Appl. Math. J. Chinese Univ.
2023, 38(1): 16-26

Dispersive propagation of optical solitions and solitary wave solutions of Kundu-Eckhaus dynamical equation via modified mathematical method

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Abstract. In this research work, we constructed the optical soliton solutions of nonlinear complex Kundu-Eckhaus (KE) equation with the help of modified mathematical method. We obtained the solutions in the form of dark solitons, bright solitons and combined dark-bright solitons, travelling wave and periodic wave solutions with general coefficients. In our knowledge earlier reported results of the KE equation with specific coefficients. These obtained solutions are more useful in the development of optical fibers, dynamics of solitons, dynamics of adiabatic parameters, dynamics of fluid, problems of biomedical, industrial phenomena and many other branches. All calculations show that this technique is more powerful, effective, straightforward, and fruitfulness to study analytically other higher-order nonlinear complex PDEs involves in mathematical physics, quantum physics, Geo physics, fluid mechanics, hydrodynamics, mathematical biology, field of engineering and many other physical sciences.

§1 Introduction

In this research work the complex KE equation is take [1–3] as

$$iq_t + \alpha\delta q_{xx} + \gamma|q|^4 q + \delta(|q|^2)_x q = 0. \tag{1}$$

In Eq.(1), the $q(x,t)$ is a dependent variable which denotes to complex values of wave structure. The variable x denotes to spatial variable and t denotes to temporal variable. In Eq.(1), the first term becomes from nonlinear wave evolution of temporal, where α, δ and γ are real constants which are denoted to GVD (group velocity dispersion), nonlinear of quintic and effect of nonlinear, respectively. In KE equation, when $\alpha = 1, \delta = 1$ and $\gamma = 2$, some authors earlier studies have done with these specific values of constants in [1]. In the past KE equation was also studied with general values of coefficients in [2, 3].

In previous research the optical soliton solutions have been investigated by many authors [1–16]. The optical solitons have many applications in ultra-fast pluses processing system and

Received: 2019-07-13. Revised: 2020-06-11.
MR Subject Classification: 35J05, 35J10, 35K05, 35L05.
Keywords: Kundu-Eckhaus equation, modified mathematical method, solitons and solitary wave solutions.
Digital Object Identifier(DOI): https://doi.org/10.1007/s11766-023-3861-2.
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telecommunication due to their a lot of research have been done in nonlinear optics. The optical solitons obtained to balance nonlinear effect and dispersion effect of group velocity which arise due to the changing in nonlinear phenomena in the index of refractions [17–19]. In physics and various areas of applied Mathematics, solitons are the self-reinforcing solitary waves, packets or wave pluses, when it travels with constant speed that does not change its structure. Soliton is caused by the cancellations of dispersive effect and nonlinear effect in the medium.

Solitons are the solitary wave pulses which are the special waves. They have marvelous applications in optical fibers, dynamics of solitons, dynamics of adiabatic parameters, dynamics of fluid, problems of biomedical, industrial phenomena, engineering, and many other branches due to their significant stability futures. When dispersive waves inelastically scatter then hold some properties such as, same speed, shape and lose energy because of radiation phenomena and after full interaction of nonlinear then disappear the solitary waves. Such as, when optical shocks are formed when stronger radiations of laser transfer to optical fibers are regularly accompanied by the breaking wave phenomena that have marvelous applications in the transmission of light signal in fiber optics of digital system of communication.

Past few decades a lot of study have been done to investigate the optical soliton solutions of different nonlinear models by using a large variety of techniques. Some important and powerful techniques as Backlund transformation, Hirota bilinear method, Darboux transformation, Exp-function method, jacobian method, Trial equation technique, method of Simple equation, improved Fan-sub equation method, Extend mapping method, Sinh-cosh method, Extended modified rational expansion method, Direct algebraic method, improved F-expansion method, Modified extended tanh method, Racatti equation mapping method, auxiliary equation mapping method [20–42]. In this study, we constructed the optical soliton solutions of complex KE equation by applying the modified mathematical method [43–46].

This research work is systematized as fellows, we explain the introduction in section 1. We described the proposed method in section 2. We constructed the families of new optical soliton solutions of nonlinear complex KE equation by applying the modified mathematical technique in section 3. In section 4, finally this work end at the conclusion.

§2 Description of proposed method

Here we shall explain the main future of modified mathematical method for finding the solutions of solitary waves of NLPDEs. The general form of NLPDE consider as:

$$P(q, q_t, q_x, q_{tt}, q_{xx}, q_{xt}, ...) = 0, \tag{2}$$

where P is polynomial function of $q(x,t)$ and its derivatives. The features of modified mathematical technique explain as:

Step 1. Traveling wave transformations consider as:

$$q(x,t) = \sqrt{u(\zeta)}e^{i(-\sigma x+\omega t+\theta)}, \qquad \zeta = k(x + 2\alpha\sigma t). \tag{3}$$

We obtain ODE of Eq.(2) as:

$$Q(u, u^{'}, u^{''}, u^{'''}, ...) = 0, \tag{4}$$

where Q is polynomial function in $u(\zeta)$ and its derivatives.
Step 2. The trial solution of Eq.(4), consider as,

$$u(\zeta)=\sum_{i=0}^{N}a_i\Psi(\zeta)^i+\sum_{i=-1}^{-N}b_{-i}\Psi(\zeta)^i+\sum_{i=2}^{N}c_i\Psi(\zeta)^{i-2}\Psi'(\zeta)+\sum_{i=1}^{N}d_i\left(\frac{\Psi'(\zeta)}{\Psi(\zeta)}\right)^i, \tag{5}$$

where the a_i, b_i, c_i, d_i are constants which calculated be later, while $\Psi(\zeta)$ satisfying the following auxiliary equation

$$\begin{aligned}\Psi^{'}(\zeta)=&\sqrt{\beta_1\Psi^2(\zeta)+\beta_2\Psi^3(\zeta)+\beta_3\Psi^4(\zeta)},\\ \Psi^{''}(\zeta)=&\beta_1\Psi(\zeta)+\frac{3}{2}\beta_2\Psi^2(\zeta)+2\beta_3\Psi^3(\zeta).\end{aligned} \tag{6}$$

In Eq.(6), $\beta_i^{'}s$ are real constants which found to be later.
Step 3. Balancing the higher order derivative and terms of nonlinear in Eq.(4), determined N of Eq.(5).
Step 4. Substitute Eq.(6) into Eq.(5), and collect the every co-efficients of $\Psi^{'j}(\zeta)\Psi^i(\zeta)(i=1,2,3,...N;j=0,1)$, then every co-efficient make zero and get a set of equations, solve these set of equations using any computer software, the values of these parameter a_i, b_i, c_i, d_i be found.
Step 5. Substitute the parameters values which are obtaining and $\Psi(\zeta)$ in Eq.(5), then the solutions of Eq.(2) are obtained.

§3 Application of modified mathematical method for complex KE equation

Here we constructed the families of optical soliton solutions of complex KE equation. The Eq.(1), is the complex equation, the traveling wave transformation consider as:

$$q(x,t)=\sqrt{u(\zeta)}e^{i(-\sigma x+\omega t+\theta)}, \qquad \zeta=k(x+2\alpha\sigma t). \tag{7}$$

In Eq.(7), $u(\zeta)$ is the function which represents the pulse shape. Where σ is the soliton frequency, θ is constant of phase and ω is wave number of soliton. Substitute Eq.(7) into Eq.(1), separating the real and imaginary parts, we obtained couple system of equations as

$$2\alpha k^2uu^{''}+4\delta ku^2u^{'}-\alpha k^2(u^{'})^2-4(\omega+\alpha\sigma^2)u^2+4\gamma u^4=0. \tag{8}$$

Balance the nonlinear term and derivative of higher order in Eq.(8), obtain $N=1$. The trial solution of Eq.(8), take as:

$$u(\zeta)=a_0+a_1\Psi(\xi)+\frac{b_1}{\Psi(\xi)}+d_1\frac{\Psi^{'}(\xi)}{\Psi(\xi)}. \tag{9}$$

Substitute Eq.(9) into Eq.(8), and collecting each coefficients of $\Psi^{'j}(\zeta)\Psi^i(\zeta)(i=1,2,3,4,5,...N;$ $j=0,1)$, and compare each coefficient to zero. We obtained a set of equations. These set of equations we solve by using the Mathematica software, the values of constants and frequency are obtained as:

Case-1

$$a_0 = a_0, \quad a_1 = \pm\frac{\sqrt{\alpha}\sqrt{\beta_3}k}{4\sqrt{\gamma}}, \quad b_1 = 0, \quad d_1 = -\frac{\sqrt{\alpha}k}{4\sqrt{\gamma}}, \quad \omega = 4a_0^2\gamma - \alpha\sigma^2,$$
$$\delta = 2\sqrt{\alpha}\sqrt{\gamma}, \quad \beta_1 = \frac{16a_0^2\gamma}{\alpha k^2}. \tag{10}$$

Putting the Eq.(10), only for the positive value of a_1 into Eq.(9), then we obtain the solutions of Eq.(1), as:

$$q_1(x,t) = \left(\sqrt{a_0 + \left(k\sqrt{\alpha}\sqrt{\beta_1}\left(\beta_2\epsilon\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right) - \right.\right.} \right.$$
$$\sqrt{2\sqrt{\beta_1}\sqrt{\beta_3}\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)+1\right)^2\Big)\Big)\Big/}$$
$$\left.\sqrt{8\beta_2\sqrt{\gamma}\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)+1\right)\Big)}\right)e^{i(-\sigma x+\omega t+\theta)} \tag{11}$$

$$q_2(x,t) = \left(\sqrt{a_0 - \left(\sqrt{\alpha}k\left(2\sqrt{\beta_1}\epsilon\left(\eta\cosh\left(\sqrt{\beta_1}(\theta+\zeta_0)\right)+1\right)+\right.\right.}\right.$$
$$\sqrt{\sqrt{\frac{\beta_1}{\beta_3}}\sqrt{\beta_3}\left(\cosh\left(\sqrt{\beta_1}(\theta+\zeta_0)\right)+\epsilon\sinh\left(\sqrt{\beta_1}(\theta+\zeta_0)\right)+\eta\right)^2\Big)\Big)\Big/}$$
$$\sqrt{\left(8\sqrt{\gamma}\left(\cosh\left(\sqrt{\beta_1}(\theta+\zeta_0)\right)+\eta\right)\right.}$$
$$\left.\sqrt{\cosh\left(\sqrt{\beta_1}(\theta+\zeta_0)\right)+\epsilon\sinh\left(\sqrt{\beta_1}(\theta+\zeta_0)\right)+\eta\Big)}\right)e^{i(-\sigma x+\omega t+\theta)} \tag{12}$$

$$q_3(x,t) = \left(\sqrt{\left(a_0 + \left(\sqrt{\alpha}k\epsilon\left(-1-\eta\sqrt{p^2+1}\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+\right.\right.\right.}\right.$$
$$\sqrt{p\sinh\left(\sqrt{\beta_1}\left(2\alpha k\sigma t+kx+\zeta_0\right)\right)\Big)\sqrt{\beta_1}\Big)\Big/}$$
$$\sqrt{\left(4\sqrt{\gamma}\left(\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+\eta\sqrt{p^2+1}\right)\right.}$$
$$\sqrt{\left(\eta\sqrt{p^2+1}+\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+\right.}$$
$$\sqrt{\epsilon\left(\sinh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+p\right)\Big)\Big)+}$$
$$\left.\sqrt{\frac{\sqrt{\alpha}\sqrt{\beta_3}k\left(-\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}(k(2\alpha\sigma t+x)+\zeta_0)\right)+p\right)}{\cosh\left(\sqrt{\beta_1}(k(2\alpha\sigma t+x)+\zeta_0)\right)+\eta\sqrt{p^2+1}}-1\right)}{4\sqrt{\gamma}}}\right)e^{i(-\sigma x+\omega t+\theta)} \tag{13}$$

Case-II

$$
\begin{aligned}
a_0 =& a_0, \quad a_1 = \pm\frac{\sqrt{\alpha}\sqrt{\beta_3}k}{4\sqrt{\gamma}}, \quad b_1 = 0, \quad d_1 = \frac{\sqrt{\alpha}k}{4\sqrt{\gamma}}, \quad \omega = 4a_0^2\gamma - \alpha\sigma^2, \\
\delta =& -2\sqrt{\alpha}\sqrt{\gamma}, \quad \beta_1 = \frac{16a_0^2\gamma}{\alpha k^2}.
\end{aligned} \tag{14}
$$

Putting Eq.(14), only for positive value of a_1 in Eq.(9), then we obtain the solutions of Eq.(1), as:

$$
\begin{aligned}
q_4(x,t) = \Bigg(&\sqrt{a_0 - \Big(k\sqrt{\alpha}\sqrt{\beta_1}\Big(\beta_2\epsilon\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right) +} \\
&\sqrt{2\sqrt{\beta_1}\sqrt{\beta_3}\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)+1\right)^2\Big)\Big)\Big/} \\
&\sqrt{8\beta_2\sqrt{\gamma}\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)+1\right)}\Bigg)\, e^{i(-\sigma x+\omega t+\theta)}
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
q_5(x,t) = \Bigg(&\sqrt{\Big(a_0 - \Big(k\sqrt{\alpha}\Big(-2\sqrt{\beta_1}\epsilon\left(\eta\cosh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)+1\right) +} \\
&\sqrt{\Big(\eta+\cosh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right) +} \\
&\sqrt{\epsilon\sinh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)\Big)^2\ \sqrt{\frac{\beta_1}{\beta_3}}\sqrt{\beta_3}\Big)\Big)\Big/} \\
&\sqrt{\Big(8\sqrt{\gamma}\left(\eta+\cosh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)\right)} \\
&\sqrt{\Big(\eta+\cosh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)} \\
&\sqrt{+\epsilon\sinh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)\Big)\Big)\Big)}\Bigg)\, e^{i(-\sigma x+\omega t+\theta)}
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
q_6(x,t) = \Bigg(&\sqrt{\Big(a_0 + \Big(\sqrt{\alpha}k\epsilon\left(1+\eta\sqrt{p^2+1}\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right) -\right.} \\
&\sqrt{p\sinh\left(\sqrt{\beta_1}\left(2\alpha k\sigma t+kx+\zeta_0\right)\right)\right)\ \sqrt{\beta_1}\Big)\Big/} \\
&\sqrt{\Big(4\sqrt{\gamma}\left(\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+\eta\sqrt{p^2+1}\right)} \\
&\sqrt{\Big(\eta\sqrt{p^2+1}+\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right) +} \\
&\sqrt{\epsilon\left(\sinh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+p\right)\Big)\Big) +}
\end{aligned}
$$

$$\sqrt{\frac{\sqrt{\alpha}\sqrt{\beta_3}k\left(-\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}(k(2\alpha\sigma t+x)+\zeta_0)\right)+p\right)}{\cosh\left(\sqrt{\beta_1}(k(2\alpha\sigma t+x)+\zeta_0)\right)+\eta\sqrt{p^2+1}}-1\right)}{4\sqrt{\gamma}}}\Bigg) e^{i(-\sigma x+\omega t+\theta)} \tag{17}$$

Case-III

$$a_0=0,\quad a_1=\pm d_1\sqrt{\beta_3},\quad b_1=0,\quad d_1=d_1,\omega=-\alpha\sigma^2,\quad \delta=-\frac{3\alpha k}{8d_1}-\frac{2\gamma d_1}{k}. \tag{18}$$

Putting the Eq.(18), only for positive value of a_1 in Eq.(9), then we obtain the solutions of Eq.(1), as:

$$q_7(x,t)=\left(\sqrt{-\frac{\sqrt{\beta_1}d_1\epsilon\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)}{2\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+2}}-\right.$$
$$\left.\sqrt{\frac{\beta_1\sqrt{\beta_3}d_1\left(\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+1\right)}{\beta_2}}\right)e^{i(-\sigma x+\omega t+\theta)} \tag{19}$$

$$q_8(x,t)=\left(\sqrt{-\left(d_1\left(-4\sqrt{\beta_1}\epsilon\left(\eta\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+1\right)+\right.\right.}\right.$$
$$\sqrt{2\left(\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+\eta\right.}$$
$$\sqrt{\left.+\epsilon\sinh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)\right)^2\ \sqrt{\frac{\beta_1}{\beta_3}}\sqrt{\beta_3}\Big)\Big)\Big/}$$
$$\sqrt{\left(4\left(\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+\eta\right)\right.}$$
$$\sqrt{\left(\eta+\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+\right.}$$
$$\left.\sqrt{\epsilon\sinh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)\Big)\Big)}\right)e^{i(-\sigma x+\omega t+\theta)} \tag{20}$$

$$q_9(x,t)=\left(\sqrt{\left(-\left(d_1\left(\left(\epsilon\left(\left(-1-\eta\sqrt{p^2+1}\cosh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)+\right.\right.\right.\right.\right.}\right.$$
$$\sqrt{p\sinh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)\Big)\sqrt{\beta_1}+}$$
$$\sqrt{\sinh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)+p}$$
$$\sqrt{\left(2\eta\sqrt{p^2+1}+2\cosh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)+\right.}$$
$$\sqrt{\epsilon\left(\sinh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)+p\right)\Big)\ \sqrt{\beta_3}\Big)\Big)\Big/}$$

$$\sqrt{\sqrt{\beta_3}+\left(\cosh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)+\eta\sqrt{p^2+1}\right)^2\Big)\Big)\Big/}$$
$$\sqrt{\frac{\epsilon\left(\sinh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)+p\right)}{\cosh\left(\sqrt{\beta_1}\left(k(x+2\alpha\sigma t)+\zeta_0\right)\right)+\eta\sqrt{p^2+1}}+1\Bigg)}e^{i(-\sigma x+\omega t+\theta)} \tag{21}$$

Case-IV

$$\begin{aligned} a_0 =&0,\quad a_1=b_1=0,\quad d_1=\pm\frac{\sqrt{\alpha}k}{2\sqrt{\gamma}},\omega=\frac{\alpha\beta_2^2k^2}{16\beta_3}-\alpha\sigma^2,\\ &\delta=\pm2\sqrt{\alpha}\sqrt{\gamma},\quad \beta_1=\frac{\beta_2^2}{4\beta_3}. \end{aligned} \tag{22}$$

Putting Eq.(22), only for positive value of d_1 in Eq.(9), then we obtain the solutions of Eq.(1), as:

$$q_{10}(x,t)=\sqrt{-\frac{\sqrt{\alpha}\sqrt{\beta_1}k\epsilon\operatorname{csch}^2\left(\frac{1}{2}\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)}{\sqrt{\gamma}\left(4\epsilon\coth\left(\frac{1}{2}\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+4\right)}}e^{i(-\sigma x+\omega t+\theta)} \tag{23}$$

$$\begin{aligned} q_{11}(x,t)=\Bigg(&\sqrt{\sqrt{\alpha}\sqrt{\beta_1}k\epsilon\left(\eta\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+1\right)/(2\sqrt{\gamma}}\\ &\sqrt{\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+\eta\left(\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+\eta\right.}\\ &\sqrt{\left.+\epsilon\sinh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)\right)\Big)}\Bigg)e^{i(-\sigma x+\omega t+\theta)} \end{aligned} \tag{24}$$

$$\begin{aligned} q_{12}(x,t)=&\sqrt{\left(\sqrt{\alpha}k\epsilon\left(\eta\sqrt{p^2+1}\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)-\right.\right.}\\ &\sqrt{\left.p\sinh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+1\right)}\\ &\sqrt{\left.\sqrt{\beta_1}\right)\Big/\left(2\sqrt{\gamma}\left(\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+\eta\sqrt{p^2+1}\right)\right.}\\ &\sqrt{\left(\eta\sqrt{p^2+1}+\cosh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+\right.}\\ &\sqrt{\left.\left.\epsilon\left(\sinh\left(\sqrt{\beta_1}\left(k(2\alpha\sigma t+x)+\zeta_0\right)\right)+p\right)\right)\right)}\Bigg)e^{i(-\sigma x+\omega t+\theta)} \end{aligned} \tag{25}$$

§4 Conclusion

We constructed successfully the new optical soliton solutions of KE equation by using modified form of mathematical method. We obtained the solutions in the form of dark solitons, bright solitons and combined dark-bright solitons, travelling wave and periodic wave solutions with general coefficients. The all calculations show that this technique is more powerful, effective, straightforward, and fruitfulness to study analytically other higher-order nonlinear complex PDEs involves in mathematical physics, quantum physics, Geo physics, fluid mechanics, hydrodynamics, mathematical biology, field of engineering and many other physical sciences.

These obtained solutions are more useful in the development of optical fibers, dynamics of solitons, dynamics of adiabatic parameters, dynamics of fluid, problems of biomedical, industrial phenomena and many other branches.

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