Appl. Math. J. Chinese Univ.
2022, 37(2): 159-176

Unilateral global interval bifurcation for problem with mean curvature operator in Minkowski space and its applications

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Abstract. In this paper, we establish a unilateral global bifurcation result from interval for a class problem with mean curvature operator in Minkowski space with non-differentiable nonlinearity. As applications of the above result, we shall prove the existence of one-sign solutions to the following problem

$$\begin{cases} -\mathrm{div}\left(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\right) = \alpha(x)v^+ + \beta(x)v^- + \lambda a(x)f(v), & \text{in } B_R(0), \\ v(x) = 0, & \text{on } \partial B_R(0), \end{cases}$$

where $\lambda \neq 0$ is a parameter, R is a positive constant and $B_R(0) = \{x \in \mathbb{R}^N : |x| < R\}$ is the standard open ball in the Euclidean space $\mathbb{R}^N (N \geq 1)$ which is centered at the origin and has radius R. $v^+ = \max\{v, 0\}, v^- = -\min\{v, 0\}$, $a(x) \in C(\overline{B_R(0)}, (0, +\infty))$, $\alpha(x), \beta(x) \in C(\overline{B_R(0)})$, $a(x), \alpha(x)$ and $\beta(x)$ are radially symmetric with respect to x; $f \in C(\mathbb{R}, \mathbb{R})$, $sf(s) > 0$ for $s \neq 0$, and $f_0 \in [0, \infty]$, where $f_0 = \lim_{|s|\to 0} f(s)/s$. We use unilateral global bifurcation techniques and the approximation of connected components to prove our main results. We also study the asymptotic behaviors of positive radial solutions as $\lambda \to +\infty$.

§1 Introduction

Dirichlet problem in a ball, associated to the mean curvature operator in the flat Minkowski space $\mathbb{L}^{N+1}$ with $(x_1, \cdots, x_N, t)$ and the Lorentzian metric $\sum_{i=1}^N (dx_i)^2 - (dt)^2$ are of interest in differential geometry and in general relativity [1, 2].

We first consider the following problem with mean curvature operator in Minkowski space

$$\begin{cases} -\mathrm{div}\left(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\right) = \lambda a(x)v + g(x, v, \lambda), & \text{in } B_R(0), \\ v(x) = 0, & \text{on } \partial B_R(0), \end{cases} \tag{1}$$

where λ is a parameter, R is a positive constant and $B_R(0) = \{x \in \mathbb{R}^N : |x| < R\}$ is the standard open ball in the Euclidean space $\mathbb{R}^N (N \geq 1)$ which is centered at the origin and

Received: 2017-11-29. Revised: 2021-03-11.
MR Subject Classification: 35J65, 34C10, 34C23, 35B40.
Keywords: interval bifurcation, unilateral global bifurcation, one-sign solutions, mean curvature operator.
Digital Object Identifier(DOI): https://doi.org/10.1007/s11766-022-3580-0.
Supported by the 'Kaiwu' Innovation Team Support Project of Lanzhou Institute of Technology(2018KW-03), the NSFC(11561038).

has radius R. Here the nonlinear function $g \in C(\overline{B_R(0)} \times \mathbb{R}^2, \mathbb{R})$ and $a(x)$ is a weighted function. Some important and interesting results for this type of problems have been obtained, see [3-4] for zero or constant curvature, and [5-9] for variable curvature. Some specialists [10-12] have also studied problem (1). Among them, Treibergs [11] studied the problem (1) with $\lambda a(x)u + g(x, u, \lambda) \equiv C$. López [12] studied the problem (1) with $a \equiv 0, g(x, u, \lambda) = kv + \lambda$. Recently, Bereanu et al. [13, 14] have proved existence of classical positive radial solutions for the problem (1) by Leray-Schauder degree argument and critical point theory. As in [13, 14], we can easily show that the radially symmetric solutions of the problem (1) satisfy the following boundary value problem

$$\begin{cases} -\left(r^{N-1}\frac{u'}{\sqrt{1-u'^2}}\right)' = \lambda r^{N-1}a(r)u + r^{N-1}g(r, u, \lambda), \quad r \in (0, R), \\ u'(0) = u(R) = 0, \end{cases} \tag{2}$$

where $r = |x|$ and $u(r) = v(|x|)$. By a solution to the problem (2), we mean a function $u = u(r) \in C^1[0, R]$ with $\|u'\|_\infty < 1$, such that $r^{N-1}u'/\sqrt{1-u'^2}$ is differentiable and (2) is satisfied. Here $\|\cdot\|_\infty$ denotes the usual sup-norm. In 2016, Ma et al. [15] and Dai et al. [16, 17] studied the existence of radial positive solutions and nodal solutions to the problem (1) (where $\lambda au + g = \lambda f(x, v)$) by bifurcation techniques, respectively.

On the other hand, among the above papers, the nonlinearities are differentiable at the origin. In [18], Berestycki established an important global bifurcation theorem from intervals for a class of second-order problems involving non-differentiable nonlinearity. The main difficulties caused by non-differentiable nonlinearity when dealing with this problem lie in the bifurcation results of [16] which cannot be applied directly to obtain our results. In [19], the result in [18] has been improved partially by Schmitt and Smith. Recently, Ma and Dai [20] improved Berestycki's result (in [18]). Later, Dai et al. [21] and [22, 23] considered interval bifurcation problem for a class of second-order and high-dimensional problems involving non-differentiable nonlinearity, respectively.

Motivated by above papers, in this paper, we shall establish a global bifurcation result from interval for problem with mean curvature operator in Minkowski space with nondifferentiable nonlinearity

$$\begin{cases} -\mathrm{div}\left(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\right) = \lambda a(x)v + F(x, v, \lambda), \text{ in } B_R(0), \\ v(x) = 0, \qquad\qquad\qquad\qquad \text{on } \partial B_R(0). \end{cases} \tag{3}$$

It is clear that the radial solutions of (3) is equivalent to the solutions of the following problem

$$\begin{cases} -\left(r^{N-1}\frac{u'}{\sqrt{1-u'^2}}\right)' = \lambda r^{N-1}a(r)u + r^{N-1}F(r, u, \lambda), \quad r \in (0, R), \\ u'(0) = u(R) = 0, \end{cases} \tag{4}$$

where $\lambda \neq 0$ is a parameter, $r = |x|$ with $x \in \overline{B_R(0)}$, the nonlinear term F has the form $F = f + g$, where $f, g \in C([0, R] \times \mathbb{R}^2)$ are radially symmetric, and a, f, g satisfying the following conditions:

$(H1)$ $a(r) \in C([0, R], (0, +\infty))$ is radially symmetric with respect to r.

$(H2)$ $\left|\frac{f(r,s,\lambda)}{s}\right| \leq M_1$, for all $r \in [0, R]$, $0 < |s| \leq R$ and all $\lambda \in \mathbb{R}$, where M_1 is a positive constant.

$(H3)$ $g(r, s, \lambda) = o(|s|)$ near $s = 0$ uniformly in $r \in [0, R]$ and λ on bounded sets.

$(H4)$ There exists a function $h(t, u) \in C([0, R] \times [-R, R], \mathbb{R})$ with $h(t, s)s > 0$ for any $r \in [0, R]$ and $s \neq 0$, such that $g(t, u, \lambda) = \lambda h(t, u)$.

Under the above assumptions, we shall show that $[\lambda_1 - d_1, \lambda_1 + d_1]$ is a bifurcation interval of problem (4) and there are two distinct unbounded sub-continua, $\mathscr{D}^+$ and $\mathscr{D}^-$, consisting of the bifurcation branch $\mathscr{D}$ from $[\lambda_1 - d_1, \lambda_1 + d_1]$, where $d_1 = M_1/a_0$, $a_0 = \min_{r\in[0,1]} a(r)$(see Theorem 3.1), λ_1 be the first eigenvalue for the following linear eigenvalue problem

$$\begin{cases} -(r^{N-1}u')' = \lambda r^{N-1}a(r)u, \quad r \in (0, R), \\ u'(0) = u(R) = 0. \end{cases} \tag{5}$$

It is well-known that λ_1 is simple, isolated and the associated eigenfunction has fixed sign in [0, R) (see for example [24] or [25, p. 269]).

On the basis of the unilateral global interval bifurcation result (Theorem 3.1), we shall study the following problem

$$\begin{cases} -\text{div}\left(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\right) = \lambda a(x)v + \alpha(x)v^+ + \beta(x)v^- + g(x, v, \lambda), & \text{in } B_R(0), \\ v(x) = 0, & \text{on } \partial B_R(0). \end{cases} \tag{6}$$

It is clear that the radial solutions of (6) is equivalent to the solutions of the following problem

$$\begin{cases} -\left(r^{N-1}\frac{u'}{\sqrt{1-u'^2}}\right)' = \lambda r^{N-1}a(r)u + \alpha(r)r^{N-1}u^+ + \beta(r)r^{N-1}u^- \\ +r^{N-1}g(r, u, \lambda), \quad r \in (0, R), \\ u'(0) = u(R) = 0, \end{cases} \tag{7}$$

where $\lambda \neq 0$ is a parameter, $a(r)$ satisfies $(H1)$, $u^+ = \max\{u, 0\}, u^- = -\min\{u, 0\}$, g satisfies (H3) and (H4), $\alpha(r), \beta(r)$ satisfy:

$(H5)$ $\alpha(r), \beta(r) \in C([0, R])$ are radially symmetric.

Furthermore, we shall investigate the existence of one-sign solutions for the following problem

$$\begin{cases} -\text{div}\left(\frac{\nabla v}{\sqrt{1-|\nabla v|^2}}\right) = \alpha(x)v^+ + \beta(x)v^- + \lambda a(x)f(v), & \text{in } B_R(0), \\ v(x) = 0, & \text{on } \partial B_R(0). \end{cases} \tag{8}$$

It is clear that the radial solutions of (8) is equivalent to the solutions of the following problem

$$\begin{cases} -\left(r^{N-1}\frac{u'}{\sqrt{1-u'^2}}\right)' = \alpha(r)r^{N-1}u^+ + \beta(r)r^{N-1}u^- + \lambda a(r)r^{N-1}f(u), \quad r \in (0, R), \\ u'(0) = u(R) = 0, \end{cases} \tag{9}$$

where $\lambda \neq 0$ is a parameter, $a(r)$ satisfies $(H1)$, $\alpha(r), \beta(r)$ satisfy $(H5)$, f satisfy condition in section 5.

The rest of this paper is arranged as follows. In Section 2, we give some Preliminaries. In Section 3, we establish the global bifurcation result from the interval for the problem (4). In Section 4, on the basis of the global interval bifurcation result(see Theorem 3.1), we shall establish unilateral global bifurcation result for the problem (7) with jumping nonlinearity (see Theorem 4.2). In Section 5, we shall investigate the existence of one-sign solutions for a class of the problems (9) with jumping nonlinearity.

§2 Preliminaries

Let $Y = C[0, R]$ with the norm $\|u\|_\infty = \max_{r\in[0,R]} |u(r)|$. Let $E := \{u(r) \in C^1[0, R] | u'(0) = u(R) = 0\}$ with the usual norm $\|u\| = \max\{\|u\|_\infty, \|u'\|_\infty\}$. Let $P^+ = \{u \in E : u(r) > 0, r \in (0, R)\}$ and set $P^- = -P^+$ and $P = P^+ \cup P^-$. Let $K^\pm = \mathbb{R} \times P^\pm$ under the product topology, $\mathbb{R} = (-\infty, +\infty)$. Let $C^\pm$ denote the closure in $K^\pm$ of the set of nontrivial solutions of (2).

We consider the following auxiliary problem

$$\begin{cases} -(r^{N-1}u')' = r^{N-1}h(r), \quad r \in (0,R), \\ u'(0) = u(R) = 0 \end{cases} \tag{10}$$

for a given $h \in Y$. By a solution of the problem (10), we understand a function $u \in E$ with $r^{N-1}u'$ absolutely continuous which satisfies (10).

We have known that for every given $h \in Y$, there is a unique solution u to the problem (10) (see [26]). Let $L_N(h)$ denote the unique solution to (10), which can be equivalently written as

$$L_N(h) = \int_0^R G(r,s)s^{N-1}h(s)ds,$$

where $G(r,s)$ be the Green's function associated with the operator $L_N(u) := -(r^{N-1}u')'$ with the same boundary condition as in problem (10) (see [26]). And $L_N : Y \to E$ is linear completely continuous (see [26, (2.5)-p.502 to Line 4-p.503]).

Similar the process of obtaining [16, (2.1)-p.60] or [17, (2.1)-p.469], when a, g satisfying $(H1)$ and $(H3)$, if u is a solution of problem (2), then for any $r \in (0,R)$, one has that

$$\begin{aligned} r^{N-1}[\lambda a(r)u + g(r,u,\lambda)] &= -(r^{N-1}u')'\frac{1}{\sqrt{1-u'^2}} - r^{N-1}u'^2u''\frac{1}{(1-u'^2)\sqrt{1-u'^2}} \\ &= -(N-1)r^{N-2}u'\frac{1}{\sqrt{1-u'^2}} - r^{N-1}u''\frac{1}{\sqrt{1-u'^2}}\cdot\frac{1}{1-u'^2}. \end{aligned} \tag{11}$$

By (11), one obtains that

$$-u'' = [\lambda a(r)u + g(r,u,\lambda)](1-u'^2)^{\frac{3}{2}} + \frac{N-1}{r}u'(1-u'^2). \tag{12}$$

Substituting (12) into (11), it follows that

$$-(r^{N-1}u')' = r^{N-1}[\lambda a(r)u + g(r,u,\lambda)](1-u'^2)^{\frac{3}{2}} - (N-1)r^{N-2}u'^3.$$

Thus, the problem (2) is equivalent to

$$\begin{cases} -(r^{N-1}u')' = \lambda r^{N-1}a(r)u(1-(u')^2)^{\frac{3}{2}} + r^{N-1}g(r,u,\lambda)(1-(u')^2)^{\frac{3}{2}} \\ \qquad\qquad -(N-1)r^{N-2}(u')^3, \quad r \in (0,R), \\ u'(0) = u(R) = 0 \end{cases} \tag{13}$$

as a bifurcation problem from the trivial solution axis.

Define the Nemitskii operator $H : \mathbb{R} \times E \to E$ by

$$H(\lambda,u)(r) := \lambda r^{N-1}a(r)u(r) + K_1(r,u,\lambda),$$

where

$$K_1(r,u,\lambda) = r^{N-2}\left[\lambda a(r)ru\left((1-(u')^2)^{\frac{3}{2}} - 1\right) + rg(r,u,\lambda)\left(1-(u')^2\right)^{\frac{3}{2}} - (N-1)(u')^3\right].$$

Furthermore, it is clear that the problem (2) can be equivalently written as

$$u = \lambda L_N u + L_N \circ K_1(u) = L_N \circ H(\lambda,u) = F(\lambda,u),$$

where

$$L_N u = \int_0^R G(t,s)s^{N-1}a(s)u(s)ds, L_N \circ K_1(u) = \int_0^R G(t,s)K_1(s,u(s),\lambda)ds.$$

And $L_N \circ K_1 : \mathbb{R} \times E \to E$ is completely continuous (see [16, 17, 26]). Moreover, F is completely continuous from $\mathbb{R} \times E \to E$ and $F(\lambda,0) = 0, \forall\lambda \in \mathbb{R}$.

Let

$$\overline{g}(r,u,\lambda) = \max_{0\le|s|\le u}|g(r,s,\lambda)| \text{ for } r \in [0,R] \text{ and } \lambda \text{ on bounded sets,}$$

then $\overline{g}$ is nondecreasing and

$$\lim_{u\to 0^+}\frac{\overline{g}(r,u,\lambda)}{u}=0 \tag{14}$$

uniformly for $r\in[0,R]$ and λ on bounded sets. Further it follows from (14) that

$$\frac{|g(r,u,\lambda)|}{\|u\|}\le\frac{\overline{g}(r,u,\lambda)}{\|u\|}\le\frac{\overline{g}(r,\|u\|_\infty,\lambda)}{\|u\|}\le\frac{\overline{g}(r,\|u\|,\lambda)}{\|u\|}\to 0\ as\ \|u\|\to 0 \tag{15}$$

uniformly for $r\in[0,R]$ and λ on bounded sets. By (15), it follows that

$$\frac{|g(r,u,\lambda)(1-(u')^2)^{\frac{3}{2}}|}{\|u\|}\to 0\ as\ \|u\|\to 0 \tag{16}$$

uniformly for $r\in[0,R]$ and λ on bounded sets.

By the proof of Theorem 1.1 of [16 or 17], we have that

$$\frac{(N-1)(u')^3}{r\|u\|}\to 0,\ \frac{\lambda a(r)u((1-(u')^2)^{\frac{3}{2}}-1)}{\|u\|}\to 0\ as\ \|u\|\to 0. \tag{17}$$

By (16) and (17), we have that

$$\frac{K_1(r,u,\lambda)}{\|u\|}\to 0\ as\ \|u\|\to 0 \tag{18}$$

uniformly for $r\in[0,R]$ and λ on bounded sets.

Applying the similar proof of Theorem 1.1 in [16], by [26, Theorem 1.3], we may obtain the following result.

Lemma 2.1(see [16, Theorem 1.1]). Assume $(H1)$ and $(H3)$ hold. Then $(\lambda_1,0)$ is a bifurcation point of the problem (2). Moreover, there exists an unbounded component C of the set of solution of problem (2) in $\mathbb{R}\times E$ bifurcating from $(\lambda_1,0)$ such that $C\subset((\mathbb{R}^+\times P)\cup\{(\lambda_1,0)\})$ and $\lim_{\lambda\to\infty}\|u_\lambda\|=\max\{1,R\}$ for $(\lambda,u_\lambda)\in C\setminus\{(\lambda_1,0)\}$. In addition, $(\lambda_1,0)$ is the unique bifurcation point on $\mathbb{R}\times\{0\}$ of solutions of problem (2).

Furthermore, by Dancer [27, Theorem 2], using the similar method to prove [17, Theorem 1.1-p.471] with obvious changes, one can obtain that the problem (2) has two distinct unbounded sub-continua C^+ and C^-, consisting of the bifurcation branch C emanating from $(\lambda_1,0)$, which satisfy:

Lemma 2.2. Both C^+ and C^- are unbounded and

$$C^\nu\subset((\mathbb{R}\times P^\nu)\cup\{(\lambda_1,0)\}),$$

where $\nu\in\{+,-\}$.

Next, we give an important lemma which will be used later.

Lemma 2.3 (see[28, p.382]). Let $L[y]=(r^{N-1}y')'$, $L[z]=(r^{N-1}z')'$, where $y,z\in C^1[0,R]$, $z\neq 0$ in $(0,R)$. Then we have the following identity:

$$\frac{d}{dt}\left[\frac{yr^{N-1}}{z}(zy'-yz')\right]=r^{N-1}\left[|y'|^2+|\frac{y}{z}z'|^2-2y'(\frac{yz'}{z})\right]+\left[\frac{y}{z}(zL[y]-yL[z]\right].$$

Remark 2.1 (see [29]). By Young's inequality, we get

$$r^{N-1}\left[|y'|^2+|\frac{y}{z}z'|^2-2y'(\frac{yz'}{z})\right]\ge 0$$

and the equality holds if and only if $(\frac{y}{z})'=0, t\in(0,R)$, i.e. $u=kv$ for some constant k in each component of $(0,R)$.

By Lemma 2.3 and Remark 2.1, we have the following result:

Lemma 2.4. Let $L[y]$ is given in Lemma 2.3, $y'(0) = y(R) = 0$, $y(t) > 0, z(t) > 0, t \in (0, R)$. $z(t)$ is a solution for the problem (5), we have

$$\int_0^R \frac{y}{z}\left(zL[y] - yL[z]\right) dr \leq 0.$$

In order to treat the problems with non-asymptotic nonlinearity at 0 and ∞, we use Whyburn type superior limit theorems. From [30], if the collection of the infinite sequence of sets is unbounded, the Whyburn's limit theorem [31, Theorem 9.1] cannot be used directly because the collection may not be relatively compact (where the definitions of superior limit and inferior limit, reference see [30, line 11 to line 16]). Dai [30] overcomed this difficulty and established the following results. In order to treat the problems with non-asymptotic nonlinearity at 0 and ∞, we shall need the following lemmas.

Lemma 2.5 (see [30, Lemma 2.5]). Let X be a normal space and let $\{C_n | n = 1, 2, ...\}$ be a sequence of unbounded connected subsets of X. Assume that:

(1) there exists $z^* \in \liminf_{n\to+\infty} C_n$ with $\|z^*\| < +\infty$;

(2) for every $R > 0$, $\left(\cup_{n=1}^{+\infty} C_n\right) \cap \overline{B}_R$ is a relatively compact set of X, where

$$B_R = \{x \in X | \|x\| \leq R\}.$$

Then $\mathbb{D} := \limsup_{n\to\infty} C_n$ is unbounded, closed and connected.

Lemma 2.6 (see [30, Theorem 1.2]). Let X be a normal vector space and let $\{C_n | n = 1, 2, ...\}$ be a sequence of unbounded connected subsets of X. Assume that:

(1) there exists $z^* \in \liminf_{n\to+\infty} C_n$ with $\|z^*\| = +\infty$;

(2) There exists a homeomorphism $T : X \to X$ such that $\|T(z^*)\| < +\infty$ and $\{T(C_n)\}$ be a sequence of unbounded connected subsets in X ;

(3) for every $R > 0$, $\left(\cup_{n=1}^{+\infty} C_n\right) \cap \overline{B}_R$ is a relatively compact set of X, where

$$B_R = \{x \in X | \|x\| \leq R\}.$$

Then $\mathbb{D} := \limsup_{n\to\infty} C_n$ is unbounded, closed and connected.

Lemma 2.7 (see [30, Lemma 2.6]). Let (X, ρ) be a metric space. If $\{C_i\}_{i\in\mathbb{N}}$ is a sequence of sets whose limit superior is L and there exists a homeomorphism $T : X \to X$ such that for every $R > 0$, $\left(\cup_{i=1}^{+\infty} T(C_i)\right) \cap \overline{B}_R$ is a relatively compact set, then for each $\epsilon > 0$ there exists an m such that for every $n > m, C_n \subset V_\epsilon(L)$, where $V_\epsilon(L)$ denotes the set of all points p with $\rho(p, x) < \epsilon$ for any $x \in L$.

§3 Unilateral global interval bifurcation

Similar to the process of obtaining (13), when a, f, g satisfy $(H1)$, $(H2)$, $(H3)$ and $(H4)$, we obtain that the problem (4) is equivalent to

$$\begin{cases} -(r^{N-1}u')' = \lambda r^{N-1}a(r)u + r^{N-1}K_2(r, u, \lambda), & r \in (0, R), \\ u'(0) = u(R) = 0, \end{cases} \tag{19}$$

where

$$K_2(r,u,\lambda) = \lambda a(r)u((1-(u')^2)^{\frac{3}{2}}-1) + f(r,u,\lambda)(1-(u')^2)^{\frac{3}{2}} + g(r,u,\lambda)(1-(u')^2)^{\frac{3}{2}} - \frac{(N-1)}{r}u'^3.$$

Let $\mathscr{S}^{\pm}$ denote the closure in $K^{\pm}$ of the set of nontrivial solutions of (4).

Using the similar method to prove [17, Lemma 2.1], by (19), we may easily obtain the following result.

Lemma 3.1 (see [17, Lemma 2.1]). If (λ, u) is a nontrivial solution of (4) under assumptions $(H1)$, $(H2)$ and $(H3)$ and u has a double zero, then $u \equiv 0$.

The first main result for (4) is the following theorem.

Theorem 3.1 Let $(H1)$, $(H2)$, $(H3)$ and $(H4)$ hold. Let $d_1 = M_1/a_0$, where $a_0 = \min_{r\in[0,1]} a(r)$, and let $I = [\lambda_1 - d_1, \lambda_1 + d_1]$. The component $\mathscr{C}^{\nu}$ of $\mathscr{S}^{\nu} \cup (I\times\{0\})$ contains $I\times\{0\}$, for $\nu = +,-$ and such that

(*i*) $\mathscr{C}^{\nu} \subset (K^{\nu} \cup (I\times\{0\}))$;

(*ii*) $\mathscr{C}^{\nu}$ is unbounded;

(*iii*) $\lim_{\lambda\to\infty}\|u_\lambda\| = \max\{1, R\}$ for $(\lambda, u_\lambda) \in \mathscr{C}^{\nu} \setminus (I\times\{0\})$.

To prove Theorem 3.1, we introduce the following auxiliary approximate problem:

$$\begin{cases} -\left(r^{N-1}\frac{u'}{\sqrt{1-u'^2}}\right)' = \lambda r^{N-1}a(r)u + r^{N-1}f(r,u|u|^{\epsilon},\lambda) + r^{N-1}g(r,u,\lambda), \quad r\in(0,R), \\ u'(0) = u(R) = 0. \end{cases} \tag{20}$$

By (19), the problem (20) is equivalent to

$$\begin{cases} -(r^{N-1}u')' = \lambda r^{N-1}a(r)u + r^{N-1}K_3(r,u,\lambda), \quad r\in(0,R), \\ u'(0) = u(R) = 0, \end{cases}$$

where

$$K_3(r,u,\lambda) = \lambda a(r)u((1-(u')^2)^{\frac{3}{2}}-1) + f(r,u|u|^{\epsilon},\lambda)(1-(u')^2)^{\frac{3}{2}} + g(r,u,\lambda)(1-(u')^2)^{\frac{3}{2}} - \frac{(N-1)}{r}u'^3.$$

To prove Theorem 3.1, the next lemma will play a key role.

Lemma 3.2. Let ϵ_n, $0<\epsilon_n<1$, be a sequence converging to 0. If there exists a sequence $(\lambda_n, u_n) \in \mathbb{R}\times P^{\nu}$ such that (λ_n, u_n) is a nontrivial solution of problem (20) corresponding to $\epsilon = \epsilon_n$, and (λ_n, u_n) converges to $(\lambda, 0)$ in $\mathbb{R}\times E$, then $\lambda \in I$.

Proof. Without loss of generality, we may assume that $\|u_n\| \le 1$. Let $w_n = u_n/\|u_n\|$, then w_n satisfies the problem

$$\begin{cases} -(r^{N-1}w_n')' = \lambda_n r^{N-1}a(r)w_n + \frac{r^{N-1}K_3(r,u_n,\lambda_n)}{\|u_n\|}, \quad r\in(0,R), \\ w_n'(0) = w_n(R) = 0. \end{cases}$$

Clearly, $(H2)$ implies that

$$\frac{|f(r,u_n|u_n|^{\epsilon_n},\lambda_n)(1-(u_n')^2)^{\frac{3}{2}}|}{\|u_n\|} \le \frac{|f(t,u_n|u_n|^{\epsilon_n},\lambda_n)|}{u_n|u_n|^{\epsilon_n}}\cdot\frac{u_n|u_n|^{\epsilon_n}}{\|u_n\|} \le M_1\cdot|u_n|^{\epsilon_n} \to M_1 \tag{21}$$

uniformly in $r\in[0,R]$.

Note that $\|w_n\| = 1$ implies $\|w_n\|_\infty \le 1$. By (15) and (21), we have that $\lambda_n r^{N-1} a(r) w_n + r^{N-1} K_3(r, u_n, \lambda_n)/\|u_n\|$ is bounded in $C^2[0, R]$ for n large enough. The compactness of L_N implies that w_n is convergent in E. Without loss of generality, we may assume that $w_n \to w$ in E with $\|w\| = 1$. Clearly, we have $w \in \overline{P^\nu}$. We claim that $w \in P^\nu$. On the contrary, suppose that $w \in \partial P^\nu$, by Lemma 3.1, then $w \equiv 0$, which is a contradiction with $\|w\| = 1$.

Now, we deduce the boundedness of λ. Let $\varphi^\nu \in P^\nu$ be an eigenfunction of problem (5) corresponding to λ_1.

We can assume without loss of generality that $\nu = +$. By the Lemma 2.4, it follows that

$$\begin{aligned} &\int_0^R \frac{w_n}{\varphi^+} \left(\varphi L[w_n] - w_n L[\varphi^+]\right) dr \\ &= \int_0^R (\lambda_1 - \lambda_n) a(r) r^{N-1} (\varphi^+)^2 dr - \int_0^R \frac{r^{N-1} K_3(r, u_n, \lambda_n)}{\|u_n\|} (\varphi^+)^2 dr \le 0. \end{aligned} \tag{22}$$

Similarly, we can also show that

$$\int_0^R (\lambda_n - \lambda_1) a(r) r^{N-1} w_n^2 dr + \int_0^R \frac{r^{N-1} K_3(r, u_n, \lambda_n)}{\|u_n\|} w_n^2 dr \le 0. \tag{23}$$

If $\lambda \le \lambda_1$, considering (15), (21) and (22), we have that

$$\int_0^R (\lambda_1 - \lambda) a(r) r^{N-1} (\varphi^+)^2 \le \lim_{n\to\infty} \int_0^R \frac{r^{N-1} K_3(r, u_n, \lambda_n)}{\|u_n\|} (\varphi^+)^2 dr \le \int_0^R M_1 r^{N-1} (\varphi^+)^2 dr.$$

Hence, we get that

$$\int_0^R (\lambda_1 - \lambda) a_0 r^{N-1} (\varphi^+)^2 \le \int_0^R M_1 r^{N-1} (\varphi^+)^2 dr,$$

which implies $\lambda \ge \lambda_1 - d$.

If $\lambda \ge \lambda_1$, considering (15), (21) and (23), we have that

$$\int_0^R (\lambda - \lambda_1) a(r) r^{N-1} w_n^2 dr \le \int_0^R M_1 r^{N-1} w^2 dr.$$

Hence, we get that $\lambda \le \lambda_1 + d$. Therefore, we have that $\lambda \in I$.

Proof of Theorem 3.1. We only prove the case of $\mathscr{C}^+$ since the case of $\mathscr{C}^-$ is similar. Let $\mathscr{C}^+$ be the component of $\mathscr{S}^+ \cup (I \times \{0\})$, containing $I \times \{0\}$.

We divide the proofs into the following several steps.

(i) We show that $\mathscr{C}^\nu \subset (K^\nu \cup (I \times \{0\}))$. Using the similar method to prove Theorem 1.1 of [21, Line 13-20 of p.107] with obvious changes, we may prove that $\mathscr{C}^+ \subset (\mathbb{R} \times P^+) \cup (I \times \{0\})$.

(ii) We prove that $\mathscr{C}^+$ is unbounded.

Suppose on the contrary that $\mathscr{C}^+$ is bounded. Using a proof similar to that of Theorem 1 of [18] with obvious changes, we can find a neighborhood $\mathcal{O}$ of $\mathscr{C}^+$ such that $\mathscr{S}^+ \cap \partial\mathcal{O} = \emptyset$.

In order to complete the proof of this theorem, we consider the problem (20). For $\epsilon > 0$, it is easy to show that nonlinear term $f(r, u|u|^\epsilon, \lambda) + g(r, u, \lambda)$ satisfies the condition $(H3)$. Let

$$\mathscr{S}_\epsilon = \overline{\{(\lambda, u) : (\lambda, u) \text{ satisfies } (20) \text{ and } u \not\equiv 0\}}^{\mathbb{R}\times E}.$$

By Lemma 2.2, there exists an unbounded continuum $\mathscr{C}_\epsilon^\nu$ of $\mathscr{S}_\epsilon^\nu$ bifurcating from $(\lambda_1, 0)$ such that

$$\mathscr{C}_\epsilon^\nu \subset (\mathbb{R} \times P^\nu) \cup \{(\lambda_1, 0)\}, \text{ for } \nu = + \text{ and } -.$$

So there exists $(\lambda_\epsilon, u_\epsilon) \in \mathscr{C}_\epsilon^+ \cap \partial\mathcal{O}$ for all $\epsilon > 0$. Since $\mathcal{O}$ is bounded in $\mathbb{R} \times P^+$, Eq. (20) shows that $(\lambda_\epsilon, u_\epsilon)$ is bounded in $\mathbb{R} \times C^2$ independently of ϵ. By the compactness of L_N, one can find a sequence $\epsilon_n \to 0$ such that $(\lambda_{\epsilon_n}, u_{\epsilon_n})$ converges to a solution (λ, u) of (4). So, $u \in \overline{P^\nu}$, if $u \in \partial P^\nu$, then from Lemma 3.1 follows that $u \equiv 0$. By Lemma 3.2, $\lambda \in I$, which contradicts the definition of $\mathcal{O}$. If $u \in P^+$, then $(\lambda, u) \in \mathscr{S}^+ \cap \partial\mathcal{O}$ which contradicts $\mathscr{S}^+ \cap \partial\mathcal{O} = \emptyset$.

(iii) Now, we shall prove that $\lim_{\lambda\to\infty} \|u\| = \max\{1, R\}$. For any $(\lambda_n, u_n) \in \mathscr{C}^+ \setminus (I \times \{0\})$ with $\lambda_n \to +\infty$ as $n \to +\infty$.

Same as the proof of Theorem 1.1 of [16, Line 8 of p.62 to Line 19 of p.63], there exist a positive constant $\tau_0 > 0$ and $\rho_n \in (0, R)$ such that $u_n(\rho) \geq \sigma_0$ for n large enough, where $\rho \in (0, \rho_*)$, $\rho_* = \liminf_{n\to+\infty} \rho_n$, and one may obtain that $u_n(r) \in [\sigma_0, R]$ for any $r \in [\rho/4, \rho]$.

By (H4), letting $h_1 = \min_{[\rho/4,\rho]\times[\tau_0,R]} h(t, u_n)$. It follows that $g(t, u_n, \lambda_n) \geq \lambda_n h_1$. Now, integrating the first equation of problem (4) from $\rho/2$ to r for any $r \in [\rho/2, \rho]$ and n large enough, we get that

$$r^{N-1}\frac{u_n'}{\sqrt{1-u_n'^2}} = -\int_{\rho/2}^{r} t^{N-1}[\lambda_n a(t)u_n + f(t, u_n, \lambda_n) + g(t, u_n, \lambda_n)]dt.$$

Set $a_0 = \min_{t\in[\rho/2,\rho]} a(t)$. By simple computation, we can show that

$$|\lambda_n a(t)u_n + f(t, u_n, \lambda_n) + g(t, u_n, \lambda_n)| \geq |\lambda_n\tau_0(a_0 + h_1) - RM| \text{ for all } t \in [0, R].$$

Moreover, we have that

$$\begin{aligned}
\frac{1}{\sqrt{1-\|u_n'\|_\infty^2}} &\geq \frac{1}{\sqrt{1-u_n'^2}} \geq |\frac{u_n'}{\sqrt{1-u_n'^2}}| \\
&= |\frac{1}{r^{N-1}}\int_{\rho/4}^{r} t^{N-1}[\lambda_n a(t)u + f(t, u_n, \lambda_n) + g(t, u_n, \lambda_n)]dt| \\
&\geq \frac{|\lambda_n\tau_0(a_0 + h_1) - RM|}{Nr^{N-1}}\int_{\rho/4}^{r} t^{N-1}dt \\
&= \frac{|\lambda_n\tau_0(a_0 + h_1) - RM|}{r^{N-1}}(r^N - (\rho/4)^N) \\
&\geq \frac{|\lambda_n\tau_0(a_0 + h_1) - RM|}{N\rho^{N-1}}((\rho/2)^N - (\rho/4)^N) \geq \frac{|\lambda_n\tau_0(a_0 + h_1) - RM|\rho}{N2^N}(1 - \frac{1}{2^N})
\end{aligned}$$

By $\lim_{n\to+\infty} \lambda_n = +\infty$, it follows that $\lim_{n\to+\infty} \|u_n'\|_\infty = 1$. Noting that

$$|u_n(r)| = |\int_R^r u_n(t)dt| \leq \int_R^r |u_n(t)'|dt \leq \|u_n'\|_\infty R.$$

Thus, one may obtain that $\lim_{\lambda\to\infty} \|u\| = \max\{1, R\}$.

From Theorem 3.1 and its proof, we can easily get a corollary.

Corollary 3.1. There exist two sub-continua $\mathscr{C}^+$ and $\mathscr{C}^-$ of solutions of (4) in $\mathbb{R} \times E$, bifurcating from $I \times \{0\}$, for $\nu = +, -$ and such that

(i) $\mathscr{C}^\nu \subset ((\mathbb{R} \times P^\nu) \cup (I \times \{0\}))$;

(ii) $\mathscr{C}^\nu$ is unbounded;

(iii) $\lim_{\lambda\to\infty} \|u_\lambda\| = \max\{1, R\}$ for $(\lambda, u_\lambda) \in \mathscr{C}^\nu \setminus (I \times \{0\})$.

Remark 3.1. If $K_2 = f + g$ satisfies the conditions $(H2)$ and $(H3)$, then the conclusions of Theorem 3.1 and Corollary 3.1 are valid for the problem (19).

§4 Unilateral global bifurcation for the problem (7)

Consider the following auxiliary problem

$$\begin{cases} -(r^{N-1}u')' = \lambda r^{N-1}a(r)u + \alpha r^{N-1}u^+ + \beta r^{N-1}u^-, \quad r \in (0,R), \\ u'(0) = u(R) = 0. \end{cases} \tag{24}$$

By a similar argument of [23, Theorem 2], we can obtain the following theorem.

Theorem 4.1. There exist two simple half-eigenvalues λ^+ and λ^- for problem (24). The corresponding half-linear solutions are in $\{\lambda^+\} \times P^+$ and $\{\lambda^-\} \times P^-$. Furthermore, aside from λ^+ and λ^-, there is no other half-eigenvalue with positive or negative eigenfunction.

Proof. By Theorem 3.1 and Remark 3.1, we know that there exists at least one solution of problem (24), $(\lambda^\nu, u^\nu) \in \mathbb{R} \times P^\nu$, for every $\nu = +$ and $\nu = -$. The positive homogeneous of problem (24) implies that $\{(\lambda^\nu, cu^\nu), c > 0\}$ are half-linear solutions in $\{\lambda^\nu\} \times P^\nu$. Lemma 3.1 implies that any nontrivial solution u of problem (24) lies in some P^ν. We claim that for any solution (λ, u) of problem (24) with $u \in P^\nu$,

Using the similar method to prove [23, Theorem 2], we may prove that $\lambda = \lambda^\nu$ and $u = cu^\nu$ for some positive constant c. Similar the process of obtaining (13), one may get that the problem (7) is equivalent to

$$\begin{cases} -(r^{N-1}u')' = \lambda r^{N-1}a(r)u + r^{N-1}(\alpha u^+ + \beta u^-) + r^{N-1}K_4(r,u,\lambda), \quad r \in (0,R), \\ u'(0) = u(R) = 0, \end{cases} \tag{25}$$

where

$$\begin{aligned} K_4(r,u,\lambda) =& (\lambda a(r)u + \alpha u^+ + \beta u^-)\left((1-(u')^2)^{\frac{3}{2}} - 1\right) \\ &+ g(r,u,\lambda)(1-(u')^2)^{\frac{3}{2}} - \frac{N-1}{r}u'^3. \end{aligned}$$

Furthermore, it is clear that the problem (25) can be equivalently written as

$$u = \lambda L_N u + L_N \circ K_5(u) = F_1(\lambda, u),$$

where

$$K_5(r,u,\lambda) = \alpha u^+ + \beta u^- + K_4(r,u,\lambda), L_N u = \int_0^R G(t,s)s^{N-1}a(s)u(s)ds,$$

$$L_N \circ K_5 = \int_0^R G(t,s)s^{N-1}K_5(s,u(s),\lambda)ds.$$

And $L_N \circ K_5 : \mathbb{R} \times E \to E$ is completely continuous (see [16, 17, 26]). Moreover, F_1 is completely continuous from $\mathbb{R} \times E \to E$ and $F_1(\lambda, 0) = 0, \forall \lambda \in \mathbb{R}$.

Similar to Lemma 2.2 in [17], we may get the following lemmas:

Lemma 4.1. For fixed $\lambda > 0$, if $\{u_k\}$ is a sequence of solutions of problem (7) satisfies $\lim_{k\to+\infty} u_k = 0$ uniformly in $r \in [0,R]$, then $\lim_{k\to+\infty} u_k' = 0$ and $\lim_{k\to+\infty} u_k'' = 0$ uniformly in $r \in [0,R]$.

Proof. Integrating the first equation of problem (7) from 0 to r for any $r \in [0,R]$, we get that

$$r^{N-1}\frac{u_k'}{\sqrt{1-u_k'^2}} = -\int_0^r [\alpha t^{N-1}u_k^+ + \beta t^{N-1}u_k^- + t^{N-1}g(t,u_k,\lambda)]dt.$$

By (H3), we have that $\lim_{k\to+\infty} g(t,u_k,\lambda)=0$ uniformly $t\in[0,R]$ and λ on bounded sets. It follows that $\lim_{k\to+\infty} u_k'=0$ uniformly in $r\in[0,R]$.

Similar to method of obtaining (12), for any $r\in(0,R)$, one has that

$$-u_k''=[\lambda a(r)u_k+\alpha u_k^+ +\beta u_k^- +g(r,u_k,\lambda)](1-u_k'^2)^{\frac{3}{2}}+\frac{N-1}{r}u_k'(1-u_k'^2). \tag{26}$$

By $\lim_{k\to+\infty} u_k=0$ and $\lim_{k\to+\infty} u_k'=0$ uniformly in $r\in[0,R]$ together with (26), it follows that $\lim_{k\to+\infty} u_k''=0$ uniformly in $r\in(0,R)$.

Taking the limit $r\to 0^+$ on both sides of the (26), together with $\lim_{r\to 0^+} u_k(r)=0$ and $\lim_{r\to 0^+} u_k'(r)=0$, by L'Hospital's rule, we have that

$$\begin{aligned}-\lim_{r\to 0^+} u_k''(r)&=\lim_{r\to 0^+}\frac{(N-1)u_k'(r)(1-u_k'^2(r))}{r}\\ &=\lim_{r\to 0^+}\frac{(N-1)[u_k''(r)(1-u_k'^2)(r))+u_k'(-2)u_k'u_k'']}{1}=\lim_{r\to 0^+}(N-1)u_k''(r).\end{aligned}$$

We obtain that $\lim_{r\to 0^+} u_k''(r)=0$.

By $\lim_{r\to R^-} u_k(r)=\lim_{r\to R^-} u_k'(r)=0$, taking the limit $r\to R^-$ on both sides of the (26), we may get that $\lim_{r\to R^-} u_k''(r)=0$.

Moreover, it follows that $\lim_{k\to+\infty} u_k''=0$ uniformly in $r\in[0,R]$.

Lemma 4.2. For fixed $\lambda>0$, if u is a solutions of problem (7) satisfies $u\to 0$ uniformly in $r\in[0,R]$, then

$$\lim_{u\to 0^+}\frac{(1-(u')^2)^{\frac{3}{2}}-1}{u}=0,\ \lim_{u\to 0^+}\frac{(N-1)(u')^3}{u}=0.$$

Proof. From Lemma 4.1, we have that $\lim_{u\to 0} u'=0$ and $\lim_{u\to 0} u''=0$ uniformly in $r\in[0,R]$.

By L'Hospital's rule, we have

$$\lim_{u\to 0^+}\frac{(1-(u')^2)^{\frac{3}{2}}-1}{u}=\lim_{u\to 0^+}\frac{\frac{3}{2}(1-(u')^2)^{\frac{1}{2}}\cdot(-2u'u'')}{u'}=\lim_{u\to 0^+}\frac{3}{2}(1-(u')^2)^{\frac{1}{2}}\cdot(-2u'')=0$$

and

$$\lim_{u\to 0^+}\frac{(N-1)(u')^3}{u}=\lim_{u\to 0^+}\frac{(N-1)3(u')^2u''}{u'}=\lim_{u\to 0^+}3(N-1)u'u''=0.$$

Using a similar method to prove [21, Theorem 3.3](or [22, Theorem 3.2]), we may obtain the following result.

Theorem 4.2. For $\nu=+,-$, $(\lambda^\nu,0)$ is a bifurcation point for problem (7). Moreover, there exists an continuum $\mathscr{D}^\nu$ of solutions of problem (7), for $\nu=+,-$ and such that

(i) $\mathscr{D}^\nu\subset((\mathbb{R}\times P^\nu)\cup\{(\lambda^\nu,0)\})$;

(ii) $\mathscr{D}^\nu$ is unbounded;

(iii) $\mathscr{D}^\nu\cap(\mathbb{R}\times\{0\})=(\lambda^\nu,0)$;

(iv) $\lim_{\lambda\to\infty}\|u_\lambda\|=\max\{1,R\}$ for $(\lambda,u_\lambda)\in(\mathscr{D}^\nu\setminus\{(\lambda^\nu,0)\})$.

Proof. By (14), we have

$$\frac{|g(r,u,\lambda)|}{|u|}\le\frac{\overline{g}(r,u,\lambda)}{|u|}\le\frac{\overline{g}(r,|u|,\lambda)}{|u|}\to 0\ as\ |u|\to 0$$

uniformly for $r \in [0,R]$ and λ on bounded sets. It follows that

$$\lim_{|u|\to 0} \frac{g(r,u,\lambda)(1-(u')^2)^{\frac{3}{2}}}{|u|} = 0. \tag{27}$$

By Lemma 4.2, one obtain that

$$\begin{aligned} &\frac{(\lambda a(r)u + \alpha u^+ + \beta u^-)\left((1-(u')^2)^{\frac{3}{2}} - 1\right)((1-(u')^2)^{\frac{3}{2}} - 1)}{|u|} \to 0, \\ &\frac{(N-1)(u')^3}{r|u|} \to 0 \ as\ |u| \to 0. \end{aligned} \tag{28}$$

By (27) and (28), we have that

$$\frac{K_4(r,u,\lambda)}{|u|} \to 0 \ as\ |u| \to 0 \tag{29}$$

uniformly for $r \in [0,R]$ and λ on bounded sets. Let $\alpha^0 := \max_{r\in[0,R]} |\alpha(r)|$ and $\beta^0 := \max_{r\in[0,R]} |\beta(r)|$. For $0 < |u| \le R$, one get that $|(\alpha u^+ + \beta u^-)/u| \le \alpha^0 + \beta^0$. Let

$$I_0 = \left[\lambda_1 - \frac{\alpha^0+\beta^0}{a_0}, \lambda_1 + \frac{\alpha^0+\beta^0}{a_0}\right].$$

Corollary 3.1 and Remark 3.1 show that there exist two unbounded sub-continua $\mathscr{D}^+$ and $\mathscr{D}^-$ of solutions of (25) in $\mathbb{R}\times E$, bifurcating from $I_0\times\{0\}$, and $\mathscr{D}^\nu \subset (\mathbb{R}\times P^\nu)\cup(I_0\times\{0\})$ for $\nu = +$ and $\nu = -$, in other words, (i) and (ii) hold.

(iii) Let us show that $\mathscr{D}^\nu \cap (\mathbb{R}\times\{0\}) = (\lambda^\nu, 0)$, i.e. $(\lambda^\nu, 0)$ is the unique bifurcation point for problem (7). Otherwise, there exists (λ_n, u_n) be a sequence of solutions of problem (7) such that $\lambda_n \to \lambda$ and $u_n \to 0$. By $\|u_n\|$ for the two side of (25) and letting $v_n = \frac{u_n}{\|u_n\|}$, we have that v_n should be a solution of problem

$$v_n = L_N\left[\lambda r^{N-1}a(r)v_n + \alpha(r)r^{N-1}v_n^+ + \beta(r)r^{N-1}v_n^- + \frac{r^{N-1}K_4(r,u_n,\lambda_n)}{\|u_n\|}\right].$$

Same as method of obtaining (18), we have that

$$\frac{K_4(r,u_n,\lambda_n)}{\|u_n\|} \to 0 \ as\ \|u_n\| \to 0 \tag{30}$$

uniformly for $r \in [0,R]$ and λ on bounded sets.

By (30) and the compactness of L_N, we obtain that for some convenient subsequence $v_n \to v_0$ as $n \to +\infty$. Now v_0 verifies the equation

$$-(r^{N-1}v_0')' = \lambda a(r)r^{N-1}v_0 + \alpha(r)r^{N-1}v_0^+ + \beta(r)r^{N-1}v_0^- \tag{31}$$

and $\|v_0\| = 1$. By (31), it follows that $\lambda = \lambda^\nu$ for $\nu \in \{+,-\}$.

(iv) Now, we shall prove that $\lim_{\lambda\to\infty} \|u_\lambda\| = \max\{1,R\}$. For any $(\lambda_n, u_n) \in \mathscr{C}^+ \setminus \{(\lambda^+, 0)\}$ with $\lambda_n \to +\infty$ as $n \to +\infty$.

Same as the proof of Theorem 3.1, integrating the first equation of problem (7) from $\rho/2$ to r, for any $r \in [\rho/2, \rho]$ and and n large enough, we get that

$$r^{N-1}\frac{u_n'}{\sqrt{1-u_n'^2}} = -\int_{\rho/2}^{r} t^{N-1}[\lambda_n a(t)u_n + \alpha u_n^+ + \beta u_n^- + g(t,u_n,\lambda_n)]dt.$$

Set $\alpha^0 := \max_{t\in[\rho/4,\rho]} |\alpha(t)|$ and $\beta^0 := \max_{t\in[\rho/4,\rho]} |\beta(t)|$. Similar to the proof of Theorem 3.1, we have that

$$\begin{aligned}\frac{1}{\sqrt{1-\|u_n'\|_\infty^2}} &\geq \frac{1}{\sqrt{1-u_n'^2}} \geq |\frac{u_n'}{\sqrt{1-u_n'^2}}| \\ &= |\frac{1}{r^{N-1}}\int_{\rho/2}^{r} t^{N-1}[\lambda_n a(t)u+\alpha u_n^+ +\beta u_n^- + g(t,u_n,\lambda_n)]dt| \\ &\geq \frac{|\lambda_n\sigma_0(a_0+h_1)-R(\alpha^0+\beta^0)|}{Nr^{N-1}}\int_{\rho/2}^{r} t^{N-1}dt \\ &\geq \frac{|\lambda_n\sigma_0(a_0+h_1)-R(\alpha^0+\beta^0)|\rho}{N2^N}((1-\frac{1}{2^N}).\end{aligned}$$

By $\lim_{n\to+\infty}\lambda_n=+\infty$, it follows that $\lim_{n\to+\infty}\|u_n'\|_\infty=1$. Noting that

$$|u_n(r)| = |\int_R^r u_n(t)dt| \leq \int_R^r |u_n(t)'|dt \leq \|u_n'\|_\infty R.$$

Thus, one may obtain that $\lim_{\lambda\to\infty}\|u\| = \max\{1,R\}$.

Remark 4.1. Theorem 4.2 indicates that the bifurcation interval $I_0=\{\lambda^+,\lambda^-\}$, i.e., for problem (7), the bifurcation interval I_0 is a finite point set. What conditions can ensure that the component indeed bifurcating from an interval is still an open problem for the problems with mean curvature operator in Minkowski space.

§5 Radial one-sign solutions for the problem (9)

Following Theorem 4.2, we shall investigate the existence of one-sign solutions for the problem (9), where $a(r)$, $\alpha(r)$ and $\beta(r)$ satisfying the condition $(H1)$ and $(H5)$, respectively. We assume that f satisfies the following assumptions:

$(H6)$ $sf(s)>0$ for $s\neq 0$.

$(H7)$ $f_0\in(0,\infty)$.

$(H8)$ $f_0=\infty$.

$(H9)$ $f_0=0$.

where

$$f_0=\lim_{|s|\to 0}\frac{f(s)}{s}.$$

Applying Theorem 4.2 to problem (9), we have the following result.

Theorem 5.1. Let $(H1)$, $(H5)$, $(H6)$ and $(H7)$ hold. For $\nu=+,-$, $(\frac{\lambda^\nu}{f_0},0)$ is a bifurcation point for problem (9). Moreover, there exists an continuum $\mathscr{D}^\nu$ of solutions of problem (9), for $\nu=+,-$ and such that

(i) $\mathscr{D}^\nu\subset((\mathbb{R}\times P^\nu)\cup\{(\frac{\lambda^\nu}{f_0},0)\})$;

(ii) $\mathscr{D}^\nu$ is unbounded;

(iii) $\mathscr{D}^\nu\cap(\mathbb{R}\times\{0\})=(\frac{\lambda^\nu}{f_0},0)$;

(iv) $\lim_{\lambda\to\infty}\|u_\lambda\|=\max\{1,R\}$ for $(\lambda,u_\lambda)\in(\mathscr{D}^\nu\setminus\{(\frac{\lambda^\nu}{f_0},0)\})$.

Proof of Theorem 5.1. Let $\zeta\in C(\mathbb{R},\mathbb{R})$ be such that

$$f(u)=f_0u+\zeta(u)$$

with $\lim_{|u|\to 0}\frac{\zeta(u)}{u}=0$.

Same the process of getting (13), one obtain that the problem (9) is equivalent to

$$\begin{cases} -(r^{N-1}u')' = \lambda r^{N-1}a(r)f_0u + \alpha r^{N-1}u^+ + \beta r^{N-1}u^- \\ \qquad\qquad + \lambda r^{N-1}K_6(r,\lambda,u), \quad r\in(0,R), \\ u'(0)=u(R)=0, \end{cases} \tag{32}$$

where

$$\begin{aligned} K_6(r,\lambda,u) =& (\lambda a(r)f_0u + \alpha u^+ + \beta r^{N-1}u^-)((1-(u')^2)^{\frac{3}{2}}-1) \\ & + a(r)\zeta(u)(1-(u')^2)^{\frac{3}{2}} - \frac{N-1}{r}u'^3. \end{aligned}$$

Using the similar method to prove (29), one get

$$\frac{K_6(r,\lambda,u)}{|u|} \to 0 \ as \ |u|\to 0$$

uniformly for $r\in(0,R)$ and λ on bounded sets.

Let us consider the problem (32) as a bifurcation problem from the trivial solution $u\equiv 0$.

Applying Theorem 4.2 to problem (32), we have the following result.

For $\nu=+,-$, $(\frac{\lambda^\nu}{f_0},0)$ is a bifurcation point for problem (32). Moreover, there exists an unbounded continuum $\mathscr{D}^\nu$ of solutions of problem (32), such that $\mathscr{D}^\nu \subset ((\mathbb{R}\times P^\nu)\cup\{(\frac{\lambda^\nu}{f_0},0)\})$, i.e., (i) and (ii) hold.

By Theorem 4.2 (iii), (iv), we may get the results of Theorem 5.1 (iii) and (iv).

Theorem 5.2. Let $(H1)$, $(H5)$, $(H6)$ and $(H8)$ hold. For $\nu=+,-$, $(0,0)$ is a bifurcation point for problem (9). Moreover, there exists an continuum $\mathscr{D}^\nu$ of solutions of problem (9), for $\nu=+,-$ and such that

(i) $\mathscr{D}^\nu$ is unbounded;

(ii) $\mathscr{D}^\nu$ joins $(0,0)$ to $(\infty,\max\{1,R\})$;

(iii) $\mathscr{D}^\nu\subset((\mathbb{R}\times P^\nu)\cup\{(0,0)\})$;

(iv) $\mathscr{D}^\nu\cap(\mathbb{R}\times\{0\})=\{(0,0)\}$.

Proof of Theorem 5.2. Inspired by the idea of [32], we define the cut-off function of f as the following

$$f^{[n]}(s) := \begin{cases} ns, & s\in[-\frac{1}{n},\frac{1}{n}], \\ \left[f(\frac{2}{n})-1\right](ns-2)+f(\frac{2}{n}), & s\in(\frac{1}{n},\frac{2}{n}), \\ -\left[f(-\frac{2}{n})+1\right](ns+2)+f(-\frac{2}{n}), & s\in(-\frac{2}{n},-\frac{1}{n}), \\ f(s), & s\in(-\infty,-\frac{2}{n}]\cup[\frac{2}{n},+\infty). \end{cases}$$

We consider the following problem

$$\begin{cases} -\left(r^{N-1}\frac{u'}{\sqrt{1-u'^2}}\right)' = \alpha r^{N-1}u^+ + \beta r^{N-1}u^- + \lambda r^{N-1}a(r)f^{[n]}(u), \quad r\in(0,R), \\ u'(0)=u(R)=0. \end{cases} \tag{33}$$

Clearly, we can see that $\lim_{n\to+\infty} f^{[n]}(s)=f(s)$, $(f^{[n]})_0=n$.

Similar the proof of Theorem 5.1, there exists an unbounded continuum $\mathscr{D}^{\nu[n]}$ of solutions of the problem (33) emanating from $(\frac{\lambda^\nu}{n},0)$, such that $\mathscr{D}^{\nu[n]}\subset((\mathbb{R}\times P^\nu)\cup\{(\frac{\lambda^\nu}{n},0)\})$ and $(\infty,\max\{1,R\})\in\mathscr{D}^{\nu[n]}$.

Taking $z^*=(0,0)$, we easily obtain that $z^*\in\liminf_{n\to+\infty}\mathscr{D}^{\nu[n]}$. So condition (1) in Lemma 2.5 is satisfied with $z^*=(0,0)$.

Since F_1 is completely continuous from $\mathbb{R}\times E\to E$, we have that $\left(\cup_{n=1}^{+\infty}\mathscr{D}^{\nu[n]}\right)\cap\overline{B}_R$ is s pre-compact,and accordingly (2) in Lemma 2.5 holds.

Therefore, by Lemma 2.5, $\mathscr{D}^{\nu} = \limsup_{n\to\infty} \mathscr{D}^{\nu[n]}$ is unbounded closed connected such that $z^* \in \mathscr{D}^{\nu}$, and $(\infty, \max\{1, R\}) \in \mathscr{D}^{\nu}$. Clearly, u is the solution of problem (9) for any $(\lambda, u) \in \mathscr{D}^{\nu}$. From the definition of superior limit (see [31, P. 7]), we can easily see that $\mathscr{D}^{\nu} \subseteq \cup_{n=1}^{\infty} \mathscr{D}^{\nu[n]}$. So one has that $\mathscr{D}^{\nu} \subset ((\mathbb{R}\times P^{\nu}) \cup \{(0,0)\})$. In other words, (i), (ii) and (iii) hold.

(iv) We may claim that $z^* = (0, 0)$, is the unique bifurcation point of $\mathscr{D}^{\nu}$.

Suppose on the contrary that there exists a sequence $(\lambda_n, u_n) \in \mathscr{D}^{\nu} \setminus \{(0, 0)\} = \limsup_{n\to\infty} \mathscr{D}^{\nu[n]} \setminus \{(0, 0)\}$ such that $\lim_{n\to\infty} \lambda_n = \mu \neq 0$ and $\lim_{n\to\infty} u_n = 0$. Hence, for any $N_0 \in \mathbb{N}$, there exists $n_0 \geq N_0$ such that $(\lambda_n, u_n) \in \mathscr{D}^{\nu[n_0]}$. By (33), it follows that $\lambda_{n_0} = \frac{\lambda^{\nu}}{n_0}$ for $n_0 \geq N_0$. From the arbitrary of N_0, it implies that $n_0 \to \infty$, i.e., $\mu = 0$, which contradicts the assumption of $\mu \neq 0$.

Theorem 5.3. Let $(H1)$, $(H5)$, $(H6)$ and $(H9)$ hold. $(\infty, 0)$ is a bifurcation point for problem (9). Moreover, there exists an continuum $\mathscr{D}^{\nu}$ of solutions of problem (9), for $\nu = +, -$ and such that

(i) $\mathscr{D}^{\nu}$ is unbounded;

(ii) $\mathscr{D}^{\nu}$ joins $(\infty, 0)$ to $(\infty, \max\{1, R\})$;

(iii) $\mathscr{D}^{\nu} \subset (\mathbb{R} \times P^{\nu})$;

(iv) $\mathtt{Proj}_{\mathbb{R}}(\mathscr{D}^{\nu}) \neq \emptyset$.

Proof of Theorem 5.3. Inspired by the idea of [32], we define the cut-off function of f as the following

$$f^{[n]}(s) := \begin{cases} \frac{1}{n}s, & s \in [-\frac{1}{n}, \frac{1}{n}], \\ \left[f(\frac{2}{n}) - \frac{1}{n^2}\right](ns - 2) + f(\frac{2}{n}), & s \in (\frac{1}{n}, \frac{2}{n}), \\ -\left[f(-\frac{2}{n}) + \frac{1}{n^2}\right](ns + 2) + f(-\frac{2}{n}), & s \in (-\frac{2}{n}, -\frac{1}{n}), \\ f(s), & s \in (-\infty, -\frac{2}{n}] \cup [\frac{2}{n}, +\infty). \end{cases}$$

We consider the following problem

$$\begin{cases} -\left(r^{N-1}\frac{u'}{\sqrt{1-u'^2}}\right)' = \alpha r^{N-1}u^+ + \beta r^{N-1}u^- + \lambda r^{N-1}a(t)f^{[n]}(u), \quad r \in (0, R), \\ u'(0) = u(R) = 0. \end{cases} \tag{34}$$

Clearly, we can see that $\lim_{n\to+\infty} f^{[n]}(s) = f(s)$, $(f^{[n]})_0 = \frac{1}{n}$.

Similar the proof of Theorem 5.1, there exists an unbounded continuum $\mathscr{D}^{\nu[n]}$ of solutions of the problem (34) emanating from $(\lambda^{\nu}n, 0)$, such that $\mathscr{D}^{\nu[n]} \subset ((\mathbb{R} \times P^{\nu}) \cup \{(\lambda^{\nu}n, 0)\})$ and $(\infty, \max\{1, R\}) \in \mathscr{D}^{\nu[n]}$.

Taking $z^* = (\infty, 0)$, we easily obtain that $z^* \in \liminf_{n\to+\infty} \mathscr{D}^{\nu[n]}$ with $\|z^*\|_{\mathbb{R}\times E} = +\infty$ So condition (1) in Lemma 2.6 is satisfied with $z^* = (+\infty, 0)$.

Define a mapping $T : \mathbb{R} \times X \to \mathbb{R} \times X$ such that

$$T(\lambda, u) = \begin{cases} (\frac{1}{\lambda}, u), & \lambda \in (-\infty, 0) \cup (0, +\infty), \\ (0, u), & \lambda = \infty, \\ (\infty, u), & \lambda = 0. \end{cases}$$

It is easy to verify that T is a homeomorphism and $\|T(z^*)\|_{\mathbb{R}\times X} = 0$. Obviously, $\{T(\mathscr{D}^{\nu[n]})\}$ be a sequence of unbounded connected subsets in X. So (2) in Lemma 2.6 holds.

Since F_1 is completely continuous from $\mathbb{R} \times E \to E$, we have that $\left(\cup_{n=1}^{+\infty} T(\mathscr{D}^{\nu[n]})\right) \cap \overline{B}_R$ is s pre-compact,and accordingly (3) in Lemma 2.6 holds.

Therefore, by Lemma 2.6, $\mathscr{D}^{\nu} = \limsup_{n\to\infty} \mathscr{D}^{\nu[n]}$ is unbounded closed connected such that

$z^* \in \mathscr{D}^\nu$ and $(\infty, \max\{1, R\}) \in \mathscr{D}^\nu$, i.e., (i) and (ii) hold.

(iii) Obviously, u is the solution of problem (9) for any $(\lambda, u) \in \mathscr{D}_k^\nu$. From the definition of superior limit (see [31, P. 7]), we can easily see that $\mathscr{D}^\nu \subseteq \cup_{n=1}^\infty \mathscr{D}^{\nu[n]}$. So one has that and $\mathscr{D}^\nu \subset (\mathbb{R} \times (P^\nu \cup \{0\}))$.

Next, we show that $\mathscr{D}^\nu \cap (\mathbb{R} \times \{0\}) = \emptyset$. Suppose on the contrary that there exists a sequence $\{(\lambda_n, u_n)\} \subseteq \mathscr{D}^\nu$ such that $\lim_{n\to\infty} \lambda_n = \mu$ and $\lim_{n\to\infty} \|u_n\| = 0$. From (11) we can easily get that

$$u_n = \int_0^R G(r,s)s^{N-2}[(s\alpha u_n^+ + s\beta u_n^- + \lambda_n a(s)sf(u_n))(1-(u')^2)^{\frac{3}{2}} - (N-1)u'^3]ds.$$

Letting $v_n = u_n/\|u_n\|$. we have that

$$v_n = \int_0^R G(r,s)s^{N-2}\left[(s\alpha v_n^+ + s\beta v_n^- + \lambda_n a(s)s\frac{f(u_n)}{\|u_n\|})(1-(u')^2)^{\frac{3}{2}} - (N-1)\frac{u'^3}{\|u_n\|}\right]ds.$$

Similar to (15), we can show that

$$\lim_{n\to\infty} \frac{f(u_n)}{\|u_n\|} = 0. \tag{35}$$

By (17), (35), and the compactness of L_N, we obtain that for some convenient subsequence $v_n \to v$ as $n \to \infty$. Letting $n \to \infty$, we obtain that

$$|v| \le M \int_0^R G(r,s)s^{N-1}|v|ds,$$

where $M = \max\{\alpha^0, \beta^0\}$. By the Gronwall-Bellman inequality [33, Lemma 2.1], we obtain that $\|v\| = 0$.This contradicts the fact of $\|v\| = 1$. Hence, we have that $\mathscr{D}^\nu \subset (\mathbb{R} \times P^\nu)$.

(iv) Next we show that the projection of $\mathscr{D}^\nu$ on $\mathbb{R}$ is nonempty. By Theorem 5.1 (ii) and (iv), we have known that $\mathscr{D}^{\nu[n]}$ has unbounded projection on $\mathbb{R}$ for any fixed $n \in \mathbb{N}$. By Lemma 2.7, for each fixed $\epsilon > 0$ there exists an m such that for every $n > m, \mathscr{D}^{\nu[n]} \subset V_\epsilon(\mathscr{D}^\nu)$. This implies that

$$(\lambda^\nu n, \infty) \subseteq \mathtt{Proj}_{\mathbb{R}}(\mathscr{D}^{\nu[n]}) \subseteq \mathtt{Proj}_{\mathbb{R}}(V_\epsilon(\mathscr{D}^\nu)),$$

where $\mathtt{Proj}_{\mathbb{R}}(\mathscr{D}^\nu)$ denotes the projection of $\mathscr{D}^\nu$ on $\mathbb{R}$. It follows that the projection of $\mathscr{D}^\nu$ is nonempty on $\mathbb{R}$.

From Theorems 5.1-5.3, we can easily derive the following corollary, which gives the ranges of parameter guaranteeing problem (9) has zero, one or two one-sign radial solutions.

Corollary 5.1 Assume that $(H1)$, $(H5)$, $(H6)$ and $(H7)$ hold. We may get the following results.

(i) If $\lambda^\nu > 0$, then there exists $\mu_1 \in (0, \frac{\lambda^\nu}{f_0})$ such that problem (9) has no radial solution for all $\lambda \in (0, \mu_1)$; has at least two radial one-sign solutions for all $\lambda \in (\mu_1, +\infty)$.

(ii) If $\nu\lambda^\nu > 0$, then there exist $\mu_1^+ \in (0, \frac{\lambda^+}{f_0})$ and $\mu_1^- \in (\frac{\lambda^-}{f_0}, 0)$ such that problem (9) has no radial solution for all $\lambda \in (\mu_1^-, 0) \cup (0, \mu_1^+)$; has at least two radial one-sign solutions for all $\lambda \in (-\infty, \mu_1^-) \cup (\mu_1^+, +\infty)$.

(iii) If $\nu\lambda^\nu < 0$, then there exist $\mu_1^- \in (0, \frac{\lambda^-}{f_0})$ and $\mu_1^+ \in (\frac{\lambda^+}{f_0}, 0)$ such that problem (9) has no radial solution for all $\lambda \in (\mu_1^+, 0) \cup (0, \mu_1^-)$; has at least two radial one-sign solutions for all $\lambda \in (-\infty, \mu_1^+) \cup (\mu_1^-, +\infty)$.

(iv) If $\lambda^\nu < 0$, then there exists $\mu_1 \in (\frac{\lambda^\nu}{f_0}, 0)$ such that problem (9) has no radial solution for all $\lambda \in (\mu_1, 0)$; has at least two radial one-sign solutions for all $\lambda \in (-\infty, \mu_1)$.

Corollary 5.2 Assume that $(H1)$, $(H5)$, $(H6)$ and $(H8)$ hold. We may get the following results.

(*i*) If $\lambda^\nu > 0$, then the problem (9) has at least two radial one-sign solutions for all $\lambda \in (0, +\infty)$.

(*ii*) If $\nu\lambda^\nu \neq 0$, then the problem (9) has at least two radial one-sign solutions for all $\lambda \in (-\infty, 0) \cup (0, +\infty)$.

(*iii*) If $\lambda^\nu < 0$, then the problem (9) has at least two radial one-sign solutions for all $\lambda \in (-\infty, 0)$.

Corollary 5.3 Assume that $(H1)$, $(H5)$, $(H6)$ and $(H9)$ hold. We may get the following results.

(*i*) If $\lambda^\nu > 0$, then there exists $0 < \mu_2 \leq \mu_3$ such that problem (9) has no radial solution for all $\lambda \in (0, \mu_2)$; has at least four radial one-sign solutions for all $\lambda \in (\mu_3, +\infty)$.

(*ii*) If $\nu\lambda^\nu \neq 0$, then there exists $0 \neq \nu\mu_2^\nu \leq \nu\mu_3^\nu$ such that problem (9) has no radial solution for all $\lambda \in (\mu_2^{-\nu}, 0) \cup (0, \mu_2^\nu)$; has at least four radial one-sign solutions for all $\lambda \in (-\infty, \mu_3^{-\nu}) \cup (\mu_3^\nu, +\infty)$.

(*iii*) If $\lambda^\nu < 0$, then there exists $\mu_3 \leq \mu_2 < 0$ such that problem (9) has no radial solution for all $\lambda \in (\mu_2, 0)$; has at least four radial one-sign solutions for all $\lambda \in (-\infty, \mu_3)$.

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