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Optical dromions for complex Ginzburg Landau model with nonlinear media

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Abstract. This manuscript studies the optical dromions with beta derivative (BD) applied to the Complex Ginzburg Landau equation (CGLE) with Kerr law, parabolic law, cubic quintic septic law and quadratic cubic law. We obtain bright dromians by using the sine-cosine method (SCM). We will also obtain domain walls with the assistance of Bernoulli equation approach (BEA). Constraint conditions are also listed.

§1 Introduction

Optical solitons have far-reaching utilization in electromagnetic and telecommunications, especially in the dynasty of ultrafast signal processing systems and optical soliton communications. The nonlinear Schrödinger equation (NLSE) is a universal nonlinear model that explains many physical nonlinear systems. It can be used in nonlinear optics, quantum condensates, nonlinear acoustics, hydrodynamics, plasma physics, heat pulses in solids and in many nonlinear instability phenomena; There are so many methods to get optical solitons like as [1–10]. The NLSE has been used to explain a variety of effects in the propagation of optical pulses. The balance between the self-phase modulation and group velocity dispersion leads to the so-called soliton solutions for the NLSE [11–20]. This behavior of the pulse propagation offered the potential for understanding pulse transmission over very long distances. The importance of studying optical solitons comes from the fact that they have potential applications in optical transmission and all-optical processing. Solitary wave solutions have been known to exist in a variety of nonlinear and dispersive media for many years. Solitons propagation is explicated by nonlinear Schrödinger equation (NLSE) with non-Kerr law nonlinearities. There are power law, log law, dual power law, parabolic law, triple power law [25] etc. Recently, some new non-Kerr laws have been observed. These are anti-cubic law [23], cubic quintic septic law [21],

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quadratic cubic law [21]. A lot of work has been done on these nonlinearities [1-13]. The NLSE is transformed into cubic complex Ginzburg Landau (CGLC) equation after the linear and nonlinear gains and losses. This paper genuinely examine the Kerr law and some non-Kerr laws for beta-derivative (BD) to the optical solitons for CGLE with the aid of two integration norms. The constraint conditions are also reported for optical solitons. We will use SCM to get bright soliton and BEA to retrieve dark solitons.

Elucidation of the effect of memory in modelling has been a serious concern for quite a long time. Classical techniques were insufficient for the exploration of memory [34–36]. Several scientists contemplated that fractional derivatives are favorites to overcome the problem of memory effect. Khalil et al. introduced a modish definition of derivative known as comformable derivative (CD) [37]. Antangana gave introduction of a new derivative called beta derivative, after the evaluation of CD through definitions and theorems [38]. The BD can be stated as [33]:

$$ {}_0^A D_x^\beta(g(x)) = \lim_{\delta\to 0} \frac{g(x+\delta(x+\frac{1}{\Gamma(\beta)})-g(x)}{\delta}. $$

Here we are listing the various properties of BD

$$ {}_0^A D_x^\beta(ag(x)+bh(x)) = a{}_0^A D_x^\beta g(x)_0^A + b{}_0^A D_x^\beta h(x)_0^A, \tag{1} $$

$$ D_x^\beta(c) = 0, \tag{2} $$

for any constant c

$$ {}_0^A D_x^\beta(g(x)*h(x)) = h(x){}_0^A D_x^\beta g(x) + g(x){}_0^A D_x^\beta h(x), \tag{3} $$

$$ {}_0^A D_x^\beta\left(\frac{g(x)}{h(x)}\right) = \frac{h(x){}_0^A D_x^\beta g(x) - g(x){}_0^A D_x^\beta h(x)}{h^2(x)}, \tag{4} $$

Considering $\delta = (x+\frac{1}{\Gamma(\beta)})^{\beta-1}f$, $f\to 0$ when $\delta\to 0$ therefore we have

$$ {}_0^A D_x^\beta(g(x)) = \left(x+\frac{1}{\Gamma(\beta)}\right)^{1-\beta}\frac{dg(x)}{dx}, \tag{5} $$

with

$$ \eta = \frac{l(x+\frac{1}{\Gamma(\beta)})^\beta}{\beta}, \tag{6} $$

where l is a constant.

$$ {}_0^A D_x^\beta\left(\frac{g(\eta)}{h(x)}\right) = l\frac{dg(\eta)}{d\eta}. \tag{7} $$

§2 Mathematical Model

The CGLE equation is given by [33].

$$ i{}_0^E D_t^\beta q + a{}_0^E D_x^{2\beta} q + bF(|q|^2)q = \frac{1}{|q|^2 q^*}\left[\epsilon{}_0^E D_x^{2\beta}(|q|^2)|q|^2 - B({}_0^E D_x^\beta q)^2\right] + Aq, \tag{8} $$

where $q(x,t)$ is the normalized electric feild, ${}_0^E D_t^\beta$ and ${}_0^E D_x^\beta$ are the beta derivatives [38], 0 $< \beta \leq 1$, describing the order of fractional derivatives and a, b, A, B and ϵ are the real constants.

We assume the following transformation,

$$q(x,t) = q(\eta)e^{i\psi(x,t)}, \tag{9}$$

$q(x,t)$ represents the shape of the pulse so that,

$$\eta = \frac{(x+\frac{1}{\Gamma(\beta)})^{\beta}}{\beta} - \frac{v(t+\frac{1}{\Gamma(\beta)})^{\beta}}{\beta}, \tag{10}$$

and the phase component is given by

$$\psi(x,t) = \frac{-k(x+\frac{1}{\Gamma(\beta)})^{\beta}}{\beta} + \frac{w(t+\frac{1}{\Gamma(\beta)})^{\beta}}{\beta} + \phi(\eta), \tag{11}$$

where soliton frequency, wave number, phase function and speed of soliton are represented by k, w, $\phi(\eta)$ and v respectively. Now substituting Eq. (9)- Eq. (11) into Eq. (8), we get the real and imaginary parts

$$wq + a(q'' - k^2 q) + bF(q^2)q = 2(\epsilon - 2B)\frac{q'^2}{q} + 2\epsilon q'' + Aq, \tag{12}$$

and

$$v = -2ak, \tag{13}$$

v denotes the soliton velocity. Setting $\epsilon = 2\beta$ in Eq. (12), we get

$$(a - 4B)q'' - (w + ak^2 + A)q + bF(q^2)q = 0. \tag{14}$$

In the coming section, we will find bright soliton by virtue of SCM and dark soliton with aid of BEA with Kerr law.

§3 Kerr law

Kerr law can be written as [21]

$$F(q) = q, \tag{15}$$

Thus Eq. (14) becomes

$$(a - 4B)q'' - (w + ak^2 + A)q + bq^3 = 0. \tag{16}$$

3.1 SCM

We consider the following solutions [31]

$$q(\eta) = \lambda cos^{\beta}(\mu\eta), \tag{17}$$

$$q'(\eta) = -\lambda\beta cos^{\beta-1}(\mu\eta)sin(\mu\eta), \tag{18}$$

$$q''(\eta) = \lambda\mu^2\beta(\beta-1)cos^{\beta-2}(\mu\eta) - \lambda\beta^2\mu^2 cos^{\beta}(\mu\eta), \tag{19}$$

By using Eq. (17) - Eq. (19) into Eq. (16) and comparing different powers of $cos(\mu\eta)$, we get some equations, which provide the following solutions

$$\beta = -1,\ \mu = \sqrt{\frac{w + ak^2 + A}{4B - a}}, \tag{20}$$

$$\lambda = \sqrt{\frac{2(w + ak^2 + A)}{b}}, \tag{21}$$

Where a, b, B, k, A and w are the constants, also $(w+ak^2+A)(4B-a) > 0$ and $(w+ak^2+A)b > 0$. As a result, the bright soliton for Eq. (8) can be shown as.

$$q_{11}(x,t) = \sqrt{\frac{2(w + ak^2 + A)}{b}} sech\left[\sqrt{\frac{w + ak^2 + A}{4B - a}}\left(\frac{(x + \frac{1}{\Gamma(\beta)})^\beta}{\beta} - \frac{v(t + \frac{1}{\Gamma(\beta)})^\beta}{\beta}\right)\right] e^{i\psi(x,t)}. \tag{22}$$

Fig (1a) is 3D plot and Fig (1b) shows 2D plot for bright dromians for the given values of parameters.

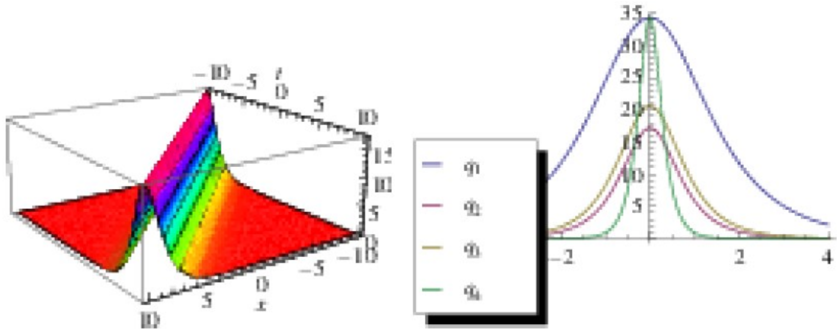

Figure 1. The graphical description of $q_{11}(x,t)$ in Eq. (22) given by these parameters $\omega = 5, \theta = 1, \beta = 1, v = 1, \kappa = 3, a = 10, b = 2, A = 200, B = 100$ in interval (-10,10) and (-10,10). Fig. (a) shows 3D graph of $q_{11}(x,t)$ and Fig. (b) shows 2D plot of $q_{11}(x,t)$ in the interval $-4 \leq x \leq 4$.

3.2 BEA

For BEA, we use the following substitution [31],

$$q(\eta) = A_0 + A_1 u(\eta), \tag{23}$$

where

$$u(\eta) = \frac{\delta}{2}\left[1 + tanh\left(\frac{\delta}{2}(\eta)\right)\right], \tag{24}$$

where A_0 and A_1 are constants and u satisfies

$$u'(\eta) = \delta u(\eta) - u^2(\eta), \tag{25}$$

By using Eq. (23)- Eq. (25) in Eq. (16), we procure various equations, which gives the specified solutions

$$\delta = \frac{bA_0A_1}{(a - 4B)}, \tag{26}$$

$$A_1 = \sqrt{\left(\frac{(2a - 8B)}{b}\right)}, \quad A_0 = \sqrt{\left(\frac{(w + ak^2 + A)}{b}\right)}, \tag{27}$$

where κ, A_0, A_1 are arbitrary constants, also $(2a-8b)b>0$. After using above values in Eq. (24) we obtain,

$$u(\eta)=\frac{bA_0A_1}{2(a-4B)}\left[1\pm tanh(\frac{bA_0A_1}{2(a-4B)}\left(\frac{(x+\frac{1}{\Gamma(\beta)})^{\beta}}{\beta}-\frac{v(t+\frac{1}{\Gamma(\beta)})^{\beta}}{\beta}\right))\right],$$

Thus the dark soliton solutions for Eq. (8) can be shown as

$$q_{12}(x,t)$$
$$=\left[A_0+\frac{bA_0A_1^2}{2(a-4B)}\left(1\pm tanh(\frac{bA_0A_1}{2(a-4B)}\left(\frac{(x+\frac{1}{\Gamma(\beta)})^{\beta}}{\beta}-\frac{v(t+\frac{1}{\Gamma(\beta)})^{\beta}}{\beta}\right))\right)\right]e^{i\psi(x,t)}. \tag{28}$$

Fig (2a) is 3D plot and Fig (2b) shows 2D plot for domain walls for the given values of parameters.

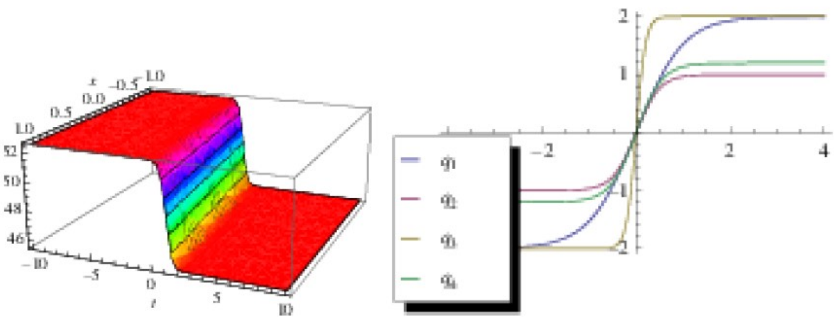

Figure 2. The graphical description of $q_{12}(x,t)$ in Eq. (28) given by these parameters $\omega=5,\theta=1,\beta=1,v=1,k=3,b=2,B=10$. in interval (-1, 1) and (-10, 10). Fig. (a) shows 3D graph of $q_{12}(x,t)$ and Fig. (b) shows 2D plot of $q_{12}(x,t)$.

In the upcoming section, we will retain bright soliton with the help of SCM and dark soliton by the aid of BEA with parabolic law.

§4 Parabolic law

For parabolic, we assume [31]

$$F(q)=c_1q+c_2q^2, \tag{29}$$

Thus Eq. (14) becomes

$$(a-4B)q''-(w+ak^2+A)q+b(c_1q^3+c_2q^5)=0. \tag{30}$$

4.1 SCM

By using Eq. (17) - Eq. (19) into Eq. (30) and comparing different powers of $cos(\mu\eta)$, we get so many equations, which provide the following solutions.

$$\beta=\frac{-1}{2},\quad \mu=\sqrt{\frac{(w+ak^2+A)}{4(4B-a)}}, \tag{31}$$

$$\lambda = \left(\frac{3(w + ak^2 + A)}{2bc_2}\right)^{\frac{1}{4}}, \tag{32}$$

Where a, b, B, k, A and w are the constants, also $(w + ak^2 + A)4(4B - a) > 0$ and $(w + ak^2 + A)bc_2 > 0$. Thus we retrieve bright soliton solution for Eq. (8).

$$q_{21}(x,t) = \left(\frac{3(w+ak^2+A)}{2bc_2}\right)^{\frac{1}{4}} sech^{\frac{1}{2}}\left[\sqrt{\frac{(w+ak^2+A)}{4(a-4B)}}\left(\frac{(x+\frac{1}{\Gamma(\beta)})^{\beta}}{\beta} - \frac{v(t+\frac{1}{\Gamma(\beta)})^{\beta}}{\beta}\right)\right]e^{i\psi(x,t)}. \tag{33}$$

Fig (3a) is 3D plot and Fig (3b) shows 2D plot for bright dromians for the given values of parameters.

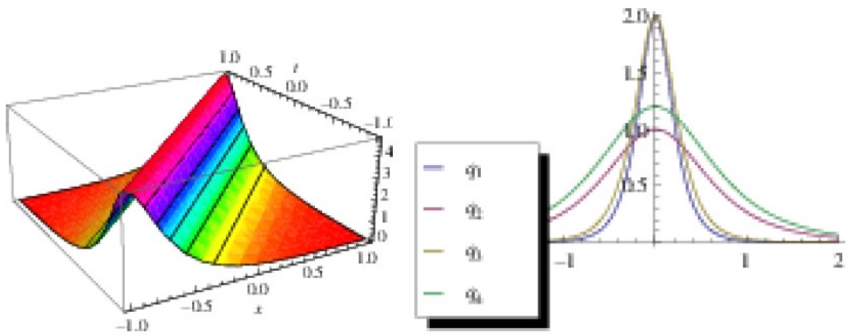

Figure 3. The graphical description of $q_{21}(x,t)$ in Eq. (33) given by these parameters $\omega = 5, \theta = 1, \beta = 1, v = 1, k = 10, A = 300, a = 10, b = 2, B = 15, c_2 = 2$ in interval (-1,1) and (-1,1). Fig. (a) shows 3D graph of $q_{21}(x,t)$ and Fig. (b) shows 2D plot of $q_{21}(x,t)$.

4.2 BEA

Here we assume that [31],

$$q(\eta) = A_1(u(\eta))^{\frac{1}{2}}, \tag{34}$$

where

$$u(\eta) = \frac{\delta}{2}\left[1 + tanh\left(\frac{\delta}{2}(\eta)\right)\right], \tag{35}$$

where A_1 is a constant and u satisfies

$$u'(\eta) = \delta u(\eta) - u^2(\eta). \tag{36}$$

By using Eq. (34)- Eq. (36) into Eq. (30), we achieve a system of equations, which gives the following solutions.

$$\delta = \sqrt{\frac{4w + ak^2 + A}{(a - 4B)}}, \tag{37}$$

$$A_1 = \left(\frac{3(4 - aB)}{4bc_3}\right)^{\frac{1}{4}}, \tag{38}$$

where k, A_1 are arbitrary constants, also $(4w + ak^2 + A)(a - 4B) > 0$. Now after using above values in Eq. (35) we gain,

$$u(\eta) = \sqrt{\frac{4w + ak^2 + A}{4(a - 4B)}}\left[1 \pm tanh(\sqrt{\frac{4w + ak^2 + A}{4(a - 4B)}}\eta)\right],$$

Thus, the dark soliton for Eq. (8) is given by

$$q_{22} = A_1\left[\sqrt{\frac{4w+ak^2+A}{4(a-4B)}}\left[1 \pm tanh\left(\sqrt{\frac{4w+ak^2+A}{4(a-4B)}}\left(\frac{(x+\frac{1}{\Gamma(\beta)})^\beta}{\beta} - \frac{v(t+\frac{1}{\Gamma(\beta)})^\beta}{\beta}\right)\right)\right]\right]^{\frac{1}{2}} e^{i\psi(x,t)}. \quad (39)$$

Fig. (4a) is 3D plot and Fig. (4b) shows 2D plot for domain walls for the given values of parameters.

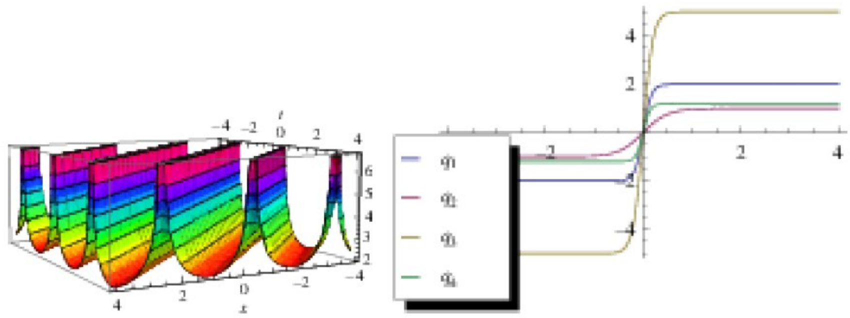

Figure 4. The graphical description of $q_{22}(x,t)$ in Eq. (39) given by these parameters $\omega = 10, \theta = 2, \beta = 1, v = 1, k = 3, A = 10, a = 10, b = 2, c_3 = 10, B = 10$ in interval (-4, 4) and (-4, 4). Fig. (a) shows 3D graph of $q_{22}(x,t)$ and Fig. (b) shows 2D plot of $q_{22}(x,t)$.

In the upcoming section, we will acquire bright soliton with the help of SCM and dark soliton by the aid of BEA with cubic quintic septic law.

§5 Cubic quintic septic law

For cubic quintic septic law, we consider [21]

$$F(q) = c_1 q + c_2 q^2 + c_3 q^3, \quad (40)$$

Thus Eq. (14) becomes

$$(a - 4B)q'' - (w + ak^2 + A)q + b(c_1 q^3 + c_2 q^5 + c_3 q^7) = 0. \quad (41)$$

5.1 SCM

By using Eq. (17) - Eq. (19) in Eq. (41), we gather system of equations, possessing following solution.

$$\beta = \frac{-1}{3}, \mu = \sqrt{\frac{(w + ak^2 + A)}{9(4B - a)}}, \quad (42)$$

$$\lambda = \left(\frac{12(w + ak^2 + A)}{9bc_3}\right)^{\frac{1}{6}}, \tag{43}$$

Where a, b, B, k, A and w are the constants, also $(w + ak^2 + A)(4B - a) > 0$. Thus the bright soliton for Eq. (8) is given by

$$q_{31}(x,t)$$
$$= \left(\frac{12(w+ak^2+A)}{9bc_3}\right)^{\frac{1}{6}} sech^{\frac{1}{3}}\left[\sqrt{\frac{(w+ak^2+A)}{9(4B-a)}}\left(\frac{(x+\frac{1}{\Gamma(\beta)})^{\beta}}{\beta} - \frac{v(t+\frac{1}{\Gamma(\beta)})^{\beta}}{\beta}\right)\right]e^{i\psi(x,t)}. \tag{44}$$

Fig(5a) is 3D plot and Fig(5b) shows 2D plot for bright dromion for the given values of parameters.

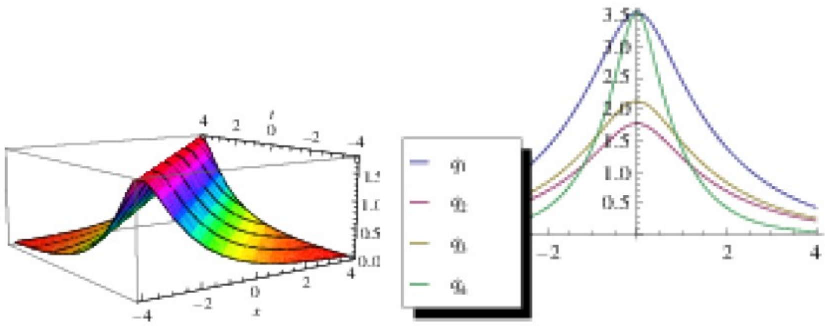

Figure 5. The graphical description of $q_{31}(x,t)$ in Eq. (44) given by these parameters $\omega = 5, \theta = 1, \beta = 1, v = 1, k = 30, a = 10, b = 20, A = 300, B = 150, c = 20$ in interval (-4, 4) and (-4, 4). Fig. (a) shows 3D graph of $q_{31}(x,t)$ and Fig. (b) shows 2D plot of $q_{31}(x,t)$.

5.2 BEA

Here we consider [31],

$$q(\eta) = A_1(u(\eta))^{\frac{1}{3}}, \tag{45}$$

where

$$u(\eta) = \frac{\delta}{2}\left[1 + tanh\left(\frac{\delta}{2}(\eta)\right)\right], \tag{46}$$

where A_1 is a constant and u satisfies

$$u'(\eta) = \delta u(\eta) - u^2(\eta). \tag{47}$$

By using Eq. (51)-Eq. (53) into Eq. (44) and equating the coefficients, we retrieve few algebraic equations, which produce the given solution.

$$\delta = \sqrt{\frac{9(w + ak^2 + A)}{(a - 4B)}}, \tag{48}$$

$$A_1 = \left(\frac{-4(a - 4B)}{abc_3}\right)^{\frac{1}{6}}, \tag{49}$$

where k, A_1 are arbitrary constants.Also $(w + ak^2 + A)(4B - a) > 0$. After using the above values into Eq. (46) we gain,

$$u(\eta) = \sqrt{\frac{9(w + ak^2 + A)}{4(a - 4B)}}\left[1 \pm tanh(\sqrt{\frac{9(w + ak^2 + A)}{4(a - 4B)}}\eta)\right],$$

Hence, the dark soliton for Eq. (8) can be written as

$$q_{32}(x,t)$$

$$= A_1\left[\sqrt{\frac{9(w+ak^2+A)}{4(a-4B)}}\left[1 \pm tanh(\sqrt{\frac{9(w+ak^2+A)}{4(a-4B)}}\left(\frac{(x+\frac{1}{\Gamma(\beta)})^\beta}{\beta} - \frac{v(t+\frac{1}{\Gamma(\beta)})^\beta}{\beta}\right))\right]\right]^{\frac{1}{3}} e^{i\psi(x,t)}. \quad (50)$$

Fig(6a) is 3D plot and Fig(6b) shows 2D plot for domain walls for the given values of parameters.

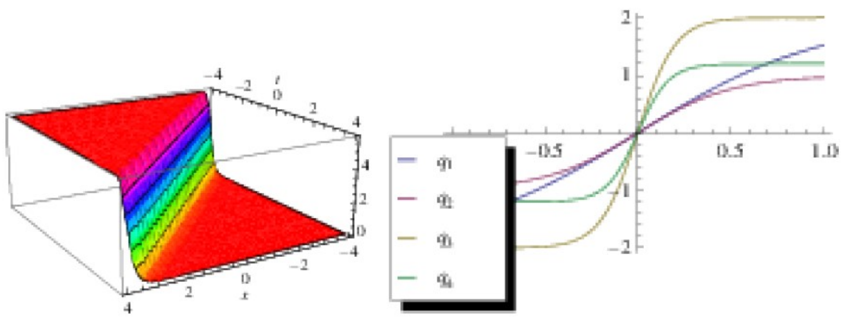

Figure 6. The graphical description of $q_{32}(x,t)$ in Eq. (50) given by these parameters $\omega = 5, \theta = 2, \beta = 1, v = 1, k = 2, b = 2, c_3 = 10, a = 10, A = 10, B = 2$ in interval (-4, 4) and (-4, 4). Fig. (a) shows 3D graph of $q_{32}(x,t)$ and Fig. (b) shows 2D plot of $q_{32}(x,t)$.

In the next section, we will procure bright soliton and dark soliton with the help of SCM and BEA respectively under quadratic cubic law.

§6 Quadratic cubic law

For quadratic cubic law, we assume that [21]

$$F(q) = c_1\sqrt{q} + c_2 q, \quad (51)$$

Thus Eq. (14) becomes

$$(a - 4B)q'' - (w + ak^2 + A)q + b(c_1 q^2 + c_2 q^3) = 0. \quad (52)$$

6.1 SCM

By using Eq. (17) - Eq. (19) into Eq. (52) and comparing different powers of $cos(\mu\eta)$, we get various equations, which retain the specified values.

$$\beta = -1, \mu = \sqrt{\frac{(w + ak^2 + A)}{(4B - a)}}, \quad (53)$$

$$\lambda = \sqrt{\frac{2(w + ak^2 + A)}{bc_2}}, \tag{54}$$

Where a, b, B, k, A and w are the constants, also $(w + ak^2 + A)(4B - a) > 0$. Thus, the bright soliton for Eq. (8) is given as

$$q_{41}(x,t) = \sqrt{\frac{2(w+ak^2+A)}{bc_2}}sech\left[\sqrt{\frac{(w+ak^2+A)}{(4B-a)}}\left(\frac{(x+\frac{1}{\Gamma(\beta)})^\beta}{\beta} - \frac{v(t+\frac{1}{\Gamma(\beta)})^\beta}{\beta}\right)\right]e^{i\psi(x,t)}. \tag{55}$$

Fig. (7a) is 3D plot and Fig. (7b) shows 2D plot for bright dromians for the given values of parameters.

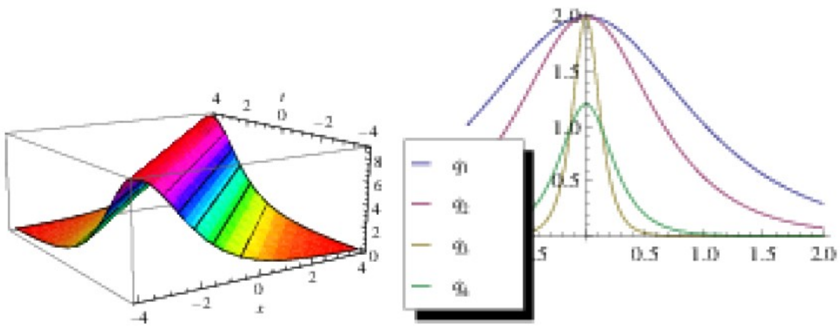

Figure 7. The graphical description of $q_{41}(x,t)$ in Eq. (55) given by these parameters $\omega = 5, \theta = 1, \beta = 1, v = 1, k = 3, a = 10, b = 2, A = 300, B = 200, c = 5$ in interval (-4, 4) and (-4, 4). Fig. (a) shows 3D graph of $q_{41}(x,t)$ and Fig. (b) shows 2D plot of $q_{41}(x,t)$.

6.2 BEA

Here we assume that [31],

$$q(\eta) = A_0 + A_1 u(\eta), \tag{56}$$

where

$$u(\eta) = \frac{\delta}{2}\left[1 + tanh\left(\frac{\delta}{2}(\eta)\right)\right], \tag{57}$$

where A_1 and A_0 are constants and u satisfies

$$u'(\eta) = \delta u(\eta) - u^2(\eta). \tag{58}$$

By using Eq. (56)- Eq. (58) in Eq. (52) and by comparing the coefficients, we retrieve some algebraic equations, which yields the solutions.

$$\delta = \frac{A_1(bc_1 + 3bc_2A_0)}{3(a - 4B)}, \tag{59}$$

$$A_1 = \left(\frac{2(a - 4B)}{bc_2}\right)^{\frac{1}{2}}, \quad A_0 = \left(\frac{(w + ak^2 + A)}{b(c_1 + c_2)}\right), \tag{60}$$

where k, A_0, A_1 are arbitrary constants, also $2(a - 4B)bc_2 > 0$. After using above equations in Eq. (57) we obtain,

$$u(\eta) = \frac{A_1(bc_1 + 3bc_2A_0)}{6(a - 4B)}\left[1 \pm tanh(\frac{A_1(bc_1 + 3bc_2A_0)}{6(a - 4B)}\eta)\right],$$

so the dark soliton for Eq. (8) is given by

$$q_{42}(x,t)$$
$$= \left[A_0 + \frac{A_1^2(bc_1+3bc_2A_0)}{6(a-4B)}\left[1 \pm tanh(\frac{A_1(bc_1+3bc_2A_0)}{6(a-4B)}\left(\frac{(x+\frac{1}{\Gamma(\beta)})^\beta}{\beta} - \frac{v(t+\frac{1}{\Gamma(\beta)})^\beta}{\beta}\right))\right]\right]e^{i\psi(x,t)}. \quad (61)$$

Fig (8a) is 3D plot and Fig (8b) shows 2D plot for domain walls for the given values of parameters.

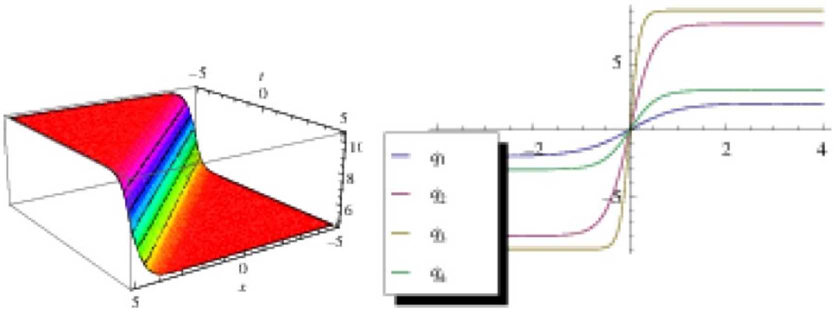

Figure 8. The graphical description of $q_{42}(x,t)$ in Eq. (61) given by these parameters $\omega = 5, \theta = 1, \beta = 1, v = 1, k = 3, a = 10, b = 2, A = 10, B = 10, b = 0.5, c_1 = 10, c_2 = 10$ in interval (-5, 5) and (-5, 5). Fig. (a) shows 3D graph of $q_{42}(x,t)$ and Fig. (b) shows 2D plot of $q_{42}(x,t)$.

§7 Results and Discussion

In this paper, we studied complex Ginzburg Landau equation with beta derivative. Yusuf et al. [33] studied this model and obtained dark and singular soliton by using generalized tanh method and generalized Bernoulli sub-ODE method with Kerr law. But they were unable to obtain bright soliton and other solitary wave solutions. We obtained bright dromion and domain walls with the help of sine-cosine method and Bernoulli equation approach respectively. Bright dromions are also referred as bell shaped solitons. When group velocity dispersion is negative, bright dromions appear in the anomalous dispersion regime. As there is no phase change for larger distance thats why bright solitons are also called non-topological solitons. In nonlinear optics, the dark solitons are also termed as topological optical solitons. For domain walls, the phase changes its form for large distance. We used four forms of nonlinearities like Kerr law, parabolic law, cubic quintic septic law and quadratic cubic law. Since we know that Kerr law nonlinearity is also known as cubic nonlinearity. This nonlinearity appears when a light wave in an optical fiber responses nonlinearly. Although the nonlinear responses are so weak, their effects emerge in several ways over long distance of propagation. Parabolic law is the generalization of Kerr law nonlinearity and it appears due to the interaction between electrons and Langmuir waves. Cubic quintic septic nonlinearity studies highly dispersive optical solitons.

To the best of our knowledge, no one used so many forms of nonlinearities for this model. Fig. (1a), Fig. (3a), Fig. (5a) and Fig. (7a) shows 3D plot while Fig. (1b), Fig. (3b), Fig. (5b) and Fig. (7b) represents 2D plot of bright dromion in Eq. (22), Eq. (33), Eq. (44) and Eq. (55) with Kerr law, parabolic law, cubic quintic septic law and quadratic cubic law respectively by using Sine cosine method. Fig. (2a), Fig. (4a), Fig. (4a) and Fig. (8a) shows 3D plot while Fig. (2b), Fig. (4b), Fig. (6b) and Fig. (8b) represents 2D plot of domain walls in Eq. (28), Eq. (39), Eq. (50) and Eq. (61) with Kerr law, parabolic law, cubic quintic septic law and quadratic cubic law respectively by using Bernaulli equation approach.

§8 Conclusions

We obtained two types of optical dromians i.e bright and dark solitons under various nonlinearities for CGLE. Previously, Yusuf et al. [33] studied CGLE equation with beta derivative for optical dromians only with Kerr law. They used generalized tanh method and sub-ODE method to obtain domain walls and singular dromians. In this paper, we used four nonlinearities those are Kerr, parabolic, cubic quintic septic and quadratic cubic law with the help of SCM and BEA. SCM retrieved bright dromians and BEA provided domain walls. Hence, we declare that our work is quite original and the obtained results might be useful in nonlinear optics to control internet bottleneck effect.

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