Appl. Math. J. Chinese Univ.
2023, 38(2): 310-316

The existence of radial k-admissible solutions for n-dimension system of k-Hessian equations

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Abstract. In this paper, we focus on a general n-dimension system of k-Hessian equations. By introducing some new suitable growth conditions, the existence results of radial k-admissible solutions of the k-Hessian system are obtained. Our approach is largely based on the well-known fixed-point theorem.

§1 Introduction

In this paper, we aim to investigate the existence of radial k-admissible solutions for the n-dimensional system by k-Hessian equations like

$$\begin{cases} S_k(\lambda(D^2u_1)) = f_1(-u_1,\cdots,-u_n) & \text{in } B, \\ S_k(\lambda(D^2u_2)) = f_2(-u_1,\cdots,-u_n) & \text{in } B, \\ \cdots & \\ S_k(\lambda(D^2u_n)) = f_n(-u_1,\cdots,-u_n) & \text{in } B, \\ u_i(x) = 0 & \text{on } \partial B, \end{cases} \tag{1}$$

where $k=1,2,\cdots,n$, $B=\{x\in\mathbb{R}^n : |x|<1\}$ is a unit ball, the nonlinearity f_i satisfies the condition:

(A) $f_i \in C([0,+\infty)\times[0,+\infty)\times\cdots\times[0,+\infty),[0,+\infty))$ $(i=1,2,\cdots,n)$.

As we all known, the k-Hessian operator $S_k(\lambda(D^2u))$ is a classical completely nonlinear partial differential operator, which is defined as follows

$$S_k(\lambda(D^2u)) = \sum_{1\le i_1<i_2<\cdots<i_k\le n} \lambda_{i_1}\lambda_{i_2}\cdots\lambda_{i_k}, \quad k=1,2,\cdots,n.$$

It is clear that $S_k(\lambda(D^2u))$ is the sum of all $k\times k$ principal minors Hessian matrix of $D^2u = \frac{\partial^2 u}{\partial x_i\partial x_j}$, where $\lambda(D^2u)=(\lambda_1,\lambda_2,\cdots,\lambda_n)$ is the vector of eigenvalues of D^2u, and $\lambda_1,\lambda_2,\cdots,\lambda_n$

Received: 2021-12-31.   Revised: 2022-06-21.
MR Subject Classification: 34B15, 34B18, 34L30, 35A01, 35A20.
Keywords: k-Hessian equations, radial k-admissible solutions, existence, fixed-point theorem.
Digital Object Identifier(DOI): https://doi.org/10.1007/s11766-023-4648-1.
Supported by the National Natural Science Foundation of China(11961060) and Graduate Research Support of Northwest Normal University(2021KYZZ01032).
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are the eigenvalues of the Hessian matrix D^2u.

It is interesting to see that the k-Hessian operator contains a family of operators: the classical Laplace operator $S_k(D^2u) = \Delta u$ for $k = 1$; the Monge-Ampère operator $S_k(D^2u) = \det(\Delta^2 u)$ for $k = n$ and other well-known operators. On the study of these operators, several authors paid more attention and obtained many excellent results, see, for instance [5,6,9,10] and the references therein.

At the same time, we notice that the discussion of solutions for k-Hessian equation has become a topic in the current study, see [1,3,4,6-15] and the references therein. In particular, Ma et al. [7] showed the existence of three radially symmetric k-admissible solutions for k-Hessian equation with 0-Dirichlet boundary condition under suitable conditions on the nonlinearity. However, to the best of our knowledge, there are few literatures study the existence of systematic solutions for the n-dimensional system by k-Hessian equations. In a recent study, Gao et al. [3] obtained the existence, uniqueness and nonexistence of radial convex solutions for some suitable constants α and β dealed for a coupled system of k-Hessian equations:

$$\begin{cases} S_k(\mu(D^2u_1)) = (-u_2)^\alpha & \text{in } B, \\ S_k(\mu(D^2u_2)) = (-u_1)^\beta & \text{in } B, \\ u_1 < 0, \quad u_2 < 0 & \text{in } B, \\ u_1 = u_2 = 0 & \text{on } \partial B, \end{cases}$$

where B is a unit ball in $\mathbb{R}^n$, $n \geq 2$, α and β are positive constants. Thus, it is inspired by the recent works, we are interested in the existence of radial k-admissible solutions for the form (1). The function $u \in C^2(B) \cap C(\bar{B})$ is called k-admissible function if $\lambda(D^2u)$ belongs to the set

$$\Gamma_k = \{\lambda \in \mathbb{R}^n | S_j(\lambda) > 0 \quad j = 1, 2, \cdots, k\}.$$

We know that k-admissible solution is subharmonic by the maximum principle, it is negative in B. Our research is an improvement and extension of [3].

Let

$$\underline{F_{i*}^0} = \liminf_{c\to 0^+} \frac{F_{i*}(c)}{c^{\alpha_i}}, \qquad \underline{F_{i*}^\infty} = \liminf_{c\to +\infty} \frac{F_{i*}(c)}{c^{\alpha_i}};$$
$$\overline{F_i^*}^0 = \limsup_{c\to 0^+} \frac{F_i^*(c)}{c^{\beta_i}}, \qquad \overline{F_i^*}^\infty = \limsup_{c\to +\infty} \frac{F_i^*(c)}{c^{\beta_i}}.$$

Then under some different suitable conditions imposed on $\underline{F_{i*}^0}$, $\underline{F_{i*}^\infty}$, $\overline{F_i^*}^0$ and $\overline{F_i^*}^\infty$ (here, we may call them α_i or β_i-asymptotic growth condition, super-α_i or β_i-asymptotic growth condition, and sub-α_i or β_i-asymptotic growth condition), and some inequalities properties imposed on α_i and β_i, we obtain the existence of radial k-admissible solutions to the system (1). It is noted that the k-asymptotic growth is the case that the constants $\alpha_i = k$ and $\beta_i = k$ of α_i or β_i-asymptotic growth. Therefore, our conditions here are more flexible than those existing results and the results here are completely new. Meanwhile, we also make the following assumption:
(B) there exist two pairs of nonnegative continuous functions F_{i*}, $F_i^*(i = 1, 2, \cdots, n)$ such that for any $-u_{i0} \in \{-u_j\}$, $u_{i0} \neq u_{k0}$ $(i \neq k)$,

$$F_{i*}(-u_{i0}) \leq f_i(-u_1, -u_2, \cdots, -u_n) \leq F_i^*(-u_{i0}).$$

The rest of the present paper is organized as follows. In section 2, we give some preliminary work for radial k-admissible solutions. In section 3, we attempt to obtain the existence results of the radial k-admissible solutions to the n-dimensional system (1).

§2 Preliminary results on radial solutions

For the radial solution $v(\tau)$ with $\tau=\sqrt{\sum_{i=1}^{n}x_i^2}$, the k-Hessian operator becomes

$$S_k(\lambda(D^2v))=\begin{cases}C_{N-1}^{k-1}(-v)''(\tau)\big(\frac{(-v)'(\tau)}{\tau}\big)^{k-1}+C_{N-1}^{k}\big(\frac{(-v)'(\tau)}{\tau}\big)^{k}, & 0<\tau<1,\\ C_N^k((-v(0))'')^k=0, & \tau=0.\end{cases}$$

Based on this, we pay attention to the following boundary value problem:

$$\begin{cases}C_{N-1}^{k-1}(-v_1)''(\tau)\big(\frac{(-v_1)'(\tau)}{\tau}\big)^{k-1}+C_{N-1}^{k}\big(\frac{(-v_1)'(\tau)}{\tau}\big)^{k}=f_1(v_1,v_2,\cdots,v_n), & 0<\tau<1,\\ C_{N-1}^{k-1}(-v_2)''(\tau)\big(\frac{(-v_2)'(\tau)}{\tau}\big)^{k-1}+C_{N-1}^{k}\big(\frac{(-v_2)'(\tau)}{\tau}\big)^{k}=f_2(v_1,v_2,\cdots,v_n), & 0<\tau<1,\\ \cdots\\ C_{N-1}^{k-1}(-v_n)''(\tau)\big(\frac{(-v_n)_n'(\tau)}{\tau}\big)^{k-1}+C_{N-1}^{k}\big(\frac{(-v_n)'(\tau)}{\tau}\big)^{k}=f_n(v_1,v_2,\cdots,v_n), & 0<\tau<1,\\ v_i'(0)=v_i(1)=0.\end{cases}\tag{2}$$

Let E be the Banach space $\underbrace{C[0,1]\times\cdots\times C[0,1]}_{n}$ with the norm $\|\overrightarrow{v}\|=\max\limits_{1\le i\le n}\{|v_i|_1\}$, where $|v_i|_1=\max\limits_{0\le\tau\le1}|v_i(\tau)|$. Define an operator $A:E\to E$ as

$$A(\overrightarrow{v})(\tau)=(A_1(\overrightarrow{v})(\tau),\cdots,A_n(\overrightarrow{v})(\tau)),$$

where

$$A_i(\overrightarrow{v})(\tau)=\int_\tau^1\left(\frac{k}{t^{N-k}}\int_0^t\frac{s^{N-1}}{C_{N-1}^{k-1}}f_i(v_1(s),\cdots,v_n(s))ds\right)^{\frac{1}{k}}dt,\quad \tau\in[0,1],\ i=1,\cdots,n.$$

Set a sub-cone $K=K_1\times\cdots\times K_n$, then $K\subset E$, and $K_i:=\Big\{v_i(\tau):\min\limits_{\frac14\le\tau\le\frac34}v_i(\tau)\ge\frac14|v_i|_1\Big\}$. It is easy to see that $A:K\to K$ is completely continuous. Then, v is the fixed-point of A if and only if v is a negative solution to problem (2). Furthermore, v is a radially symmetric k-admissible solution of system (1).

Our main tools depend on the following lemma as well as some suitable estimations.

Lemma 2.1.([2]) Let E be a real Banach space and $K\subset E$ be a cone. Assume Ω_1,Ω_2 are bounded, open subsets of E with $\theta\in\Omega_1,\overline{\Omega}_1\subset\Omega_2$, and let $A:K\cap(\overline{\Omega}_2\setminus\Omega_1)\to K$ be a completely continuous operator such that either

(i) $\|Av\|\le\|v\|, v\in K\cap\partial\Omega_1$, and $\|Av\|\ge\|v\|, v\in K\cap\partial\Omega_2$; or

(ii) $\|Av\|\ge\|v\|, v\in K\cap\partial\Omega_1$, and $\|Av\|\le\|v\|, v\in K\cap\partial\Omega_2$.

Then A has a fixed point in $K\cap(\overline{\Omega}_2\setminus\Omega_1)$.

§3 Main results for system (1)

Let

$$\Gamma=\int_{\frac14}^{\frac34}\left(\frac{k}{t^{N-k}}\int_{\frac14}^{t}\frac{s^{N-1}}{C_{N-1}^{k-1}}ds\right)^{\frac1k}dt.$$

We have the following results:

Theorem 3.1. Assume that (A), (B) hold and $0 < \alpha_i,\ \beta_i < k, i = 1, 2, \cdots, n$. If $\underline{F_{i*}^0} = +\infty$, $\overline{F_i^*}^{\infty} = 0$, then (1) has at least one radial k-admissible solution.

Proof. Since $\underline{F_{i*}^0} = +\infty$, there exist $M > 0$ and $0 < r_1 < 1$ such that

$$F_{i*}(v_{i0}) \geq M v_{i0}^{\alpha_i}, \qquad 0 \leq v_{i0} \leq r_1,$$

where M satisfies

$$\min_{1\leq i\leq n}\left\{\Gamma M^{\frac{1}{k}} (\tfrac{1}{4})^{\frac{\alpha_i}{k}}\right\} \geq 1.$$

Then for any $\overrightarrow{v} \in K \cap \partial\Omega_{r_1}$, we have

$$\begin{aligned} A_i(\overrightarrow{v})(\tfrac{1}{4}) &= \int_{\frac{1}{4}}^{1}\left(\frac{k}{t^{N-k}}\int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} f_i(v_1(s), \cdots, v_n(s))ds\right)^{\frac{1}{k}} dt \\ &\geq \int_{\frac{1}{4}}^{\frac{3}{4}}\left(\frac{k}{t^{N-k}}\int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} F_{i*}(v_{i0})ds\right)^{\frac{1}{k}} dt \\ &\geq \int_{\frac{1}{4}}^{\frac{3}{4}}\left(\frac{k}{t^{N-k}}\int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} M v_{i0}^{\alpha_i} ds\right)^{\frac{1}{k}} dt \\ &\geq \int_{\frac{1}{4}}^{\frac{3}{4}}\left(\frac{k}{t^{N-k}}\int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} M (\tfrac{1}{4}|v_{i0}|_1)^{\alpha_i} ds\right)^{\frac{1}{k}} dt \\ &\geq \Gamma M^{\frac{1}{k}} (\tfrac{1}{4})^{\frac{\alpha_i}{k}} |v_{i0}|^{\frac{\alpha_i}{k}} \\ &\geq |v_{i0}|^{\frac{\alpha_i}{k}}. \end{aligned}$$

Furthermore, there exists an index v_{i_0} such that

$$\|A(\overrightarrow{v})\| > \max_{1\leq i\leq n}\left\{|v_{i0}|_1^{\frac{\alpha_i}{k}}\right\} = r_1^{\frac{\alpha_i}{k}} \geq r_1,$$

that is for any $\overrightarrow{v} \in K \cap \partial\Omega_{r_1}, \|A(\overrightarrow{v})\| > \|\overrightarrow{v}\|$.

In addition, it follows from $\overline{F_i^*}^{\infty} = 0$ that there exist a sufficient small $\gamma > 0$ and $r_2 > 0$ such that for any $v_{i0} > r_2$, we get $F_i^*(v_{i0}) \leq \gamma v_{i0}^{\beta_i}$.

Let

$$M_i = \max_{0\leq v_{i0}\leq \overline{\varrho}} F_i^*(v_{i0}).$$

Then we have

$$f_i(\overrightarrow{v}) \leq F_i^*(v_{i0}) \leq \gamma v_{i0}^{\beta_i} + M_i.$$

Moreover, we can get

$$\begin{aligned} A_i(\overrightarrow{v})(\tau) &= \int_{\tau}^{1}\left(\frac{k}{t^{N-k}}\int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} f_i(v_1(s), \cdots, v_n(s))ds\right)^{\frac{1}{k}} dt \\ &\leq \int_0^1\left(\frac{k}{t^{N-k}}\int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} (\gamma v_{i0}^{\beta_i} + M_i)ds\right)^{\frac{1}{k}} dt \\ &\leq \frac{1}{2}\left(\gamma v_{i0}^{\beta_i} + M_i\right)^{\frac{1}{k}} \left(\tfrac{k}{N C_{N-1}^{k-1}}\right)^{\frac{1}{k}}. \end{aligned}$$

Combining this with $\beta_i \le k$, then there exists a sufficient large constant

$$R :\ge \max_{1\le i\le n}\left\{\frac{1}{2}\left(\gamma\overline{R}^{\beta_i}+M_i\right)^{\frac{1}{k}}\left(\frac{k}{NC_{N-1}^{k-1}}\right)^{\frac{1}{k}}\right\}$$

such that for any $\overrightarrow{v} \in K\cap\partial\Omega_R$, we have

$$\begin{aligned}\|A(\overrightarrow{v})\| &\le \max_{1\le i\le n}\left\{\frac{1}{2}\left(\gamma|v_{i0}|^{\beta_i}+M_i\right)^{\frac{1}{k}}\left(\frac{k}{NC_{N-1}^{k-1}}\right)^{\frac{1}{k}}\right\}\\ &\le \max_{1\le i\le n}\left\{\frac{1}{2}\left(\gamma\overline{R}^{\beta_i}+M_i\right)^{\frac{1}{k}}\left(\frac{k}{NC_{N-1}^{k-1}}\right)^{\frac{1}{k}}\right\}\\ &\le R=\|\overrightarrow{v}\|.\end{aligned}$$

By the Lemma 2.1, one can infer that A has at least one fixed point in $K\cap(\overline{\Omega}_R\setminus\Omega_{r_1})$. □

Theorem 3.2. Assume that (A), (B) hold and $\alpha_i,\beta_i \ge k$. If $\overline{F_i^*}^0 = 0$ and $\underline{F_{i*}^\infty} = +\infty$, then (1) has at least one radial k-admissible solution.

Proof. By the definition of $\overline{F_i^*}^0 = 0$, there exist $r_3 \in (0,1)$ and a sufficient small $\delta > 0$ with $\frac{1}{2}\left(\frac{k\delta}{NC_{N-1}^{k-1}}\right) \le 1$ such that

$$F_i^*(v_{i0}) \le \delta v_{i0}^{\beta_i}, \qquad 0 \le v_{i0} \le r_3.$$

For any $\overrightarrow{v} \in K\cap(\partial\Omega_{r_3})$, we have

$$\begin{aligned}A_i(\overrightarrow{v})(\tau) &= \int_\tau^1\left(\frac{k}{t^{N-k}}\int_0^t\frac{s^{N-1}}{C_{N-1}^{k-1}}f_i(v_1(s),\cdots,v_n(s))ds\right)^{\frac{1}{k}}dt\\ &\le \int_\tau^1\left(\frac{k}{t^{N-k}}\int_0^t\frac{s^{N-1}}{C_{N-1}^{k-1}}\delta v_{i0}^{\beta_i}ds\right)^{\frac{1}{k}}dt\\ &\le \frac{1}{2}\left(\frac{k\delta}{NC_{N-1}^{k-1}}\right)^{\frac{1}{k}}|v_{i0}|_1^{\frac{\beta_i}{k}}\\ &\le |v_{i0}|_1^{\frac{\beta_i}{k}}.\end{aligned}$$

Combining this with $\beta_i \ge k$, we have

$$\|A(\overrightarrow{v})\| < \max_{1\le i\le n}\left\{|v_{i0}|_1^{\frac{\beta_i}{k}}\right\} \le r_3,$$

which implies that for any $\overrightarrow{v} \in K\cap(\partial\Omega_{r_3})$, $\|A(\overrightarrow{v})\| < \|\overrightarrow{v}\|$.

Besides, by the definition of $\underline{F_{i*}^\infty} = +\infty$, there exist $\widehat{M} > 0$ and $r_4 > r_3$ such that

$$F_{i*}(v_{i0}) \ge \widehat{M}v_{i0}^{\alpha_i}, \quad v_{i0} \ge r_4,$$

where $\widehat{M}$ satisfies

$$\min_{1\le i\le n}\left\{\Gamma\widehat{M}^{\frac{1}{k}}\left(\tfrac{1}{4}\right)^{\frac{\alpha_i}{k}}\right\} \ge 1.$$

Set $\widetilde{R} = 4r_4 + 1$, and let

$$D_i = \min_{0\le v_{i0}\le\widetilde{\varrho}} F_{i*}(v_{i0}).$$

If $\|\overrightarrow{v}\| = |v_{i0}|_1 = \widetilde{R}$, then for $\overrightarrow{v} \in K \cap \partial\Omega_{\widetilde{R}}$, we can get

$$\begin{aligned}
A_i(\overrightarrow{v})(\tfrac{1}{4}) &= \int_{\frac{1}{4}}^{1} \left(\frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} f_i(v_1(s), \cdots, v_n(s)) ds \right)^{\frac{1}{k}} dt \\
&\geq \int_{\frac{1}{4}}^{\frac{3}{4}} \left(\frac{k}{t^{N-k}} \int_{\frac{1}{4}}^t \frac{s^{N-1}}{C_{N-1}^{k-1}} F_{i*}(v_{i0}) ds \right)^{\frac{1}{k}} dt \\
&\geq \int_{\frac{1}{4}}^{\frac{3}{4}} \left(\frac{k}{t^{N-k}} \int_{\frac{1}{4}}^t \frac{s^{N-1}}{C_{N-1}^{k-1}} \widehat{M} v_{i0}^{\alpha_i} ds \right)^{\frac{1}{k}} dt \\
&\geq \int_{\frac{1}{4}}^{\frac{3}{4}} \left(\frac{k}{t^{N-k}} \int_{\frac{1}{4}}^t \frac{s^{N-1}}{C_{N-1}^{k-1}} \widehat{M} (\tfrac{1}{4}|v_{i0}|_1)^{\alpha_i} ds \right)^{\frac{1}{k}} dt \\
&\geq \Gamma \widehat{M}^{\frac{1}{k}} (\tfrac{1}{4})^{\frac{\alpha_i}{k}} |v_{i0}|_1^{\frac{\alpha_i}{k}} \\
&\geq |v_{i0}|_1^{\frac{\alpha_i}{k}}
\end{aligned}$$

and for $i \neq j$, we also have

$$\begin{aligned}
A_j(\overrightarrow{v})(\tfrac{1}{4}) &= \int_{\frac{1}{4}}^{1} \left(\frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{k-1}} f_j(v_1(s), \cdots, v_n(s)) ds \right)^{\frac{1}{k}} dt \\
&\geq \int_{\frac{1}{4}}^{\frac{3}{4}} \left(\frac{k}{t^{N-k}} \int_{\frac{1}{4}}^t \frac{s^{N-1}}{C_{N-1}^{k-1}} F_{j*}(v_{j0}) ds \right)^{\frac{1}{k}} dt \\
&\geq \int_{\frac{1}{4}}^{\frac{3}{4}} \left(\frac{k}{t^{N-k}} \int_{\frac{1}{4}}^t \frac{s^{N-1}}{C_{N-1}^{k-1}} D_j ds \right)^{\frac{1}{k}} dt \\
&\geq \Gamma \widehat{D_j}^{\frac{1}{k}}.
\end{aligned}$$

Moreover, we get that

$$\begin{aligned}
\max_{1 \leq i \leq n} \left\{ A_i(\overrightarrow{v})(\tfrac{1}{4}) \right\} > \max & \left\{ |\overrightarrow{v}_i 0|_1^{\frac{\alpha_i}{k}}, \Gamma \widehat{D_j}^{\frac{1}{k}} \right\} \\
&\geq |\overrightarrow{v}_i 0|_1^{\frac{\alpha_i}{k}} = \widetilde{R}^{\frac{\alpha_i}{k}} \geq \widetilde{R} = \|v\|.
\end{aligned}$$

Combining this with Lemma 2.1, A has at least one fixed point in $K \cap (\overline{\Omega}_{\widetilde{R}} \setminus \Omega_{r_4})$. □

Acknowledgement

We would like to thank the Editor and the anonymous referees for their critical comments and thoughtful suggestions, which led to a much improved version of this paper.

Declarations

Conflict of interest The authors declare no conflict of interest.

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