline{\beta_1}(\mathcal{K}_1), \underline{\beta_2}(\mathcal{K}_1))) = ((0.86, 0.95), (0.88, 0.96)),$$
$$((\overline{\beta_1}(\mathcal{K}_2), \overline{\beta_2}(\mathcal{K}_2)),(\underline{\beta_1}(\mathcal{K}_2), \underline{\beta_2}(\mathcal{K}_2))) = ((0.84, 0.94), (0.91, 0.96)),$$
$$((\overline{\beta_1}(\mathcal{K}_3), \overline{\beta_2}(\mathcal{K}_3)),(\underline{\beta_1}(\mathcal{K}_3), \underline{\beta_2}(\mathcal{K}_3))) = ((0.84, 0.94), (0.88, 0.95)),$$
$$((\overline{\beta_1}(\mathcal{K}_4), \overline{\beta_2}(\mathcal{K}_4)),(\underline{\beta_1}(\mathcal{K}_4), \underline{\beta_2}(\mathcal{K}_4))) = ((0.92, 0.99), (0.92, 0.99)).$$

Step 5: Finally, we calculate the score values to prioritize the alternatives for decision-making based on combined belief degrees obtained at step 4 and one of the score functions shown in Eq. (20), (21), and (22).

The final order of ranking of the alternatives according to the largest value given by the score functions is $\mathcal{K}_4 > \mathcal{K}_1 > \mathcal{K}_3 > \mathcal{K}_2$. So the medical support alternative $\mathcal{K}_4$ be the better choice to be safe and avoid any sort of viral infections from the COVID-19 disease. Our experimentation shows that the ranking of the COVID-19 alternatives shown in Table 13 and also graphically shown in Fig. 5 using all the three score functions (ESF, QSF, and SF) are similar.

4.2 Sustainable energy planning management

Energy sources such as inexhaustible and unsustainable energy assets play a significant part in the development of the world economy and ecological equilibrium. Energy resources like electricity, thermal, tidal, wind energies, and so forth are likewise important for ecological balance. Specifically, electricity is one of the most extensive energy resources in different sectors.

This section shows the implementation of the proposed approach for emergency decision-

Table 14. Description of the alternatives.

Symbol	Description	Resources
$\mathcal{K}_1$	In this circumstance, the selection of clean coal is preferable.	Indigenous coal, fuel, gas, etc.
$\mathcal{K}_2$	In this case, the parameters of produc -tivity and preservation are examined.	Production strategy and capacity recycling
$\mathcal{K}_3$	In this case, the current state policy and plan are followed	In accordance with the state's goals and strategies.
$\mathcal{K}_4$	This choice favors sustainable energy and technology.	Environmental energy, hydro -electricity, sunlight, wind, and bio -mass sources

Table 15. A q-RLDFS Decision matrix D=$((\mu_F(\mathcal{K}_i),\ \nu_F(\mathcal{K}_i)),(\alpha,\ \beta))$, $q = 4$.

	C_1	C_2	C_3	C_4
$\mathcal{K}_1$	((0.73,0.41), (0.31,0.13))	((0.63,0.53), (0.13,0.23))	((0.73,0.41), (0.23,0.15))	((0.63,0.53), (0.31,0.36))
$\mathcal{K}_2$	((0.71,0.34), (0.51,0.31))	((0.63,0.51), (0.43,0.39))	((0.71,0.41), (0.31,0.41))	((0.69,.38), (0.41,0.31))
$\mathcal{K}_3$	((0.63,0.59), (0.41,0.31))	((0.78,0.43), (0.38,0.41))	((0.63,0.48), (0.28,0.17))	((0.58,0.49), (0.31,0.42))
$\mathcal{K}_4$	((0.81,0.58), (0.49,0.31))	((0.73,0.68), (0.43,0.49))	((0.69,0.73), (0.31,0.31))	((0.68,0.51), (0.43,0.21))

making in sustainable energy planning decision management. The proposed method with q-RLDFS works as a decision-making model, which suggests an optimal choice for the selection of energy policies. The problem information is represented by the decision makers using the decision matrix D based on q-RLDFS with four alternatives and four criteria which is shown in Table 15 [15]. Let $\mathcal{K}_i = \{\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3, \mathcal{K}_4\}$ be the set of four alternatives shown in Table 14. Among these alternatives, the better alternative will be selected based on four factors or criteria C_n= $\{C_1,\ C_2,\ C_3,\ C_4\}$ for precise and efficient decision-making by decision maker, where 1 $\leq$ i $\leq$ 4, 1 $\leq$ n $\leq$ 4. The criteria information is given below. C_1: Carbon dioxide emission: This parameter characterizes the carbon dioxide outflows and the expense identified with squander treatment.
C_2: Risk: It estimates the likelihood of failure.
C_3: Feasibility: This criterion determines the application of probability in the energy scenario.
C_4: Investment cost: This criterion comprises the cost of equipment, personnel, building, and infrastructure throughout the mounting of a power plant.

It is considered that for the decision matrix D, there are no weight constraints given by the decision makers, so we directly obtain the optimal weights of the attributes by executing the programming model based on the working principles of differential evolution optimization techniques. The optima weight vector $\mathcal{W}_j^* = \{0.4,\ 0.25,\ 0.14,\ 0.2\}$. For the decision matrix D given by decision maker, the combined belief degrees $\overline{\beta_1}(\mathcal{K}_i)$ $\overline{\beta_2}(\mathcal{K}_i)$ and $\underline{\beta_1}(\mathcal{K}_i), \underline{\beta_2}(\mathcal{K}_i)$ are evaluated based on steps 2.2 and 3.3 given in (13) and (14) respectively in section 3. The evaluated combined belief degrees for the decision matrix given by decision maker D are computed as $((\overline{\beta_1}(\mathcal{K}_1), \overline{\beta_2}(\mathcal{K}_1)),\ (\underline{\beta_1}(\mathcal{K}_1), \underline{\beta_2}(\mathcal{K}_1)))$ = ((0.3507, 0.4198), (0.6559, 0.5907), $((\overline{\beta_1}(\mathcal{K}_2), \overline{\beta_2}(\mathcal{K}_2)),$

$(\underline{\beta_1}(\mathcal{K}_2), \underline{\beta_2}(\mathcal{K}_2)))$ = ((0.2147, 0.3128), (0.8253, 0.7533)), $((\overline{\beta_1}(\mathcal{K}_3), \overline{\beta_2}(\mathcal{K}_3)),\ (\underline{\beta_1}(\mathcal{K}_3), \underline{\beta_2}(\mathcal{K}_3)))$ = ((0.4079, 0.4925), (0.6165, 0.5498)), $((\overline{\beta_1}(\mathcal{K}_4), \overline{\beta_2}(\mathcal{K}_4)),\ (\underline{\beta_1}(\mathcal{K}_4), \underline{\beta_2}(\mathcal{K}_4)))$ = ((0.3313, 0.3732), (0.7189, 0.6957)). Finally, we calculate the score values to prioritize the alternatives for decision-making based on the obtained final combined belief degrees and one of the score functions shown in (20), (21), and (22).

The final order of ranking of the alternatives according to the score functions is $\mathcal{K}_3 > \mathcal{K}_1 > \mathcal{K}_4 > \mathcal{K}_2$, which has been clear shown in Table 16 and Fig. 6. So, the alternative $\mathcal{K}_3$ be the better energy policy for the sustainable energy planning management.

Table 16. Score values of the energy planning policies.

Score functions	$\mathcal{K}_1$	$\mathcal{K}_2$	$\mathcal{K}_3$	$\mathcal{K}_4$
ESF	0.401	0.269	0.439	0.349
SF	-0.197	-0.461	-0.120	-0.301
QSF	-0.160	-0.369	-0.109	-0.230

4.3 Numerical Example

Assume a student aspires to communicate and publish a research article in a renowned publication with a high impact factor. His two research supervisors have also picked four journals based on the needed criteria. The suggested MADM technique employing q-RLDFS can be applied to select the best journal among the four journals for the student required for publishing.

Let $\mathcal{K} = \{\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3, \mathcal{K}_4\}$ be the four journals selected by the student and his two supervisors for publishing the research article, where $\mathcal{K}_1$ is the journal one, $\mathcal{K}_2$ is the journal two and likewise. Assume the decision makers represented the input decision matrix D in q-RLDFS as shown in Table 17 [9] with n criteria (student and supervisors) and m alternatives (journals).

For the given input decision matrix, there are no weight constraints, so directly we obtain the optimal weights by executing the programming model based on the working principles of differential evolution optimization techniques.

For the decision matrix D given by decision maker, the combined belief degrees $((\overline{\beta_1}(\mathcal{K}_1), \overline{\beta_2}(\mathcal{K}_1)),\ (\underline{\beta_1}(\mathcal{K}_1), \underline{\beta_2}(\mathcal{K}_1)))$ are evaluated based on the steps 1-4. The aggregated combined belief degrees for the decision matrix D are $((\overline{\beta_1}(\mathcal{K}_1), \overline{\beta_2}(\mathcal{K}_1)),\ (\underline{\beta_1}(\mathcal{K}_1), \underline{\beta_2}(\mathcal{K}_1)))$ = ((0.35, 0.41), (.6559,0.5907), $((\overline{\beta_1}(\mathcal{K}_2), \overline{\beta_2}(\mathcal{K}_2)),\ (\underline{\beta_1}(\mathcal{K}_2), \underline{\beta_2}(\mathcal{K}_2)))$ = ((0.2147, 0.3128), (0.8253, 0.7533)), $((\overline{\beta_1}(\mathcal{K}_3), \overline{\beta_2}(\mathcal{K}_3)),\ (\underline{\beta_1}(\mathcal{K}_3), \underline{\beta_2}(\mathcal{K}_3)))$ = ((0.4079, 0.4925), (0.6165, 0.5498)), $((\overline{\beta_1}(\mathcal{K}_4), \overline{\beta_2}(\mathcal{K}_4)),\ (\underline{\beta_1}(\mathcal{K}_4), \underline{\beta_2}(\mathcal{K}_4)))$ = ((0.3313, 0.3732), (0.7189, 0.6957)).

Finally, we calculate the score values which are shown in Table 18 to choose the journal for the publication based on obtained final combined belief degrees and one of the score functions shown in (20), (21), and (22). It is noted that the ranking of the journals for all the three score functions are similar, which clearly shows in Fig. 7.

Table 17. Decision matrix D with student and two supervisors opinions.

	Student	Supervisor-1	Supervisor-2
$\mathcal{K}_1$	((73,.68), (.41, .37))	((.83,.43), (.51,.15))	((,78,.57), (.45,.21))
$\mathcal{K}_2$	((.64,.21), (.37,.28))	((..78,.13), (.61, .15))	((.73,.18), (.51, 0.19))
$\mathcal{K}_3$	((.89,.87), (.41, .33))	((.97,.63), (.63, .11))	((.91,.71), (.49,.26))
$\mathcal{K}_4$	((.91,.73), (.46, .18))	((.96,.47), (.61, .14))	((.92,.68), (.51, .15))

Table 18. Score values of the Journals.

Score functions	$\mathcal{K}_1$	$\mathcal{K}_2$	$\mathcal{K}_3$	$\mathcal{K}_4$
ESF	0.213	0.084	0.149	0.062
SF	-0.573	-0.830	-0.701	-0.875
QSF	-0.498	-0.809	-0.673	-0.911

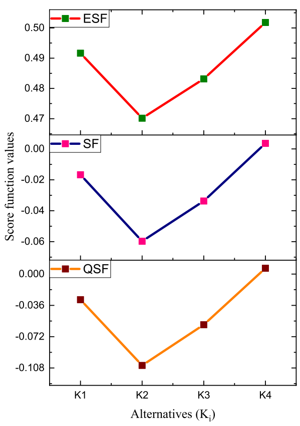

Figure 5. Ranking analysis using ESF, SF, and QSF score functions

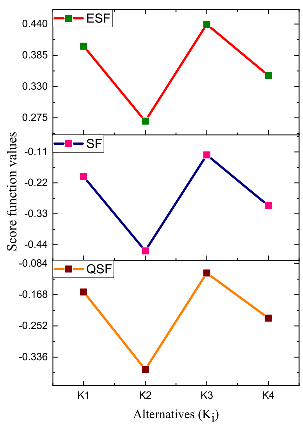

Figure 6. Energy planning policies ranking using ESF, SF, and QSF score functions

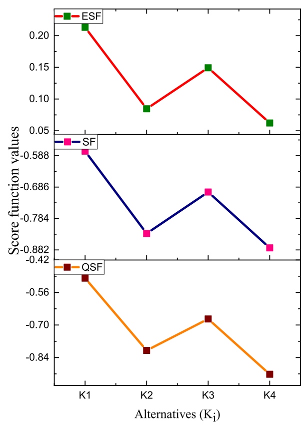

Figure 7. Ranking analysis using ESF, SF, and QSF s-core functions.

§5 Comparative analysis and discussion

This section compares the proposed approach with the other three existing approaches to justify the significance of the proposed approach. In order to evaluate the performance of the proposed approach, we have formulated two numerical examples in this section. Then we have compared the results of these examples with that of the other three approaches.

Example 5.1: Let $\mathcal{K}$=$\{\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3, \mathcal{K}_4, \mathcal{K}_5, \mathcal{K}_6, \mathcal{K}_7\}$ be the set of alternatives, C= $\{C_1$, C_2, C_3,

Table 19. A q-RLDFS Decision matrix D=(($\mu_F(\mathcal{K}_i)$, $\nu_F(\mathcal{K}_i)$),(α, β)), $q = 4$.

	C_1	C_2	C_3	C_4
$\mathcal{K}_1$	((.73,.41), (.31,.13))	((.63,.53), (.13,.23))	((.73,.41), (.23,.15))	((.63,.53), (.31,.36))
$\mathcal{K}_2$	((.63,.43), (.41,.42))	((.74,.32), (.63,.21))	((.68,.41), (.53,.21))	((.71,.41), (.43,.28))
$\mathcal{K}_3$	((.71,.34), (.51,.31))	((.63,.51), (.43,.39))	((.71,.41), (.31,.41))	(.69,.38), (.41,.31))
$\mathcal{K}_4$	((.69, .59), (.61,.21))	((.81,.51), (.31,.42))	((.83,.41), (.32,.41))	(.73,.49), (.41,.21))
$\mathcal{K}_5$	((.72,.41), (.51,.21))	((.83,.41), (.42,.31))	((.73,.41), (.31,.42))	(.83,.49), (.28,.41))
$\mathcal{K}_6$	((.63,.59), (.41,.31))	((.78,.43), (.38,.41))	((.63,.48), (.28,.17))	(.58,.49), (.31,.42))
$\mathcal{K}_7$	((.81,.58), (.49,.31))	((.73,.68), (.43,.49))	((.69,.73), (.31,.31))	((.68,.51), (.43,.21))

$C_4\}$ be the set of attributes and the corresponding decision matrix in terms of q-RLDFS given by the experts is given in Table 19 [7]. For the given decision matrix in Table 19, we obtain the optimal weights $w_j^* = \{0.4,\ 0.252,\ 0.148,\ 0.2\}$ using DEA. For the decision matrix D given by decision maker, the combined belief degrees $\overline{\beta_1}(\mathcal{K}_i)$ $\overline{\beta_2}(\mathcal{K}_i)$ and $\underline{\beta_1}(\mathcal{K}_i), \underline{\beta_2}(\mathcal{K}_i)$ are evaluated based on the steps 1-4. The aggregated combined belief degrees for the decision matrix D are

$$((\overline{\beta_1}(\mathcal{K}_1), \overline{\beta_2}(\mathcal{K}_1)),(\underline{\beta_1}(\mathcal{K}_1), \underline{\beta_2}(\mathcal{K}_1))) = ((0.350, 0.655), (0.419, 0.590)),$$
$$((\overline{\beta_1}(\mathcal{K}_2), \overline{\beta_2}(\mathcal{K}_2)),(\underline{\beta_1}(\mathcal{K}_2), \underline{\beta_2}(\mathcal{K}_2))) = ((0.161, 0.898), (0.245, 0.847)),$$
$$((\overline{\beta_1}(\mathcal{K}_3), \overline{\beta_2}(\mathcal{K}_3)),(\underline{\beta_1}(\mathcal{K}_3), \underline{\beta_2}(\mathcal{K}_3))) = ((0.214, 0.825), (0.312, 0.753)),$$
$$((\overline{\beta_1}(\mathcal{K}_4), \overline{\beta_2}(\mathcal{K}_4)),(\underline{\beta_1}(\mathcal{K}_4), \underline{\beta_2}(\mathcal{K}_4))) = ((0.169, 0.887), (0.235, 0.851)),$$
$$((\overline{\beta_1}(\mathcal{K}_5), \overline{\beta_2}(\mathcal{K}_5)),(\underline{\beta_1}(\mathcal{K}_5), \underline{\beta_2}(\mathcal{K}_5))) = ((0.203, 0.834), (0.302, 0.756)),$$
$$((\overline{\beta_1}(\mathcal{K}_6), \overline{\beta_2}(\mathcal{K}_6)),(\underline{\beta_1}(\mathcal{K}_6), \underline{\beta_2}(\mathcal{K}_6))) = ((0.407, 0.616), (0.492, 0.549)),$$
$$((\overline{\beta_1}(\mathcal{K}_7), \overline{\beta_2}(\mathcal{K}_7)),(\underline{\beta_1}(\mathcal{K}_7), \underline{\beta_2}(\mathcal{K}_7))) = ((0.331, 0.718), (0.373, 0.695)).$$

Finally, we calculate the score values shown in Table 20 to select the ranking of the alternatives based on the obtained final combined belief degrees and the score functions given in (20), (21), and (22).

Table 20. Core values of the alternatives.

Score functions	$\mathcal{K}_1$	$\mathcal{K}_2$	$\mathcal{K}_3$	$\mathcal{K}_4$	$\mathcal{K}_5$	$\mathcal{K}_6$	$\mathcal{K}_7$
ESF	0.40	0.18	0.26	0.18	0.26	0.43	0.34
QSF	-0.16	-0.52	-0.36	-0.51	-0.38	-0.10	-0.23
SF	-0.19	-0.62	0.46	-0.62	-0.47	-0.12	-0.30

Tables 21, 22 and 23 show the comparative study of the proposed approach with three other approaches in terms of the ranking of the alternatives. Our proposed approach is found to be significant in the sense that it yields similar ranking for the three different score functions (ESF, QSF, and SF) while the other approaches yields slightly different ranking for the same. Hence the proposed method ensures robust and consistent decision-making compared to the existing methods.

Table 21. Order of preference using ESF score function.

Methods	Ranking of alternatives
q-RLDFAA [10]	$\mathcal{K}_4 > \mathcal{K}_7 > \mathcal{K}_1 > \mathcal{K}_6 > \mathcal{K}_5 > \mathcal{K}_2 > \mathcal{K}_3$
q-RLDFWA [10]	$\mathcal{K}_3 > \mathcal{K}_2 > \mathcal{K}_7 > \mathcal{K}_6 > \mathcal{K}_4 > \mathcal{K}_5 > \mathcal{K}_1$
LDFS [9]	$\mathcal{K}_3 > \mathcal{K}_2 > \mathcal{K}_7 > \mathcal{K}_6 > \mathcal{K}_1 > \mathcal{K}_4 > \mathcal{K}_5$
Proposed method	$\mathcal{K}_6 > \mathcal{K}_1 > \mathcal{K}_7 > \mathcal{K}_3 > \mathcal{K}_5 > \mathcal{K}_4 > \mathcal{K}_2$

Table 22. Order of preference using QSF score function.

Methods	Ranking of alternatives
q-RLDFAA [10]	$\mathcal{K}_4 > \mathcal{K}_7 > \mathcal{K}_6 > \mathcal{K}_1 > \mathcal{K}_5 > \mathcal{K}_2 > \mathcal{K}_3$
q-RLDFWA [10]	$\mathcal{K}_3 > \mathcal{K}_2 > \mathcal{K}_7 > \mathcal{K}_6 > \mathcal{K}_5 > \mathcal{K}_4 > \mathcal{K}_1$
LDFS [9]	$\mathcal{K}_3 > \mathcal{K}_2 > \mathcal{K}_7 > \mathcal{K}_6 > \mathcal{K}_1 > \mathcal{K}_5 > \mathcal{K}_4$
Proposed method	$\mathcal{K}_6 > \mathcal{K}_1 > \mathcal{K}_7 > \mathcal{K}_3 > \mathcal{K}_5 > \mathcal{K}_4 > \mathcal{K}_2$

Table 23. Order of preference using SF score function.

Methods	Ranking of alternatives
q-RLDFAA [10]	$\mathcal{K}_4 > \mathcal{K}_7 > \mathcal{K}_1 > \mathcal{K}_6 > \mathcal{K}_5 > \mathcal{K}_2 > \mathcal{K}_3$
q-RLDFWA [10]	$\mathcal{K}_3 > \mathcal{K}_2 > \mathcal{K}_7 > \mathcal{K}_6 > \mathcal{K}_4 > \mathcal{K}_5 > \mathcal{K}_1$
LDFS [9]	$\mathcal{K}_3 > \mathcal{K}_2 > \mathcal{K}_7 > \mathcal{K}_6 > \mathcal{K}_1 > \mathcal{K}_4 > \mathcal{K}_5$
Proposed method	$\mathcal{K}_6 > \mathcal{K}_1 > \mathcal{K}_7 > \mathcal{K}_3 > \mathcal{K}_5 > \mathcal{K}_4 > \mathcal{K}_2$

Example 5.2:

Let $\mathcal{K} = \{\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3, \mathcal{K}_4\}$ be the four journals selected by the student for publishing the research article. The decision matrix in the framework of q-RLDFS is given in Table 24 [28]. For the given input decision matrix, there are no weight constraints, so directly we obtain the optimal weights based on differential evolution optimization techniques.

For the decision matrix D provided by the student, the combined belief degrees $((\overline{\beta_1}(\mathcal{K}_1), \overline{\beta_2}(\mathcal{K}_1)),$ $(\underline{\beta_1}(\mathcal{K}_1), \underline{\beta_2}(\mathcal{K}_1)))$ are evaluated based on Steps 1-4 as given in the proposed approach in Section 3. The aggregated combined belief degrees for the decision matrix D are $((\overline{\beta_1}(\mathcal{K}_1), \overline{\beta_2}(\mathcal{K}_1)),$ $(\underline{\beta_1}(\mathcal{K}_1), \underline{\beta_2}(\mathcal{K}_1))) =$ ((0.17,0.98), (0.95,0.97)), $((\overline{\beta_1}(\mathcal{K}_2), \overline{\beta_2}(\mathcal{K}_2)),$ $(\underline{\beta_1}(\mathcal{K}_2), \underline{\beta_2}(\mathcal{K}_2))) =$ ((0.47,0.59), (0.95,0.95)), $((\overline{\beta_1}(\mathcal{K}_3), \overline{\beta_2}(\mathcal{K}_3)),$ $(\underline{\beta_1}(\mathcal{K}_3), \underline{\beta_2}(\mathcal{K}_3))) =$ ((0.39,0.70), (0.96,0.96)), $((\overline{\beta_1}(\mathcal{K}_4), \overline{\beta_2}(\mathcal{K}_4)),$ $(\underline{\beta_1}(\mathcal{K}_4), \underline{\beta_2}(\mathcal{K}_4))) =$ ((0.38,0.99),(0.98,0.95)).

Now, we calculate the score values which are shown in Table 25 to choose the journal for the publication based on the obtained final combined belief degrees and one of the score functions

Table 24. Decision matrix D.

	Student	Supervisor-1	Supervisor-2
$\mathcal{K}_1$	((0.86,0.34), (.75,0.24))	((0.56,0.49), (0.5,0.37))	((0.78,0.35), (0.65,0.25))
$\mathcal{K}_2$	((0.75,0.34) (0.6,0.24))	((0.46,0.74), (0.28,0.6))	((0.45,0.41), (0.32,0.27))
$\mathcal{K}_3$	((0.56,0.44), (0.48,0.26))	((0.34,0.66), (0.25,0.53))	((0.78,0.59), (0.61,0.49))
$\mathcal{K}_4$	((0.95,0.11), (0.8,0.1))	((0.99,0.21), (0.88,0.08))	((0.86,0.35), (0.75,0.24))

shown in (20), (21), and (22). It is noted that the ranking of the journals for all of the three score functions are similar.

Table 25. Score values of the Journals.

Score functions	$\mathcal{K}_1$	$\mathcal{K}_2$	$\mathcal{K}_3$	$\mathcal{K}_4$
ESF	0.283	0.479	0.422	0.373
SF	-0.432	-0.041	-0.155	-0.252
QSF	-0.517	-0.035	-0.170	-0.330

Table 26. Order of preference using ESF score function.

Methods	Ranking of alternatives
q-RLDFAA [10]	$\mathcal{K}_2 > \mathcal{K}_4 > \mathcal{K}_3 > \mathcal{K}_1$
q-RLDFWA [10]	$\mathcal{K}_2 > \mathcal{K}_4 > \mathcal{K}_1 > \mathcal{K}_3$
LDFS [9]	$\mathcal{K}_2 > \mathcal{K}_4 > \mathcal{K}_1 > \mathcal{K}_3$
Proposed method	$\mathcal{K}_3 > \mathcal{K}_1 > \mathcal{K}_4 > \mathcal{K}_2$

Tables 26, 27 and 28 shows the comparative study of the proposed approach with three other approaches in terms of the ranking of the alternatives. Our proposed approach is found to be significant in the sense that it yields similar ranking for the three different score functions (ESF, QSF, and SF) while the q-RLDFAA and q-RLDFWA operators yields slightly different ranking for the same. Hence the proposed method ensures robust and consistent decision-making compared to the existing methods.

Table 27. Order of preference using QSF score function.

Methods	Ranking of alternatives
q-RLDFAA [10]	$\mathcal{K}_4 > \mathcal{K}_2 > \mathcal{K}_3 > \mathcal{K}_1$
q-RLDFWA [10]	$\mathcal{K}_4 > \mathcal{K}_2 > \mathcal{K}_3 > \mathcal{K}_1$
LDFS [9]	$\mathcal{K}_4 > \mathcal{K}_2 > \mathcal{K}_3 > \mathcal{K}_1$
Proposed method	$\mathcal{K}_3 > \mathcal{K}_1 > \mathcal{K}_4 > \mathcal{K}_2$

Table 28. Order of preference using SF s-core function.

Methods	Ranking of alternatives
q-RLDFAA [10]	$\mathcal{K}_2 > \mathcal{K}_4 > \mathcal{K}_3 > \mathcal{K}_1$
q-RLDFWA [10]	$\mathcal{K}_2 > \mathcal{K}_4 > \mathcal{K}_1 > \mathcal{K}_3$
LDFS [9]	$\mathcal{K}_2 > \mathcal{K}_4 > \mathcal{K}_1 > \mathcal{K}_3$
Proposed method	$\mathcal{K}_3 > \mathcal{K}_1 > \mathcal{K}_4 > \mathcal{K}_2$

The proposed approach determines the best alternative from the group of alternatives in emergency decision-making in uncertain environments and it can be justified from the following analysis:

1. *Robust and consistent decision-making:*

The proposed method provides reliable results when choosing the best option. It can generate the similar order of ranking for all of the three score functions (ESF, QSF, and SF), which has been proved in our experimental analysis. This shows the robust and consistent nature of the proposed method.

2. *Computation of optimal weights of the attributes:*

Usage of the optimal weights for the attributes is significant to solve any kinds of uncertain decision-making problems, where DE optimization ensures the process of finding the optimal weights. The optimal weights are the priorities given to the attributes by the optimization technique based on problem information in the decision-making process. It can also assist in the aggregation of the belief degrees in ER technique.

3. *Aggregation of the belief degrees using ER methodology:*

ER methodology is found to be suitable to aggregate the problem information effectively based on the belief degree structure. Unlike the fuzzy aggregation operators, the ER methodology evaluates the performance of the attributes by the evaluation grades and aggregates the grades using probability mass and combined probability mass and then calculates the aggregated values of the attributes of the alternatives. The complete evaluation is mathematically shown in section 2. The optimal aggregated alternatives can be ensured using ER methodology with the optimal weights obtained by DE. Thereafter, it can contribute to the efficiency of the decision-making approach.

It is proved that the structural space of q-RLDFS with the inclusion of q^{th} parameter, optimal weights of DE optimization technique, and the aggregation of evaluated attributes of the alternatives can collectively form an effective decision-making approach. The proposed method is consistent in choosing the best alternative for all the three variety of score functions. This also justifies the robust nature of the proposed decision-making approach.

§6 Conclusion

In this study, we have proposed a decision-making approach to solve COVID-19 emergency decision-making problem using q-RLDFS, differential evolutionary optimization techniques, and evidential reasoning methodology. The proposed approach is enhanced by removing the limitations and restrictions for defining membership and non-membership grades given by experts or decision makers with q-RLDFS. The ER methodology involves the belief structure for the purpose of obtaining belief degrees. Differential evolutionary optimization techniques ensure optimal weights for the attributes. The proposed approach ensures robust and consistent decision-making by providing a similar ranking of the alternatives for all the three mentioned score functions. For better illustrations of the proposed approach, we have used one COVID-19 related case study and one numerical example. The comparative study shows the applicability of the proposed approach. Researchers can use this concept to solve uncertain real-life problems using the various extensions of fuzzy set. In the future, researchers may also apply the proposed idea to COVID-19 vaccination information to make the necessary decision to optimize the vaccine distribution among the various states of a particular country to meet the requirements. Moreover, the proposed work may be extended for decision-making using fuzzy linear regression and fuzzy logistic regression techniques to manage the nature of the high volume of data.

Declarations

Conflict of interest The authors declare no conflict of interest.

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