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# Exact solutions of conformable time fractional Zoomeron equation via IBSEFM

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**Abstract**. The nonlinear conformable time-fractional Zoomeron equation is an important model to describe the evolution of a single scalar field. In this paper, new exact solutions of conformable time-fractional Zoomeron equation are constructed using the Improved Bernoulli Sub-Equation Function Method (IBSEFM). According to the parameters, 3D and 2D figures of the solutions are plotted by the aid of Mathematics software.The results show that IBSEFM is an efficient mathematical tool to solve nonlinear conformable time-fractional equations arising in mathematical physics and nonlinear optics.

## §1 Introduction

Seeking the exact travelling solutions of nonlinear partial differential equations is very important to understand the nonlinear process that appears in many areas of science. To find the exact solutions of these nonlinear equations, many powerful methods have been applied by mathematicians ([5],[15-17],[20]). The investigation of exact solutions for fractional differential equations (FDEs) is nowadays one of the main research topics. This importance is due to the fact that FDEs are widely used to describe several important processes in many fields, such as biology [33], physics [26], biomedicine [13], finance [10]. Several powerful methods have been applied in the literature to obtain the exact solutions of FDEs. Some of these effective methods are; $(G'/G)$ [30], fractional Riccati expansion [12], generalized projective Riccati equation [28], functional variable [23], the exp-function [8], modified Kudryashov [14], extended direct algebraic [32], modified trial equation [25], modified $exp(-\Omega(\xi))$ function [34] and Improved Bernoulli Sub-Equation Function Method (IBSEFM) ([4],[9],[36],[37]).

In [19], a new significant definition of the fractional derivative called conformable fractional derivative is introduced. The conformable fractional derivative is theoretically easier than fractional derivative to handle. In addition, the conformable fractional derivative satisfies many

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known features that can't be satisfied by the existing fractional derivatives, for instance; the chain rule [2].

The conformable fractional derivative has the weakness that the fractional derivative of differentiable function at the point zero is equal to zero. So that in [7] a suitable fractional derivative is proposed that allows us to escape the lack of the conformable fractional derivative. During the last years, many of techniques where applied to find exact solutions for conformable fractional nonlinear partial differential equations in literature ([3],[27],[29],[35],[36]).

In this paper, we obtain the exact solutions of (2+1)-dimensional conformable time fractional Zoomeron equation via IBSEFM. We consider

$$\frac{D^{2\alpha}u}{D_t^{2\alpha}u}\left[\frac{u_{xy}}{u}\right]-\frac{D^2u}{Dx^2}\left[\frac{u_{xy}}{u}\right]+2\frac{D^{\alpha}u}{D_t^{\alpha}u}\left[u^2\right]_x=0, \quad 0<\alpha\leq 1, \tag{1}$$

where $u(x,y,t)$ shows amplitude of the relevant wave model. This equation is a convenient model to present the novel phenomena related with boomerons and trappons and it describes the evolution of a single scalar field [21]. Many authors investigated the exact solutions of Zoomeron equation with different methods ([1],[6],[22],[24],[31]). Before beginning to the solution procedure, we should give some important and efficient properties of conformable fractional derivative.

## §2 Conformable Fractional Derivative

In this section, we give some basic definition, properties and theorems about the conformable fractional derivative.

The conformable derivative of order $\alpha$ with respect to the independent variable $t$ is defined as in [19]

$$D_t^a(y(t))=\lim_{\tau\to 0}\frac{y(t+\tau t^{1-\alpha})-y(t)}{\tau},\ t>0,\ \alpha\in(0,1],$$

for a function $y=y(t):[0,\infty)\to\mathbb{R}$.

**Theorem 1.** *Assume that the order of the derivative $\alpha\in(0,1]$ and suppose that $f=f(t)$ and $g=g(t)$ are $\alpha-$differentiable for all positive $t$. Then,*

***1.*** $D_t^a(c_1f+c_2g)=c_1D_t^a(f)+c_2D_t^a(g),\ \forall\ c_1,c_2\in\mathbb{R}.$

***2.***$D_t^a(t^k)=kt^{k-\alpha},\ \forall\ k\in\mathbb{R}.$

***3.*** *For all constant function* $f(t)=\lambda,\ D_t^a(\lambda)=0.$

***4.*** $D_t^a(fg)=fD_t^a(g)+gD_t^a(f).$

***5.***$D_t^a\left(\frac{f}{g}\right)=\frac{gD_t^a(f)-fD_t^a(g)}{g^2}.$

Conformable fractional differential operator satisfies certain basic features like the chain rule, Taylor series expansion and Laplace transform.

**Theorem 2.** *Let $f=f(t)$ be an $\alpha-$conformable differentiable function and assume that $g$ is differentiable and defined in the range of $f$. Then,*

$$D_t^a(f\circ g)(t)=t^{1-\alpha}g'(t)f'(g(t)).$$

The proofs of these theorems are given in [7] and in [2] respectively.

## §3 Description of the IBSEFM

In this part, let us give the fundamental properties of the IBSEFM ([4],[9],[11],[18]). We present the six main steps of the IBSEFM below the following:

**Step 1:** Let us take account of the following conformable time fractional partial differential equation of the style

$$P(v, D_t^{(\mu)}v, D_x^{(\mu)}v, D_{xt}^{(2\mu)}v, D_{xxt}^{(3\mu)}v, ...) = 0, \tag{2}$$

where $D_t^{(\mu)}$ is the conformable derivative operator, $v(x,t)$ is an unknown function, $P$ is a polynomial in $v$ and its partial derivatives contain fractional derivatives. The aim is to convert (2) with a suitable fractional transformation into the nonlinear ordinary differential equation. The wave transformation as

$$v(x,t) = V(\xi), \quad \xi = \left(x - kt^{\alpha}\alpha^{-1}\right), \tag{3}$$

where $k$ is an arbitrary constant and different from zero. Using the properties of conformable derivative, it enables us to convert (3) into a nonlinear ordinary differential equation in the form

$$N(V, V', V'', ...) = 0. \tag{4}$$

**Step 2:** If we integrate (4) term to term once or more, we acquire integration constant(s) which may be determined then.

**Step 3:** We hypothesize that the solution of (4) may be presented as below

$$V(\xi) = \frac{\sum\limits_{i=0}^{n} a_i F^i(\xi)}{\sum\limits_{j=0}^{m} b_j F^j(\xi)} = \frac{a_0 + a_1 F(\xi) + a_2 F^2(\xi) + ...a_n F^n(\xi)}{b_0 + b_1 F(\xi) + b_2 F^2(\xi) + ...b_m F^m(\xi)}, \tag{5}$$

where $a_0, a_1, ..., a_n$ and $b_0, b_1, ..., b_m$ are coefficients which will be determined later. $m \neq 0, n \neq 0$ are chosen arbitrary constants to balance principle and considering the form of Bernoulli differential equation below the following;

$$F'(\xi) = \sigma F(\xi) + dF^M(\xi),\ d \neq 0, \sigma \neq 0,\ M \in \mathbb{R}/\left\{0, 1, 2\right\}, \tag{6}$$

where $F(\xi)$ is polynomial.

**Step 4:** The positive integer $m, n, M$ (are not equal to zero) which is found according to the balance principle that is both nonlinear term and the highest order derivative term of (4).

Substituting (5) and (6) in (3) it yields us an equation of polynomial $\Theta(F)$ of $F$ as following;

$$\Theta(F(\xi)) = \rho_s F(\xi)^s + ... + \rho_1 F(\xi) + \rho_0 = 0,$$

where $\rho_i, i = 0, ..., s$ are coefficients and will be determined later.

**Step 5:** The coefficients of $\Theta(F(\xi))$ which will give us a system of algebraic equations, whole be zero.

$$\rho_i = 0, i = 0, .., s.$$

**Step 6:** When we solve (4), we get the following two cases with respect to $\sigma$ and $d$,

$$F(\xi)=\left[\frac{-de^{\sigma(\epsilon-1)}+\epsilon\sigma}{\sigma e^{\sigma(\epsilon-1)\xi}}\right]^{\frac{1}{1-\epsilon}},\quad d\neq\sigma, \tag{7}$$

$$F(\xi)=\left[\frac{(\epsilon-1)+(\epsilon+1)\tanh\left(\sigma(1-\epsilon)\frac{\xi}{2}\right)}{1-\tanh\left(\sigma(1-\epsilon)\frac{\xi}{2}\right)}\right], d=\sigma,\ \epsilon\in\mathbb{R}. \tag{8}$$

Using a complete discrimination system for polynomial of $F(\xi)$, we obtain the analytical solutions of (4) via mathematics software and categorize the exact solutions of (4). To achieve better results, we can plot two and three dimensional figures of analytical solutions by considering proper values of parameters.

## §4 Application of the Improved Bernoulli Sub-Equation Function Method (IBSEFM)

In this section, the application of the IBSEFM to the conformable time fractional Zoomeron equation is given. Let us consider the following wave transform:

$$u(x,y,t)=U(\xi),\ \xi=kx+my-l\left(\frac{t^{\alpha}}{\alpha}\right), \tag{9}$$

where $k,m,l$ are nonzero constants. Substituting (9) into (1), we obtain the following equation:

$$kml^2\left[\frac{U''}{U}\right]''-k^3m\left[\frac{U''}{U}\right]-2kl\left[U^2\right]''=0. \tag{10}$$

If we integrate the equation (10) with respect to $\xi$ twice, we get

$$km(k^2-l^2)U''+2klU^3+sU=0, \tag{11}$$

where $s$ is a nonzero constant of integration and the second constant of integration vanishes.

When we reconsider (11) for balance principle, considering between $U''$ and $U^3$ we obtain the following relationship for $m,n$ and $M$ :

$$M=n-m+1. \tag{12}$$

(12) gives us different cases of the solution of (11) and we can obtain some analytical solutions as follows:

If we take $M=n=3,m=1$ for (5) and (6), then we can write the following equations;

$$U(\xi)=\frac{a_0+a_1F(\xi)+a_2F^2(\xi)+a_3F^3(\xi)}{b_0+b_1F(\xi)}=\frac{\Upsilon(\xi)}{\Psi(\xi)}, \tag{13}$$

$$U'(\xi)=\frac{\Upsilon'(\xi)\Psi(\xi)-\Upsilon(\xi)\Psi'(\xi)}{\Psi^2(\xi)}, \tag{14}$$

and

$$U''(\xi)=\frac{\Upsilon'(\xi)\Psi(\xi)-\Upsilon(\xi)\Psi'(\xi)}{\Psi^2(\xi)}-\frac{\left[\Upsilon(\xi)\Psi'(\xi)\right]'\Psi^2(\xi)-2\Upsilon(\xi)[\Psi'(\xi)]^2\Psi(\xi)}{\Psi^4(\xi)}, \tag{15}$$

where $F'=\sigma F+dF^3$, $a_2\neq0$, $b_1\neq0$, $\sigma\neq0$, $d\neq0$. Using (13)-(15) in (11), we obtain a system of algebraic equations from coefficients of $F$.

$Constant: -2kla_0^3-sa_0b_0^2=0,$

$F: -6kla_0^2a_1 - sa_1b_0^2 - k^3m\sigma^2a_1b_0^2 + kl^2m\sigma^2b_0^2 - 2sa_0b_0b_1 + k^3ma_0b_0b_1 - kl^2m\sigma^2a_0b_0b_1 = 0,$
$F^2: -6kla_0^2a_1^2 - 6kla_0^2a_2 - sa_2b_0^2 - 4k^3m\sigma^2a_2b_0^2 + 4kl^2m\sigma^2a_2b_0^2 - 2sa_1b_0b_1 + k^3m\sigma^2a_1b_0b_1$
$-sa_0b_1^2 - k^3m\sigma^2a_0b_1^2 + kl^2m\sigma^2a_0b_1^2 = 0,$
$F^3: -2kla_1^3 - 12kla_0a_1a_2 - 6kla_0^2a_3 - 4dk^3m\sigma a_2b_0^2 + 4dkl^2m\sigma a_1b_0^2 - sa_3b_0^2 - 9k^3m\sigma^2a_3b_0^2$
$+9kl^2m\sigma^2a_3b_0^2 + 4dk^3m\sigma a_0b_0b_1 - 4dkl^2m\sigma a_0b_0b_1 - 2sa_2b_0b_1 - 3k^3m\sigma^2a_2b_0b_1$
$+3kl^2m\sigma^2a_2b_0b_1 - sa_1b_1^2 = 0,$
$F^4: -6kla_1^2a_2 - 6kla_0a_2^2 - 12kla_0a_1a_3 - 12dk^3m\sigma a_2b_0^2 + 12dkl^2m\sigma a_2b_0^2 - 2sa_3b_0b_1$
$-11k^3m\sigma^2a_3b_0b_1 + 11kl^2m\sigma^2a_3b_0b_1 - sa_2b_0b_1^2 - k^3m\sigma^2a_2b_1^2 + kl^2m\sigma^2a_2b_1^2 = 0,$
$F^5: -6kla_1a_2^2 - 6kla_1^2a_3 - 12kla_0a_2a_3 - 3d^2k^3m\sigma a_1b_0^2 + 3dkl^2m\sigma a_1b_0^2 - 24dkl^2m\sigma a_3b_0^2$
$+3d^2k^3ma_0b_0b_1 - 3d^2kl^2ma_0b_0b_1 - 12dk^3ma_2b_0b_1 + 12dkl^2m\sigma a_2b_0b_1 - sa_3b_1^2 - 4k^3m\sigma^2a_3b_1^2$
$+4kl^2m\sigma^2a_3b_1^2 = 0,$
$F^6: -2kla_2^3 - 12kla_1a_2a_3 - 6kla_0a_3^2 - 8d^2k^3ma_2b_0^2 + 8d^2kl^2ma_2b_0^2 - d^2k^3ma_1b_0b_1$
$+d^2kl^2ma_1b_0b_1 - 32dk^3m\sigma a_3b_0b_1 + 32dkl^2m\sigma a_3b_0b_1 + d^2k^3ma_0b_1^2 - d^2kl^2ma_0b_1^2$
$-4dk^3m\sigma a_2b_1^2 + 4dkl^2m\sigma a_2b_1^2 = 0,$
$F^7: -6kla_2^3a_3 - 6kla_1a_3^2 - 15d^2k^3ma_3b_0^2 + 15d^2kl^2ma_3b_0^2 - 9d^2k^3ma_2b_0b_1 + 9d^2kl^2ma_2b_0b_1$
$-12dk^3m\sigma a_3b_1^2 + 12dkl^2m\sigma a_3b_1^2 = 0,$
$F^8: -6kla_2a_3^2 - 21d^2k^3ma_3b_0b_1 + 21d^2kl^2ma_3b_0b_1 - 3d^2k^3ma_2b_1^2 + 3d^2kl^2ma_2b_1^2 = 0,$
$F^9: -2kla_3^3 - 8d^2k^3ma_3b_1^2 + 8d^2kl^2ma_3b_1^2 = 0.$

Solving the above system of equations with the help of mathematics programme, it yields us the following coefficients:

**Case 1:** For $\sigma \neq d$;

$$a_0 = \frac{i\sqrt{s}b_0}{\sqrt{2}\sqrt{k}\sqrt{l}};\, a_1 = \frac{i\sqrt{s}b_1}{\sqrt{2}\sqrt{k}\sqrt{l}};\, a_2 = \frac{i\sqrt{2}d\sqrt{s}b_0}{\sqrt{k}\sqrt{l}\sigma};\, a_3 = \frac{i\sqrt{2}d\sqrt{s}b_1}{\sqrt{k}\sqrt{l}\sigma};\, m = \frac{s}{2k(k^2-l^2)\sigma^2}.$$

Substituting these coefficients along with (7) in (13) we obtain the complex exponential function solution of the conformable time fractional Zoomeron equation as follows:

$$u_1(x,y,t) = \frac{i\sqrt{s}\left(1 + \frac{2d}{-d+\epsilon\sigma exp\left(-2kx\sigma + \frac{2lt^{\alpha}\sigma}{\alpha} - \frac{sy}{k^3\sigma - kl^2\sigma}\right)}\right)}{\sqrt{2kl}}.$$

**Case 2:** For $\sigma \neq d$;

$$a_0 = \frac{\sqrt{-k^2+l^2}\sqrt{m}\sqrt{s}b_0}{\sqrt{2}l\sqrt{k^3m-kl^2m}};\, a_1 = \frac{\sqrt{-k^2+l^2}\sqrt{m}\sqrt{s}b_1}{\sqrt{2}\sqrt{l}\sqrt{k^3m-kl^2m}};\, a_2 = -\frac{2d\sqrt{-k^2+l^2}\sqrt{m}b_0}{\sqrt{l}};$$
$$a_3 = -\frac{2d\sqrt{-k^2+l^2}\sqrt{m}b_1}{\sqrt{l}};\, \sigma = -\frac{\sqrt{s}}{\sqrt{2}\sqrt{k^3m-kl^2m}}.$$

Putting these coefficients along with (7) in (13) we obtain the exact solution of (1) as follows:

$$u_2(x,y,t) = \frac{\sqrt{-k^2+l^2}\sqrt{m}\left(\sqrt{2}\sqrt{s} - \frac{4d\sqrt{k(k-1)(k+1)m}}{\frac{\sqrt{2}d\sqrt{k(k-1)(k+1)m}}{\sqrt{s}} + \epsilon exp\left(\frac{\sqrt{2}\sqrt{s}\left(kx+my-\frac{lt^{\alpha}}{\alpha}\right)}{\sqrt{k(k-1)(k+1)m}}\right)}\right)}{2\sqrt{l}\sqrt{k(k-1)(k+1)m}}.$$

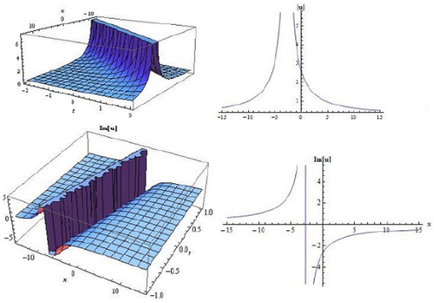

Figure 1. The 3D and 2D graphs of $|u_1(x,y,t)|$ and $u_1(x,y,t)$ considering the values $y=0.5$; $k=0.1$; $s=0.01$; $d=0.4$; $\sigma=0.5$; $\alpha=1$.

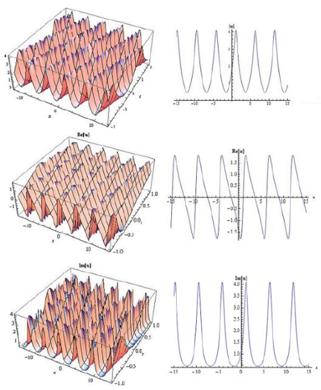

Figure 2. The 3D and 2D graphs of the solution of $|u_2(x,y,t)|$, real and imaginary part of $u_2(x,y,t)$ considering the values $y=0.2$; $k=0.2$; $s=0.3$; $d=0.6$; $m=0.7$; $l=0.4$; $\alpha=1$; $\epsilon=0.1$; $-15<x<15$, $-1<t<1$ for 3D and $t=0.2$ for 2D.

**Case 3:** For $\sigma \neq d$;

$$a_0=\frac{i\sqrt{s}b_0}{\sqrt{2kl}};a_1=\frac{i\sqrt{s}b_0}{\sqrt{2kl}};a_3=\frac{a_2b_1}{b_0};m=\frac{a_2^2}{4d^2(-k^2+l^2)b_0^2};\sigma=-\frac{i\sqrt{2}d\sqrt{s}b_0}{\sqrt{kl}a_2}.$$

Substituting above the coefficients along with (7) in (13) we obtain the exact solutions of (1) as follows:

$$u_3(x,y,t)=\frac{i\sqrt{s}}{\sqrt{2kl}}+\frac{1}{\frac{i\sqrt{kl}}{\sqrt{2s}}+\frac{e^{-\frac{2id\sqrt{2s}\left(kx-\frac{lt^\alpha}{\alpha}+\frac{lya_2^2}{4d^2(-k^2+l^2)b_0^2}\right)b_0}{\sqrt{kl}a_2}}}{a_2}\varepsilon b_0}.$$

Now, let us show the 3D and 2D figures of the solutions plotted with the help of mathematics software:

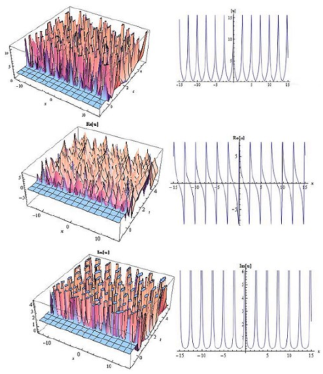

Figure 3. The 3D and 2D graphs of the solution of $|u_3(x,y,t)|$, real and imaginary part of $u_3(x,y,t)$ $y = 0.1$; $k = 0.2$; $b_0 = 1$; $d = 0.6$; $s = 0.7$; $l = 0.5$; $\alpha = 0.5$; $\epsilon = 0.2$; $-15 < x < 15$, $-1 < t < 1$ for 3D and $t = 0.1$ for 2D.

## §5 Discussion, comparision and physical explanations

In this article, we have successfully applied the IBSEFM to the nonlinear conformable time fractional Zoomeron equation to investigate some new exact solutions. It has been observed that all analytical solutions examined in this paper verify the nonlinear ordinary differential equation (11) which is obtained from nonlinear conformable time-fractional Zoomeron equation under the terms of wave transformation. All necessary computational calculations and graphs have been acquired by using mathematics software.

Figures 1-3 give a good impression about the change of amplitude and width of the soliton due to the variation of the fractional order. Remarkably, 3D graphs describe the behavior of $u(x,y,t)$ in space $x$ and $y$ at time $t$ corresponding to the value of the fractional order.The behavior represents that an increase of the fractional parameter changes the nature of the solitary wave solution. The nature of the solitary wave solution of the fractional order is confirmed by 2D line plots. Therefore, the fractional order derivative can be used to modulate the shape of the waves.

Furthermore, the nature of the waves are affected from the value of coefficients of the linear and nonlinear term of (11). According to the figures, one can see that the formats of travelling wave solutions in two and three dimensional surfaces are similar to the physical meaning of results. If we take more values of coefficients, we can obtain more travelling wave solutions.

## §6 Conclusion

By using the Improved Bernoulli Sub-Equation Function method, we obtain exact traveling wave solutions for the (2+1)-dimensional conformable timefractional Zoomeron equation under the given parameter conditions. For this equation many exact solutions have been obtained which include hyperbolic function solutions, Jacobi elliptic function solutions, trigonometric function solutions and rational function solutions in literature. Compared with the previous works, the solution method obtained in this paper has not been reported. Hence, this method is very reliable, efficient and submits new travelling wave solutions. Therefore, the IBSEFM can be applied to the other nonlinear fractional differential models in mathematical physics and nonlinear optics.

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