Appl. Math. J. Chinese Univ.
2022, 37(3): 396-403

Traveling wave for a time-periodic Lotka-Volterra model with bistable nonlinearity

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Abstract. This paper studies bistable wavefronts of a diffusive time-periodic Lotka-Volterra system. We obtain a new condition for the existence, uniqueness and stability of bistable time-periodic traveling waves. This condition is sharp and greatly improves the result established in the reference (X. Bao and Z. Wang, Journal of Differential Equations, 255(2013) 2402-2435). An example is given to demonstrate our consequence.

§1 Introduction

Consider the following Lotka-Volterra reaction-diffusion competition system

$$\begin{cases} u_t = d_1(t)u_{xx} + u(r_1(t) - a_1(t)u - b_1(t)v), \\ v_t = d_2(t)v_{xx} + v(r_2(t) - a_2(t)u - b_2(t)v), \end{cases} \quad x \in \mathbb{R},\ t > 0, \tag{1.1}$$

where $u = u(x,t)$ and $v = v(x,t)$ denote the densities of two competitive species u and v in location x at time t, respectively; the coefficient functions d_i, r_i, a_i and b_i $(i = 1, 2)$ are continuous T-periodic functions for a positive real number T. $d_1(t)$ and $d_2(t)$ are the diffusion functions, $r_1(t)$ and $r_2(t)$ represent the net birth rates, r_1/a_1 and r_2/b_2 are the carrying capacities, and a_2/r_2 and b_1/r_1 are the competitive strengths of u and v, respectively. We assume that all these functions are positive and continuous.

If all the periodic coefficients of (1.1) are positive constants, then it reduces to the classical Lotka-Volterra diffusion-competition system. In [4, 7], the authors proved the existence of traveling waves. The selection mechanism of the minimal wave speed was studied in [1,5,6].

For the periodic system (1.1) with monostable nonlinearity, Zhao and Ruan in [11] established the existence, uniqueness and asymptotic stability of time-periodic traveling waves. Alternatively importantly, we are concerned with the system (1.1) with the so-called bistable

Received: 2020-05-08. Revised: 2021-03-17.
MR Subject Classification: 35K57, 35B20, 92D25.
Keywords: existence and uniqueness, stability, bistable traveling wave, Lotka-Volterra model.
Digital Object Identifier(DOI): https://doi.org/10.1007/s11766-022-4139-9.
Supported by the National Natural Science Foundation of China(12071434, 11671359) and the NSERC discovery grant(RGPIN2016-04709).
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nonlinearity (i.e., strong-strong competition)

$$\int_0^T r_1(t) - b_1(t)q(t)\, dt < 0, \quad \int_0^T r_2(t) - a_2(t)p(t)\, dt < 0, \tag{1.2}$$

where $p(t)$ and $q(t)$ are given by

$$\begin{cases} p(t) = \dfrac{p(0)e^{\int_0^t r_1(s)ds}}{1 + p(0)\int_0^t e^{\int_0^s r_1(\tau)d\tau} a_1(s)ds}, \quad p(0) = \dfrac{e^{\int_0^T r_1(s)ds} - 1}{\int_0^T e^{\int_0^s r_1(\tau)d\tau} a_1(s)ds} > 0, \\ q(t) = \dfrac{q(0)e^{\int_0^t r_2(s)ds}}{1 + q(0)\int_0^t e^{\int_0^s r_2(\tau)d\tau} b_2(s)ds}, \quad q(0) = \dfrac{e^{\int_0^T r_2(s)ds} - 1}{\int_0^T e^{\int_0^s r_2(\tau)d\tau} b_2(s)ds} > 0. \end{cases}$$

The kinetic system of (1.1)

$$\begin{cases} \dfrac{du(t)}{dt} = u(t)(r_1(t) - a_1(t)u(t) - b_1(t)v(t)), \\ \dfrac{dv(t)}{dt} = v(t)(r_2(t) - a_2(t)u(t) - b_2(t)v(t)), \\ u(t+T) = u(t), \ v(t+T) = v(t) \end{cases} \tag{1.3}$$

has four nonnegative T-periodic solutions:

$$(0,0), \ (0, q(t)), \ (p(t), 0), \ \text{ and } (u_0(t), v_0(t)) \tag{1.4}$$

with $0 < u_0(t) < p(t)$, $0 < v_0(t) < q(t)$ for all $t \in \mathbb{R}$. Under the bistable condition (1.2), $(p(t), 0)$ and $(0, q(t))$ are linearly stable, while $(0,0)$ and $(u_0(t), v_0(t))$ are linearly unstable. For further information on the derivation of the existence and unstability of the periodic equilibrium $(u_0(t), v_0(t))$ under the condition (1.7), we refer to [2]. For convenience, we use notation

$$\overline{f} = \frac{1}{T}\int_0^T f(t)\, dt$$

to denote the average value of a function in the interval $[0, T]$, and use the following transformation

$$\phi(x,t) = \frac{u(x,t)}{p(t)}, \ \psi(x,t) = \frac{q(t) - v(x,t)}{q(t)}$$

to change the competitive system (1.1) to a cooperative system

$$\begin{cases} \phi_t = d_1(t)\phi_{xx} + \phi\,[\, a_1(t)\, p(t)\, (1-\phi) - b_1(t)\, q(t)\, (1-\psi)\,], \\ \psi_t = d_2(t)\psi_{xx} + (1-\psi)\,[\, a_2(t)\, p(t)\, \phi - b_2(t)\, q(t)\, \psi\,]. \end{cases} \tag{1.5}$$

Obviously, systems (1.5) and (1.1) are equivalent. In what follows, we will focus on system (1.5). Accordingly, the four equilibria in (1.4) become

$$\alpha_1 = (0,1), \ \mathbf{o} = (0,0), \ \beta = (1,1), \ \text{ and } \alpha^*(t) = (\phi^*(t), \psi^*(t)), \tag{1.6}$$

respectively, with $0 < \phi^*(t) < 1$, $0 < \psi^*(t) < 1$, $\phi^*(t+T) = \phi^*(t)$ and $\psi^*(t+T) = \psi^*(t)$ for $t \in \mathbb{R}$.

Bao and Wang in [2] investigated time-periodic bistable traveling waves to (1.5). Their results on the existence, uniqueness and stability are given below.

Lemma 1.1. *(Theorems 2.5, 4.4 and Corollary 4.5 in [2]) Assume that*

$$\overline{r_1} < \min_t (b_1(t)/b_2(t))\, \overline{r_2}, \quad \overline{r_2} < \min_t (a_2(t)/a_1(t))\, \overline{r_1}, \tag{1.7}$$

and

$$\overline{r_1} + \overline{r_2} > \max_t (a_2(t)/a_1(t)), \quad \overline{r_1} + \overline{r_2} > \max_t (b_1(t)/b_2(t)) \tag{1.8}$$

are true. Then

(i) there exists $c \in \mathbb{R}$ *such that (1.5) admits a* T*-periodic traveling wave* $(\phi(x,t), \psi(x,t)) = \Gamma(z,t) = (\Phi(z,t), \Psi(z,t)), z = x + ct$, *satisfying* $\Gamma(z, t+T) = \Gamma(z,t)$, $\Gamma(\infty, t) = (1,1)$ *and* $\Gamma(-\infty, t) = (0,0)$ *uniformly for all* $t > 0$. *Moreover,* $\frac{\partial}{\partial z}\Phi(z,t) > 0$ *and* $\frac{\partial}{\partial z}\Psi(z,t) > 0$ *for* $z \in \mathbb{R}$ *and* $t \in \mathbb{R}_+$*;*

(ii) the traveling wave $\Gamma(x+ct, t)$ *is stable in the sense that*

$$\lim_{t\to\infty} ||(\phi(x,t,\varrho), \psi(x,t,\varrho)) - \Gamma(x + \xi_0 + ct, t)||_{\mathcal{C}} = 0 \tag{1.9}$$

for some constant ξ_0, *where* $\varrho = (\varphi, \rho)$ *is the initial data in* $\mathcal{C}_1$ *satisfying*

$$\limsup_{x\to-\infty}(\varphi, \rho) < \delta_0(\phi_0(0), \psi_0(0)) \text{ and } \limsup_{x\to\infty}(\varphi, \rho) > (1,1) - \delta_0(\phi_1(0), \psi_1(0)) \tag{1.10}$$

for some small $\delta_0 > 0$, *with* $(\phi_0(t), \psi_0(t))$ *and* $(\phi_1(t), \psi_1(t))$ *being positive eigenfunctions of the linearized kinetic systems of (1.5) at* $(0,0)$ *and* $(1,1)$, *respectively. Here the spaces* $\mathcal{C}$ *and* $\mathcal{C}_1$ *are defined in Section 2;*

(iii) the traveling wave $(c, \Gamma(z,t))$ *is unique modulo translation in the sense that if there is another traveling wave* $(\bar{c}, \widetilde{\Gamma}(z,t))$, *then*

$$c = \bar{c} \quad \text{and } \widetilde{\Gamma}(z,t) = \Gamma(z + z_0, t), \tag{1.11}$$

for some constant z_0.

The aim of this paper is to substantially improve Lemma 1.1 by felicitously verifying the strong stability from above and below of **o** and β, respectively, only under the condition (1.2). Then we shall prove the existence, uniqueness and stability of time-periodic bistable wavefronts. The condition (1.8) will be removed. In the reference [2], (1.8) was used to prove the existence of strongly positive eigenfunctions, which implies the strong stability of two stable fixed points. Our main result is as follows.

Theorem 1.2. *Assume that (1.7) holds. Then*

(i) there exists a constant $c \in \mathbb{R}$ *such that (1.1) has a* T*-periodic traveling wave* $\Gamma(z,t) = (\Phi(z,t), \Psi(z,t)), z = x + ct$, *satisfying* $\Gamma(z, t+T) = \Gamma(z,t)$, $\Gamma(\infty, t) = (1,1)$ *and* $\Gamma(-\infty, t) = (0,0)$ *uniformly for all* $t > 0$. *Moreover,* $\frac{\partial}{\partial z}\Phi(z,t) > 0$ *and* $\frac{\partial}{\partial z}\Psi(z,t) > 0$ *for* $z \in \mathbb{R}$ *and* $t \in \mathbb{R}_+$*;*

(ii) the traveling wave $\Gamma(x+ct, t)$ *is stable in the sense that*

$$\sup_{x\in\mathbb{R}} |(\phi(x,t,\varrho), \psi(x,t,\varrho)) - \Gamma(x + \xi_0 + ct, t)| < Me^{-\mu t} \tag{1.12}$$

for some positive constants M, μ *and* ξ_0, *where* $\varrho = (\varphi, \rho)$ *is the initial data satisfying*

$$(-\delta_0, -\delta_0) < (\varphi, \rho) < (1 + \delta_0, 1 + \delta_0) \tag{1.13}$$

and

$$\limsup_{x\to-\infty} |(\varphi, \rho)| < (\delta_0, \delta_0), \text{ and } \limsup_{x\to\infty} |(1,1) - (\varphi, \rho)| < (\delta_0, \delta_0) \tag{1.14}$$

for some small positive δ_0*;*

(iii) the traveling wave $(c, \Gamma(z,t))$ *is unique modulo translation in the sense that if there is another traveling wave* $(\bar{c}, \widetilde{\Gamma}(z,t))$, *then*

$$c = \bar{c} \quad \text{and } \widetilde{\Gamma}(z,t) = \Gamma(z + z_0, t), \tag{1.15}$$

for some constant z_0.

Remark 1.1. *Since $(p(t), 0)$ and $(0, q(t))$ are solutions of (1.1), we have $\overline{r_1} = \overline{a_1 p}$ and $\overline{r_2} = \overline{b_2 q}$. Thus it is easy to see that condition (1.2) is weaker than (1.7). We conjecture that our result is true on the condition (1.2).*

This paper is organized as follows. In Section 2, we give preliminaries which will be used in the following sections. The proof of Theorem 1.2 is presented in section 3. In section 4, an example is provided to numerically demonstrate the existence, uniqueness and stability of T-periodic traveling waves to (1.1) when (1.2) is satisfied, but (1.7) and (1.8) are not true.

§2 Preliminaries

We start by proposing some notations. Suppose that χ is an ordered Banach space with the norm $\|\cdot\|_\chi$ and the positive cone χ^+. Let $\mathcal{C}$ be the set of all bounded and uniformly continuous functions from $\mathbb{R}$ to χ. We assume that $\mathrm{Int}(\chi^+)$ is not empty. For any $\xi, \varsigma \in \chi$, we say $\xi \geqslant \varsigma$ if $\xi - \varsigma \in \chi^+$, $\xi > \varsigma$ if $\xi \geqslant \varsigma$ but $\xi \neq \varsigma$, and $\xi \gg \varsigma$ if $\xi - \varsigma \in \mathrm{Int}(\chi^+)$. A subset of χ is called totally unordered provided that no two elements are ordered.

Similarly, for any $\rho, \varphi \in \mathcal{C}$, we say $\rho \geq \varphi$ if $\rho(x) > \varphi(x)$ for all $x \in \mathbb{R}$, $\rho > \varphi$ if $\rho \geq \varphi$ but $\rho \neq \varphi$, and $\rho \gg \varphi$ if $\rho(x) \gg \varphi(x)$ for all $x \in \mathbb{R}$. Let $\chi_\sigma = \{\gamma \in \chi : \sigma \geq \gamma \geq \mathbf{o}\}$ and $\mathcal{C}_\sigma = \{\varphi \in \mathcal{C} : \sigma \geq \varphi \geq \mathbf{o}\}$ for any $\sigma \in \chi$ with $\sigma > \mathbf{o}$, where $\mathbf{o}$ is the zero element in χ or $\mathcal{C}$.

Let $\beta \in \mathrm{Int}(\chi^+)$ and Q be a map from $\mathcal{C}$ to $\mathcal{C}$. Assume that E denotes the set of all fixed pints of Q restricted to χ_β.

Definition 2.1. A family of mappings $\{Q_t\}_{t\in\mathbb{R}^+}$ is called a T-periodic semiflow on a phase space $\mathcal{C}$ provided that

(i) $Q_0[\phi] = \phi$, $\forall \phi \in \mathcal{C}$,

(ii) $Q_{t+T}[\phi] = Q_t \circ Q_T[\phi]$ for all $t \geq 0$, $\phi \in \mathcal{C}$,

(iii) $Q_t[\phi]$ is jointly continuous in $(t, \phi) \in \mathbb{R}_+ \times \mathcal{C}$.

It is well known that Q_T is termed as the Poincaré map associated with $\{Q_t\}_{t\in\mathbb{R}^+}$, which is denoted by P for simplicity. Following [3], we present the following definitions and hypotheses on P.

Definition 2.2. For the map $P : \chi_\beta \to \chi_\beta$, a fixed point $\alpha \in E$ is strongly stable from above provided that there exist a number $\delta > 0$ and a positive vector $\omega_0 \in \mathrm{Int}(\chi^+)$ such that for $\eta \in (0, \delta]$,

$$P[\alpha + \eta\omega_0] \ll \alpha + \eta\omega_0. \tag{2.16}$$

Similarly, we can define strong stability from below for β by reversing all the inequality for negative η.

Definition 2.3. By a translation operator T_y on $\mathcal{C}$ for any $y \in \mathbb{R}$, we mean that

$$T_y[\phi](x) = \phi(x - y), \forall x \in \mathbb{R}, \phi \in \mathcal{C}.$$

Hypotheses on P:

(H1) (Translation invariance) $T_y \circ P[\varphi] = P \circ T_y[\varphi],\ \forall \phi \in \mathcal{C}_\beta,\ y \in \mathbb{R}$.

(H2) (Continuity) $P : \mathcal{C}_\beta \to \mathcal{C}_\beta$ is continuous with respect to the compact open topology.

(H3) (Monotonicity) P is order-preserving in the sense that $P[\rho] \geqslant P[\varphi]$ whenever $\rho \geqslant \varphi$ in $\mathcal{C}_\beta$.

(H4) (Compactness) $P : \mathcal{C}_\beta \to \mathcal{C}_\beta$ is compact with respect to the compact open topology.

(H5) (Bistability) Two fixed points $\mathbf{o}$ and β are strongly stable from above and below, respectively, for the map $P : \chi_\beta \to \chi_\beta$, and the set $E \backslash \{\mathbf{o}, \beta\}$ is totally unordered.

(H6) (Counter-propagation) For $\alpha_i \in E\backslash\{\mathbf{o}, \beta\}, c^*_-(\alpha_i, \beta) + c^*_+(0, \alpha_i) > 0, i = 1, 2$, where $c^*_-(\alpha_i, \beta)$ ($c^*_+(0, \alpha_i)$) is the leftward (rightward) spreading speed of P in the phase space $\mathcal{C}_{[\alpha_i,\beta]}$ ($\mathcal{C}_{[0,\alpha_i]}$). For the detailed definition, we refer to [3,8,9].

The lemma below is one of the main results established in [3], which will be used to prove the existence of bistable traveling waves.

Lemma 2.1. *(Theorem 3.3 in [3]) Let $\beta(t)$ be a strongly positive T-time periodic orbit of $\{Q_t\}_{t\geq 0}$ restricted to χ. If P satisfies (H1)-(H6) with $\beta = \beta(0)$, then $\{Q_t\}_{t\geq 0}$ admits a traveling wave $U(x + ct, t)$ connecting 0 to $\beta(t)$ uniformly for $t \in \mathbb{R}^+$. Furthermore, $U(x, t)$ is nondecreasing in $x \in \mathbb{R}$.*

§3 Proof of the main result

This section is devoted to proving Theorem 1.2, that is, we will show the existence, uniqueness and stability of bistable T-time periodic traveling waves to system (1.5). We denote the solution semiflow associated with system (1.5) by $\{Q_t\}_{t\geq 0}$, that is,

$$Q_t[\varrho](x) = (\phi(x, t, \varrho), \psi(x, t, \varrho)),\quad \forall \varrho \in \mathcal{C}, x \in \mathbb{R}, t \geq 0,$$

where $(\phi(x, t, \varrho), \psi(x, t, \varrho))$ is the unique solution to the Cauchy problem, i.e., system (1.5) with the initial data

$$(\phi(\cdot, 0, \varrho), \psi(\cdot, 0, \varrho)) = \varrho,\ \text{where}\ \ \varrho = (\varphi, \rho) \in \mathcal{C}.$$

Evidently, $\{Q_t\}_{t\geq 0}$ is a T-time periodic semiflow. By (1.6), the Poincaré map P associated with this periodic semiflow has four fixed points

$$\mathbf{o} = (0, 0),\ \beta = (1, 1),\ \alpha_1 = (0, 1), \text{and}\ \alpha_2 = \alpha^*(0).$$

Hence the set of all fixed points of P on χ^+ is $E = \{\mathbf{o}, \beta, \alpha_1, \alpha_2\}$. In the initial phase space $\mathcal{C}_{[\alpha_i,\beta]}, i = 1, 2$, P has the leftward spreading speed $c^*_-(\alpha_i, \beta)$, and in the initial phase space $\mathcal{C}_{[0,\alpha_i]}, i = 1, 2$, P has the rightward spreading speed $c^*_+(0, \alpha_i)$. We are now in the position to prove Theorem 1.2.

Proof. For the existence, by Lemma 2.1, we will show that P satisfies hypotheses (H1)-(H6) on $\mathcal{C}_\beta$. From system (1.5) it easily follows that P satisfies (H1)-(H4). The verification of (H6) can be found in Theorem 2.5 of [2]. Thus, under the condition (1.2), only (H5) is required to be detailedly verified.

Since $E \setminus \{\mathbf{0}, \beta\} = \{\alpha_1, \alpha_2\}$ is obviously unordered, we need only prove that $\mathbf{o}$ is strongly stable from above and β is strongly stable from below. We will use Definition 2.2 with $\alpha = \mathbf{o}$ to prove the former. The proof of the latter is similar and omitted here.

We consider the kinetic system of (1.5) over one period

$$\begin{cases} \phi' = \phi\,[\,a_1(t)\,p(t)\,(1-\phi) - b_1(t)\,q(t)\,(1-\psi)\,]\,, \\ \psi' = (1-\psi)\,[\,a_2(t)\,p(t)\,\phi - b_2(t)\,q(t)\,\psi\,]\,, \qquad\qquad t \in [0,T] \\ \phi(0) = \eta\varphi_0 > 0, \psi(0) = \eta\rho_0 > 0, \end{cases} \tag{3.17}$$

where η is a positive number. To get the strong stability on a specific direction ω_0 of the solution map $P = Q_T$ on χ^+, we first linearize system (3.17) at $\mathbf{o} = (0,0)$ to get

$$\begin{cases} \widetilde{\phi}' = [\,a_1(t)\,p(t)\, - b_1(t)\,q(t)]\,\widetilde{\phi}, \\ \widetilde{\psi}' = a_2(t)\,p(t)\,\widetilde{\phi} - b_2(t)\,q(t)\,\widetilde{\psi}, \end{cases} \quad t \in [0,T]. \tag{3.18}$$

Hence, for $\omega_0 = (\varphi_0, \rho_0) \in \mathrm{Int}(\mathbb{R}^2_+)$, define the linear Poincaré map M_T as

$$M_T(\omega_0) = (\widetilde{\phi}(T), \widetilde{\psi}(T))(\omega_0).$$

From the first equation of (3.18), we have

$$\widetilde{\phi}(T)(\omega_0) = \eta_1\varphi_0, \qquad \eta_1 = e^{\int_0^T r_1(t) - b_1(t)q(t)\ dt} < 1. \tag{3.19}$$

Using the method of variation of parameters for the second equation of (3.18), we obtain

$$\widetilde{\psi}(T)(\omega_0) = \left[\varphi_0 \int_0^T a_2(t)p(t)e^{\int_0^t a_1(s)p(s)+b_2(s)q(s)-b_1(s)q(s)ds}dt + \rho_0\right] e^{-\int_0^T b_2(t)q(t)dt}. \tag{3.20}$$

Let $\varphi_0 = \eta_2\rho_0$, where η_2 is a positive constant satisfying

$$0 < \eta_2 < \frac{e^{\int_0^T b_2(t)q(t)dt} - 1}{\int_0^T a_2(t)p(t)e^{\int_0^t a_1(s)p(s)+b_2(s)q(s)-b_1(s)q(s)ds}dt}.$$

As such, we can derive

$$\widetilde{\psi}(T)(\omega_0) = \eta_3\rho_0,\ \eta_3 = \left[\eta_2 \int_0^T a_2(t)p(t)e^{\int_0^t a_1(s)p(s)+b_2(s)q(s)-b_1(s)q(s)ds}dt + 1\right] e^{-\int_0^T b_2(t)q(t)dt} < 1.$$

Consequently, we have

$$M_T[\eta\omega_0] \le \bar{\eta}\eta\omega_0 \ll \eta\omega_0, \quad \bar{\eta} = \max\{\eta_1, \eta_3\} \tag{3.21}$$

for any positive constant η. By Definition 2.2, $\mathbf{o}$ is strongly stable from above for the Poincaré map M_T. For the Poincaré map $P : \chi_\beta \to \chi_\beta$, it is easy to know that we can choose small $\eta > 0$ such that the nonlinear map is well-approximated by the linear map near the fixed point, and following a perturbation argument we can show that there exists a small positive number δ_0 so that $P[\eta\omega_0] \ll \eta\omega_0$ for $\eta \in (0, \delta_0)$. Thus, (H5) is satisfied, and then the existence is proved.

With the idea above, we can also prove that $\mathbf{o}$ is strongly stable from below and $\beta = (1,1)$ is strongly stable from above at strongly positive directions for $P : \chi \to \chi$. Based on all these new results, by following the same idea as in Lemma 3.4 of [2], we can further prove the stability and uniqueness of bistable periodic traveling waves. Therefore, the proof is complete. □

§4 An Example

In this section, we give an example to demonstrate our conjecture proposed in Remark 1.1 and the sharpness of our result compared to that in the reference [2]. The coefficient functions of system (1.1) are taken as

$$\begin{aligned} &a_1(t) = 0.2\cos 2t + 0.7, && b_1(t) = 0.2\cos 2t + 0.42, && r_1(t) = 0.2\cos 2t + 0.5, \\ &a_2(t) = 0.2\sin 2t + 7, && b_2(t) = 0.2\sin 2t + 1.53, && r_2(t) = 0.2\sin 2t + 2.2, \\ &d_1(t) = 0.2\cos 2t + 0.4, && d_2(t) = 0.2\sin 2t + 0.25. \end{aligned} \tag{4.22}$$

Through a calculation, condition (1.2) is satisfied, but both conditions (1.7) and (1.8) do not hold. Fig.1 shows that system (1.1) admits a unique stable π-periodic traveling wave solution connecting $(0, q(t))$ to $(p(t), 0)$. In Fig.1, the numerical result has the following initial functions

$$u(x, 0) = \frac{0.7}{1 + e^{-x}}, \qquad v(x, 0) = \frac{1.44}{1 + e^{x}}. \tag{4.23}$$

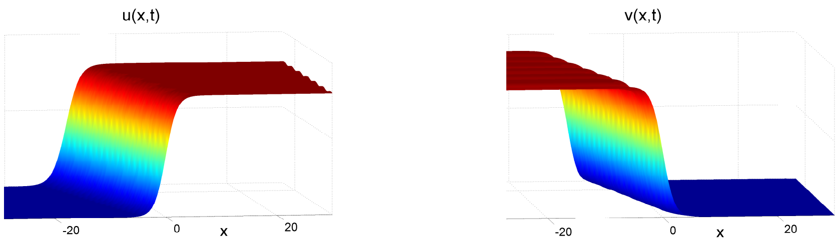

Fig 1. Wave propagation of system (1.1) with coefficients (4.22) and initial data (4.23).

§5 Conclusion

In this paper, we derived a new result, which indicates that a Lotka-Volterra system with bistable nonlinearity inherently possesses a unique stable traveling wavefront. We found positive directions near the two stable fixed points so that along them $\mathbf{o}$ is strongly stable from above (which is resulted from (3.21)) and β is strongly stable from below for the solution map. *This substantially improves the known result that requires the existence of positive eigenfunction to the linearized system around each stable fixed point.* Also based on this result, we obtain the uniqueness and stability of bistable T-periodic traveling waves by following the upper-lower solution method used in [2]. In addition, we want to point out that the result (3.21) corresponds to the assumption (H3) in [10], which was used to construct appropriate upper-and subsolutions for proving the existence of traveling curved fronts of reaction-diffusion bistable systems in two dimensional space. This motivates us to have a further presumption of the existence, uniqueness and stability of bistable T-periodic traveling waves to system (1.1) in higher dimensional space.

Acknowledgement

The authors would like to thank the referees' valuable comments, which greatly improve the exposition of the paper.

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