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Optical soliton and elliptic functions solutions of Sasa-satsuma dynamical equation and its applications

Aly R. Seadawy[1,*] Naila Nasreen[2] LU Dian-chen[2]

Abstract. The Sasa-satsuma (SS) dynamical equation interpret propagation of ultra-short and femto-second pulses in optical fibers. This dynamical model has important physical significance. In this article, two mathematical techniques namely, improved F-expansion and improved auxiliary methods are utilized to construct the several types of solitons such as dark soliton, bright soliton, periodic soliton, Elliptic function and solitary waves solutions of Sasa-satsuma dynamical equation. These results have imperative applications in sciences and other fields, and constructive to recognize the physical structure of this complex dynamical model. The computing work and obtained results show the influence and effectiveness of current methods.

§1 Introduction

Several complex physical phenomena are modeled via nonlinear partial differential equations (NLPDEs). The NLPDEs have been ussed to describe the physical structures in various fields, for example, ocean wave, chemistry, plasma physics optics, atmospheric waves, physics of condensed matters and so on [1–8]. Owing to the stability among modulation of self-Phase and dispersion of group velocity in nonlinear optics, the non-linear Schrdinger equations (NLSEs) have been utilized to describe the promulgation of soliton pulses. Due to the wide application of optical solitons, this area has gained much attention from researchers. Optical soliton pulses basically constitute for soliton communication technology such as transoceanic distances and transcontinental data transferring across the telecommunications industry, optical bers, optical communication systems and all-optical switching strategies [9–15].

These advances have provoked further inclusive research in the areas from engineering, applied mathematics. Nowadays the most prominent study area is examining these model along fibers non-linearities, for example, NLSE with nonlinearity of quadratic-cubic, Schrdinger-Hirota equation, Kaup-Newell model, Ginzburg-Landau equation, schrödinger dynamical model

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*Corresponding author.

having weak non-linearity, FokasLenells equation, Kadomtsev-Petviashvili dynamic equation, Zakharov-Kuznetsov model, KdV-Zakharov-Kuznetsov model, and many more [16–22]. Furthermore, SS equation also known as a form that explains optical soliton propagation phenomenon in optical fibers. This model is an extension of NLSE that contains dispersion of third-order, self-Steepening, moreover the effects of stimulated Raman Scattering in mono-mode optical bers. Furthermore, The SS equation portrays the dissemination pulses of femto-second in fiber optics, and also analyzes the connection and propagation of ultra-short Pulses in the femtosecond or sub pico-second system. So finding solutions of optical soliton pulses of these models becomes very attractive for the researchers.

Several powerful methods have been developed by different authors to find the exact solution of various types of NLSEs such as simple equation technique, Jacobi elliptic function expansion method, Riccati mapping equation technique, the homogeneous balancing technique, the Sine-Cosine technique, simple and modified simple equation techniques, Auxiliary equation technique, rational expansion technique, extended direct algebraic scheme, homotopy perturbation method, (G'/G)-expansion technique, Soliton ansatz technique, Backlund transform technique, Elliptic function scheme, Hirotas bilinear scheme, Extended tanh method, Rational expansion technique and many more [21–25, 27–42]. It is an observation that all these above methods have some advantages as well as some disadvantages with respect to the non-linear problems to be taken and there is no specific techniques which handle all kinds of nonlinear partial differential equations. Some authors used various techniques on the Sasa-Satsuma equation and exact solutions in different have been constructed such as Trial equation approach [17], modified simple equation methodology [25, 26] and Darboux transformation [43], F-expansion scheme [44], Lie algebra method [45, 46].

In this paper, we construct the solitons, elliptic function, and other exact solutions by utilizing two techniques namely, improved F-expansion and improved auxiliary equation methods. These solutions can be useful for physicist and mathematicians to understand the physical structures of this complex model. The remaining detail steps of achieving solutions for the aforesaid observing models are given in the following work.

§2 Governing Sasa-Satsuma Dynamical Equation

The governing model of SS [25, 26] has the form as

$$i\frac{\partial u}{\partial t} + a\frac{\partial^2 u}{\partial x^2} + b|u|^2 u + i\left(\alpha\frac{\partial^3 u}{\partial x^3} + \beta|u|^2\frac{\partial u}{\partial x} + \theta\frac{\partial |u|^2}{\partial x}u\right) = 0, \tag{1}$$

where the function u is dependent and the variables x, t are independent. In above equation,the first terms gives the temporal evolution of soliton pulses and the coefficient b describe the Kerr law non-linearity. Further, more the GVD term is given with the coefficient of a and the optical soliton pulses profile is remunerated by $u(x, t)$. Lastly, the Self-steepening, dispersion third-order sequentially, Lastly, the self-Steepening, dispersion pf third-order sequentially, in addition stimulated Raman Scattering are illustrated with the coefficients of β, α, θ respectively.

2.1 Solution of Sasa-satmasa by F-expansion Method

We assuming the solution in the wave form as

$$u(x,t) = \psi(\xi)e^{iP}, \qquad \psi(\xi) = \sum_{j=-N}^{N} b_j \left(n + F(\xi)\right)^j, \tag{2}$$

where $P = \gamma x + \nu t + \epsilon$ and $\xi = kx + \omega t$. b_j are real constants and $F(\xi)$ satisfies the below ODE

$$F'(\xi) = c_0 + c_1 F(\xi) + c_2 F^2(\xi) + c_3 F^3(\xi), \tag{3}$$

where c_0, c_1, c_2 are real constants. Putting Eq.(2) and Eq.3 into Eq.(1) and making separate into parts, got as

$$\left(3\alpha\gamma k^2 - ak^2\right)\psi''(\xi) + \left(\nu - \alpha\gamma^3 + a\gamma^2\right)\psi(\xi) + (\beta\gamma - b)\psi^3(\xi) = 0. \tag{4}$$

$$\alpha k^3 \psi^{(3)}(\xi) + \left(2a\gamma k - 3\alpha\gamma^2 k + \omega\right)\psi'(\xi) - (\beta k + 2\theta k)\psi^2(\xi)\psi'(\xi) = 0. \tag{5}$$

Integrating (5) and then the resulting equation is similar with Eq.(4), we have relation between both as follows

$$\alpha = \frac{a(\beta + 2\theta)}{3(b + 2\gamma\theta)}, \qquad \nu = -\frac{2a\gamma k(2\gamma(\theta - \beta) + 3b)^2 + 9\omega(b - \beta\gamma)(b + 2\gamma\theta)}{3k(\beta + 2\theta)(b + 2\gamma\theta)}. \tag{6}$$

By applying homogeneous balance principle on Eq.(4), we attain $m = 1$ and the solution has form as

$$\psi(x,t) = \frac{b_{-1}}{n + F(\xi)} + b_0 + b_1(n + F(\xi)). \tag{7}$$

Deputing Eqs.(7) and (3) into Eq.(4) and setting different Powers of F^i to zero, we achieved system of equations in $b_{-1}, b_0, b_1, k, \gamma, \nu, \omega, \epsilon$ and b_0, b_1, b_{-1}. the system of equations is solved via using Mathematica. The followings solution cases are as

Case 1: If $c_0 = c_3 = 0$,

$$(17)\quad b_{-1} = 0, \quad b_0 = -nb_1, \quad \gamma = -\frac{2ac_2^2k^2 + b}{2b_1^2\theta},$$

$$\omega = \frac{-12a^2c_2^4k^4(\beta - 2\theta) - 12ab\beta b_1^2c_2^2k^2 + b_1^4(\beta + 2\theta)\left(8c_0c_2\theta^2k^2 - 3b^2\right)}{24b_1^2c_2^2\theta^2 k}. \tag{8}$$

The following solitary waves of Eq.(1) are attained from (17) as:

$$u_{11}(x,t) = \left(-\frac{b_1c_1e^{c_1(\xi+d)}}{c_2e^{c_1(\xi+d)} - 1}\right)e^{i(\gamma x + \nu t + \epsilon)}, \qquad c_1 > 0. \tag{9}$$

$$u_{12}(x,t) = \left(-\frac{b_1c_1e^{c_1(\xi+d)}}{c_2e^{c_1(\xi+d)} + 1}\right)e^{i(\gamma x + \nu t + \epsilon)}, \qquad c_1 < 0. \tag{10}$$

Case 2: If $c_1 = c_3 = 0$,

Set 1:

$$b_{-1} = 0, \quad b_0 = \frac{1}{2}b_1\left(\frac{c_1}{c_2} - 2n\right),$$

$$\omega = -\frac{12a^2c_2^4k^4(\beta - 2\theta) + 12ab\beta b_1^2c_2^2k^2 + b_1^4(\beta + 2\theta)\left(3b^2 + 2c_1^2\theta^2k^2\right)}{24b_1^2c_2^2\theta^2 k}, \quad \gamma = -\frac{\frac{2ac_2^2k^2}{b_1^2} + b}{2\theta}. \tag{11}$$

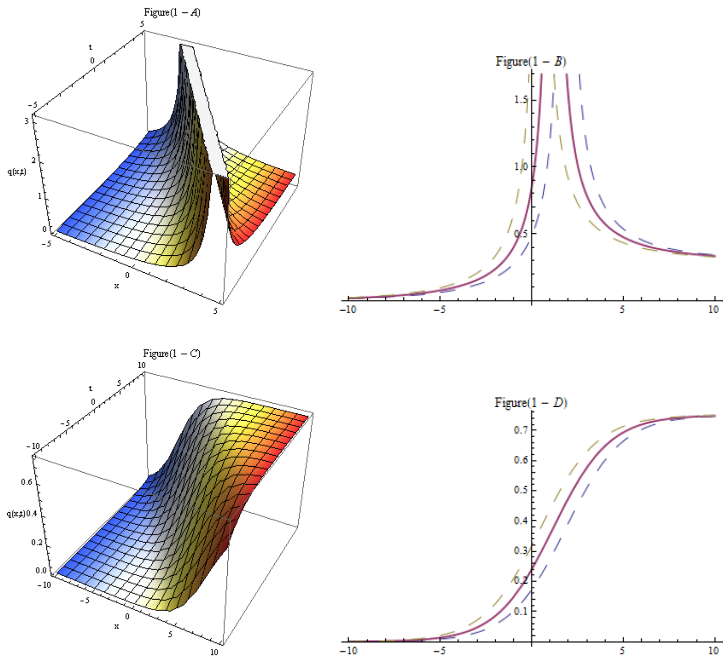

Figure 1. Solitary waves in 3-dim and 2-dim are plotted of Case 1 solutions.

Set 2:

$$b_1 = 0,\ \gamma = \frac{b}{\beta}. \tag{12}$$

The following solitary waves of Eq.(1) are attained from Set 1 as

$$u_{21}(x,t) = \left(\frac{b_1\left(2\sqrt{c_0c_2}\tan\left(\sqrt{c_0c_2}(\xi+d)\right)+c_1\right)}{2c_2}\right)e^{i(\gamma x+\nu t+\epsilon)}, \qquad c_0c_2 > 0. \tag{13}$$

$$u_{22}(x,t) = \left(\frac{b_1\left(c_1 - 2\sqrt{-c_0c_2}\tanh\left(\sqrt{-c_0c_2}(\xi+d)\right)\right)}{2c_2}\right)e^{i(\gamma x+\nu t+\epsilon)}, \qquad c_0c_2 < 0. \tag{14}$$

More solitary waves results of Eq.(1) can be obtained from set 2.

Case 3: If $c_3 = 0$,

Set 1:

$$b_{-1} = 0, \quad b_0 = \frac{\sqrt{a}k\left(c_1 - 2c_2n\right)}{\sqrt{2}\sqrt{-b-2\gamma\theta}}, \quad b_1 = \frac{\sqrt{2a}c_2k}{\sqrt{-b-2\gamma\theta}},$$
$$\omega = \frac{ak\left(6\gamma(\gamma(\beta-2\theta)-2b) + \left(c_1^2 - 4c_0c_2\right)k^2(\beta+2\theta)\right)}{6(b+2\gamma\theta)}. \tag{15}$$

Set 2:

$$b_{-1} = 0, \quad b_0 = -\frac{\sqrt{a}k\left(c_1 - 2c_2n\right)}{\sqrt{2}\sqrt{-b-2\gamma\theta}}, \quad b_1 = -\frac{\sqrt{2a}c_2k}{\sqrt{-b-2\gamma\theta}},$$
$$\omega = \frac{ak\left(6\gamma(\gamma(\beta-2\theta)-2b) + \left(c_1^2 - 4c_0c_2\right)k^2(\beta+2\theta)\right)}{6(b+2\gamma\theta)}. \tag{16}$$

The following solitons solutions of Eq.(1) from sets 1 and 2 are attained as

$$u_{31}(x,t) = \left(\frac{\sqrt{a}k\left(2c_1 - \sqrt{4c_0c_2 - c_1^2}\tan\left(\frac{\sqrt{4c_0c_2-c_1^2}}{2}(\xi+d)\right)\right)}{\sqrt{2\left(-b-2\gamma\theta\right)}} \right), \quad 4c_0c_2 > \quad c_1^2. \tag{17}$$

$$u_{32}(x,t) = \left(-\frac{\sqrt{a\left(4c_0c_2 - c_1^2\right)}k\tan\left(\frac{\sqrt{4c_0c_2-c_1^2}}{2}(\xi+d)\right)}{\sqrt{2\left(-b-2\gamma\theta\right)}} \right), \qquad 4c_0c_2 > \quad c_1^2. \tag{18}$$

Case 4: If $c_0 = c_2 = 0$,

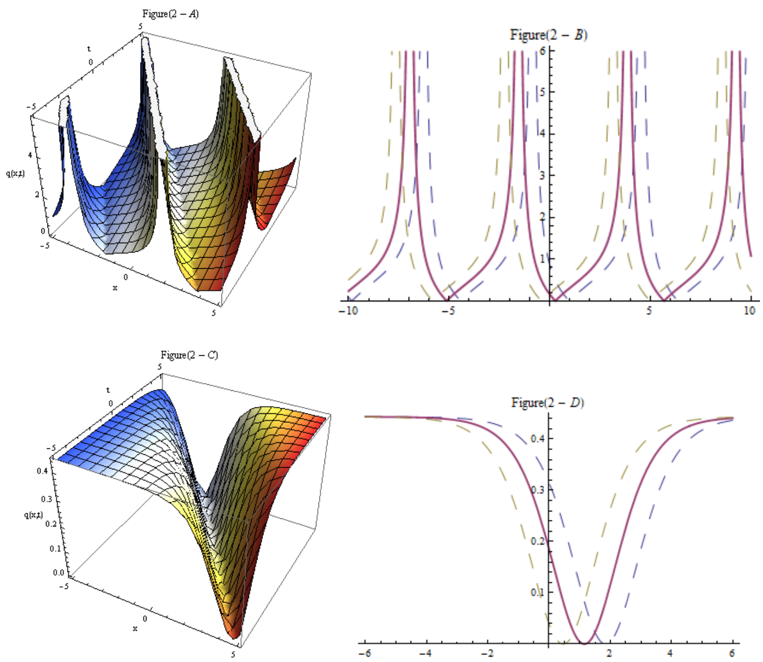

Figure 2. Solitary waves in 3-dim and 2-dim forms are plotted solutions (13) and (18).

Set 1:

$$b_1 = 0, \qquad \gamma = \frac{b}{\beta} \tag{19}$$

Set 2:

$$b_{-1} = 0, \quad \omega = -\frac{k\left(-3a\beta\gamma^2 + 6ab\gamma + 6a\gamma^2\theta + 2\beta b_0^2\gamma\theta + b\beta b_0^2 + 4b_0^2\gamma\theta^2 + 2bb_0^2\theta\right)}{3(b+2\gamma\theta)}.$$

The below solitary waves of Eq.(1) from set 1 are attained as

$$u_{41}(x,t) = \frac{b_{-1}}{\frac{e^{c_1^{3/2}\xi}}{\sqrt{1-c_3e^{2c_1\xi}}} + n} + b_0e^{i(\gamma x+\nu t+\epsilon)}, \qquad c_1 > 0. \tag{20}$$

$$q_{42}(x,t)=\left(\frac{b_{-1}}{\frac{e^{(-c_1)^{3/2}(-\xi)}}{\sqrt{1-c_3e^{-2c_1\xi}}}+n}+b_0\right)e^{i(\gamma x+\nu t+\epsilon)},\quad c_1<0. \tag{21}$$

Similarly, more solitary waves and solitons solutions of Eq.(1) from Set 2 can be obtained.

2.2 Solution of Sasa-satmasa by Auxiliary Method

Assuming the solution in wave form of this model as

$$u(x,t)=\psi(\xi)e^{iP},\quad \psi(\xi)=\sum_{i=-N}^{N}a_i\left(b+F(\xi)\right)^i, P=\gamma x+\nu t+\epsilon,\quad \xi=kx+\omega t, \tag{22}$$

where F satisfies the below ODE

$$\left(F'(\xi)\right)^2=c_0+c_1F(\xi)+c_2F^2(\xi)+c_3F^3(\xi)+c_4F^4(\xi), \tag{23}$$

where c_0,c_1,c_2 are real constants. Putting Eq.(22) and Eq.23 into Eq.(1) and making separate into parts, got as

$$\left(3\alpha\gamma k^2-ak^2\right)\psi''(\xi)+\left(\nu-\alpha\gamma^3+a\gamma^2\right)\psi(\xi)+(\beta\gamma-b)\psi^3(\xi)=0. \tag{24}$$

$$\alpha k^3\psi^{(3)}(\xi)+\left(2a\gamma k-3\alpha\gamma^2k+\omega\right)\psi'(\xi)-(\beta k+2\theta k)\psi^2(\xi)\psi'(\xi)=0. \tag{25}$$

Integrating (24) and then the resulting equation is similar with Eq.(4), we have relation between both as follows

$$\alpha=\frac{a(\beta+2\theta)}{3(b+2\gamma\theta)}\qquad \nu=-\frac{2a\gamma k(2\gamma(\theta-\beta)+3b)^2+9\omega(b-\beta\gamma)(b+2\gamma\theta)}{3k(\beta+2\theta)(b+2\gamma\theta)} \tag{26}$$

Applying balancing principle on Eq.(24), got $N=1$ and considering the solution of (24) as

$$\psi(\xi)=A_{-1}(B+F(\xi))^{-1}+A_0+A_1(B+F(\xi)). \tag{27}$$

Substituting equations (27) and (23) into (24) and arranging the coefficients of $F^i(\xi)F^{(j)}(\xi)$ to zero, we achieved a eqations system in $A_{-1},A_0,A_1,B,c_0,c_1,c_2,c_3,c_4,\gamma,k,\nu$ and ω . Mathematica Software is used for solving the equations system. The below cases of solutions as

Case 1: In this family we take $c_0=c_1=0$,

Set 1:

$$A_{-1}=-\frac{2A_0c_2}{c_3},\quad A_1=0,\quad B=\frac{2c_2}{c_3},\quad \gamma=-\frac{1}{2\theta}\left(\frac{2ac_2\left(\frac{4c_2c_4}{c_3^2}-1\right)k^2}{A_0^2}+b\right),$$

$$\omega=\frac{\frac{12a^2c_2\left(c_3^2-4c_2c_4\right)k^4(\beta-2\theta)}{c_3^2}-12aA_0^2b\beta k^2+\frac{A_0^4c_3^2(\beta+2\theta)\left(4c_2\theta^2k^2-3b^2\right)}{c_2\left(4c_2c_4-c_3^2\right)}}{24A_0^2\theta^2k}. \tag{28}$$

Set 2:

$$A_{-1}=0,\quad \gamma=\frac{b}{\beta}. \tag{29}$$

The soliton solutions of equation (1)from case 1 are obtained as follows:

$$u_{11}(\xi)=\left(\frac{A_0c_3}{\sqrt{\Delta}\cosh\left(\sqrt{c_2}\xi\right)+c_3-c_4}\right)e^{i(\gamma x+\nu t+\epsilon)},\quad c_2>0,\quad \Delta>0. \tag{30}$$

$$u_{12}(\xi)=\left(\frac{A_0c_3}{-\sqrt{\Delta}\cosh\left(\sqrt{c_2}\xi\right)+c_3-c_4}\right)e^{i(\gamma x+\nu t+\epsilon)},\ c_2>0,\ \Delta>0. \tag{31}$$

$$u_{13}(\xi) = \left(\frac{A_0 c_3}{\sqrt{-\Delta} \sinh\left(\sqrt{c_2}\xi\right) + c_3 - c_4} \right) e^{i(\gamma x + \nu t + \epsilon)}, \ c_2 > 0, \ \Delta < 0. \tag{32}$$

$$u_{14}(\xi) = \left(\frac{A_0 c_3}{-\sqrt{-\Delta} \sinh\left(\sqrt{c_2}\xi\right) + c_3 - c_4} \right) e^{i(\gamma x + \nu t + \epsilon)}, \ c_2 > 0, \ \Delta < 0. \tag{33}$$

where $\xi = kx + \omega t$ and $\Delta = c_3^2 - 4c_2c_4$.
Similarly, we can obtained more general solutions of equation (1) from set 2.

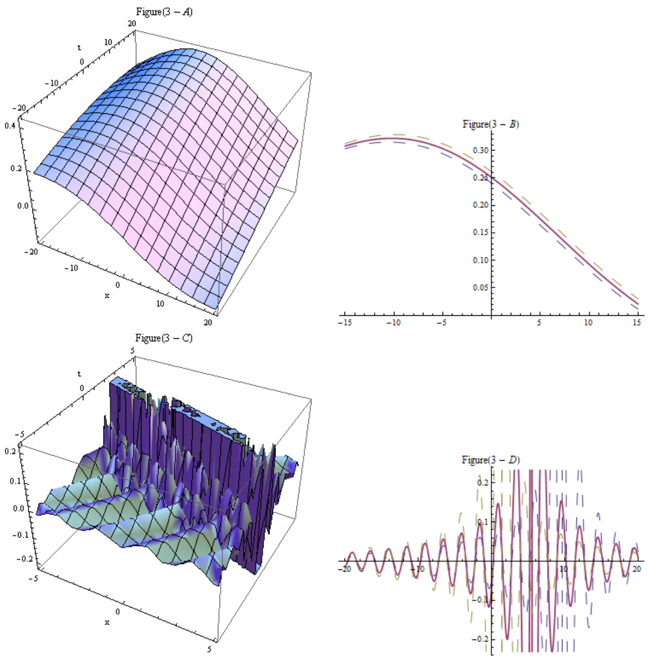

Figure 3. Exact solutions are plotted in various form of solutions (30) and (32).

Case 2: In this family we take $c_0 = c_1 = c_3 = 0$,
Set 1

$$A_{-1} = 0, \quad A_0 = A_1(-B), \quad \gamma = -\frac{\frac{2ac_4k^2}{A_1^2} + b}{2\theta}$$
$$\omega = \frac{-\frac{12a^2c_4k^4(\beta - 2\theta)}{A_1^2} - 12ab\beta k^2 + \frac{A_1^2(\beta + 2\theta)\left(4c_2\theta^2k^2 - 3b^2\right)}{c_4}}{24\theta^2 k}. \tag{34}$$

Set 2

$$A_{-1} = 0, \ \gamma = \frac{b}{\beta}. \tag{35}$$

The solitons of equation (1) from solution (34) are obtained as follows:

$$u_{21}(\xi) = \left(A_1 \sqrt{-\frac{c_2}{c_4}} \sec\left(\sqrt{-c_2}\xi\right) \right) e^{i(\gamma x + \nu t + \epsilon)}, \quad c_2 < 0, \ c_4 > 0. \tag{36}$$

$$u_{22}(\xi)=\left(A_1\sqrt{-\frac{c_2}{c_4}}\operatorname{sech}\left(\sqrt{c_2}\xi\right)\right)e^{i(\gamma x+\nu t+\epsilon)}\quad c_2>0,\ c_4<0. \tag{37}$$

Similarly, we can obtained more general solutions of equation (1) from set 2.

Case 3: Here we take parameters as $c_0=c_1=c_4=0$,

Set 1:

$$A_{-1}=\frac{2\sqrt{2a}c_2^{3/2}k}{c_3\sqrt{b+2\gamma\theta}},\quad A_0=-\frac{\sqrt{2ac_2}k}{\sqrt{b+2\gamma\theta}},\quad A_1=0,$$
$$B=\frac{2c_2}{c_3},\quad \omega=-\frac{ak\left(3\gamma(-\beta\gamma+2b+2\gamma\theta)+c_2k^2(\beta+2\theta)\right)}{3(b+2\gamma\theta)}. \tag{38}$$

Set 2:

$$A_{-1}=-\frac{2\sqrt{2a}c_2^{3/2}k}{c_3\sqrt{b+2\gamma\theta}},\quad A_0=\frac{\sqrt{2ac_2}k}{\sqrt{b+2\gamma\theta}},$$
$$A_1=0,\ B=\frac{2c_2}{c_3},\ \omega=-\frac{ak\left(3\gamma(-\beta\gamma+2b+2\gamma\theta)+c_2k^2(\beta+2\theta)\right)}{3(b+2\gamma\theta)}. \tag{39}$$

The solitons solutions of equation (1) fron set 1 and 2 are constructed as

$$u_{31}(\xi)=\left(\frac{\sqrt{2ac_2}k\sec\left(\sqrt{-c_2}\xi\right)}{\sqrt{b+2\gamma\theta}}\right)e^{i(\gamma x+\nu t+\epsilon)},\quad c_2<0. \tag{40}$$

$$u_{32}(\xi)=\left(\frac{\sqrt{2ac_2}k\operatorname{sech}\left(\sqrt{c_2}\xi\right)}{\sqrt{b+2\gamma\theta}}\right)e^{i(\gamma x+\nu t+\epsilon)},\quad c_2>0. \tag{41}$$

Similarly, we can obtained more general solutions of equation (1) from case 2.

Case 4: In this family we take $c_0=\ c_1=\ c_2=\ 0$,

$$A_{-1}=0,\quad \gamma=\frac{b}{\beta}. \tag{42}$$

The soliton solutions in the following form of equation (1) from case 1 of case 4 solutions are obtained as

$$u_{41}(\xi)=\left(A_1\left(B+\frac{4c_3}{c_3^2\xi^2-4c_4}\right)+A_0\right)e^{i(\gamma x+\nu t+\epsilon)},\quad c_2>0,c_4>0. \tag{43}$$

Case 5: In this family we take $c_0=\frac{c_2^2}{4c_4},\ c_1=\ c_3=\ 0$,

Set 1

$$A_{-1}=0,\quad A_0=0,\quad A_1=\frac{\sqrt{2ac_4}k}{\sqrt{-b-2\gamma\theta}},$$
$$B=0,\quad \omega=-\frac{ak\left(3\gamma(-\beta\gamma+2b+2\gamma\theta)+c_2k^2(\beta+2\theta)\right)}{3(b+2\gamma\theta)}. \tag{44}$$

Set 2

$$A_{-1}=\frac{\sqrt{a}c_2k}{\sqrt{2c_4(-(b+2\gamma\theta))}},\quad A_0=0,\quad A_1=-\frac{\sqrt{2ac_4}k}{\sqrt{-b-2\gamma\theta}},\quad B=0$$

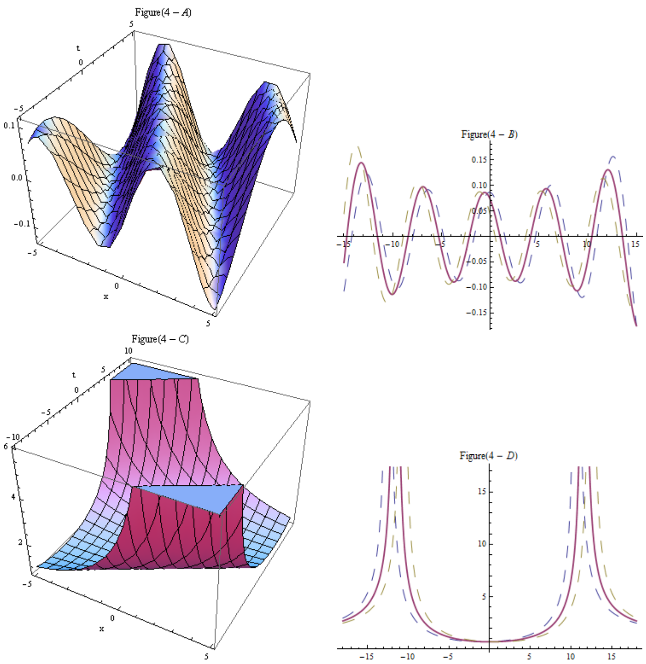

Figure 4. Exact waves are plotted in various form of solutions (40) and (43).

$$\omega = \frac{ak\left(3c_2k^2(\beta+2\theta)\sqrt{c_4(-(b+2\gamma\theta))}+\sqrt{c_4}\sqrt{-b-2\gamma\theta}\left(3\gamma(-\beta\gamma+2b+2\gamma\theta)+c_2k^2(\beta+2\theta)\right)\right)}{3\sqrt{c_4}(-b-2\gamma\theta)^{3/2}}. \tag{45}$$

The soliton solutions in the following form of equation (1) from case 1 of family 5 solutions are obtained as

$$u_{51}(\xi) = \pm\frac{\sqrt{ac_2}k\tan\left(\frac{\sqrt{c_2}}{\sqrt{2}}\xi\right)}{\sqrt{-b-2\gamma\theta}}e^{i(\gamma x+\nu t+\epsilon)}, \quad c_2>0, c_4>0. \tag{46}$$

$$u_{52}(\xi) = \pm\frac{\sqrt{-ac_2}k\tanh\left(\frac{\sqrt{-c_2}}{\sqrt{2}}\xi\right)}{\sqrt{-b-2\gamma\theta}}e^{i(\gamma x+\nu t+\epsilon)}, \quad c_2<0, c_4>0. \tag{47}$$

More novel solutions in generalized form of equation (1) from case 2 can be constructed in same way. **Case 6:** we take in this family $c_1=c_3=0, c_0=\frac{c_2^2r^2}{c_4(r^2+1)^2}$, & $0<r<1$,

set 1

$$A_{-1}=0,\quad A_0=A_1(-B),\quad \omega=\frac{A_1^2\left(3\gamma(-\beta\gamma+2b+2\gamma\theta)+c_2k^2(\beta+2\theta)\right)}{6c_4k},\quad a=-\frac{A_1^2(b+2\gamma\theta)}{2c_4k^2}.$$

set 2

$$A_{-1}=0,\ A_0=0,\ B=0,\ \omega=\frac{A_1^2\left(3\gamma(-\beta\gamma+2b+2\gamma\theta)+c_2k^2(\beta+2\theta)\right)}{6c_4k},\ a=-\frac{A_1^2(b+2\gamma\theta)}{2c_4k^2}.$$

The exact solution in the form of Jacobi elliptic function of equation (1) from case 1 are constructed as

$$u_{61}(\xi)=\left(\pm A_1\sqrt{-\frac{c_2^2r^2}{c_4\left(r^2+1\right)}}\mathrm{sn}\left(\xi\sqrt{-\frac{c_2}{r^2+1}}\middle|r\right)\right)e^{i(\gamma x+\nu t+\epsilon)},\quad h_2<0,h_4>0. \tag{48}$$

$$u_{62}(\xi)=A_1\sqrt{\frac{c_2^2r^2}{c_4-2c_4r^2}}\mathrm{cn}\left(\xi\sqrt{\frac{c_2}{2r^2-1}}\middle|r\right)e^{i(\gamma x+\nu t+\epsilon)},\quad h_4<0. \tag{49}$$

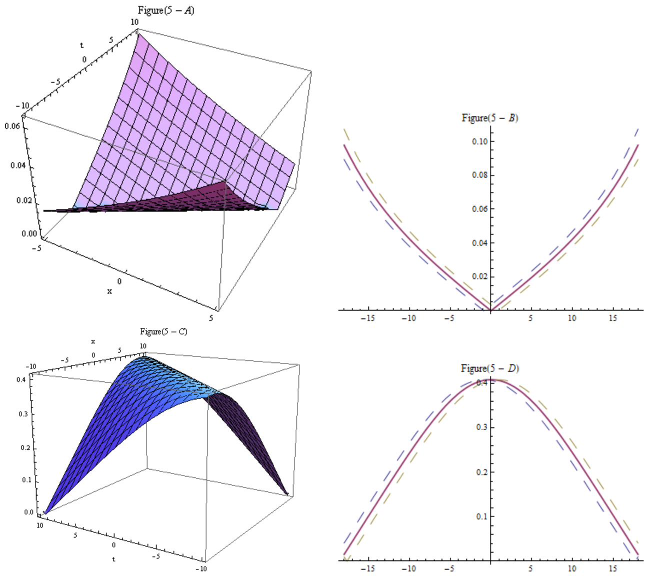

Figure 5. Exact waves are plotted in various form of solutions (40) and (43).

§3 Results and Discussions

The exact results constructed from present methods are different and novel from other techniques which are utilized by other researchers. The exact results are constructed via employing improved F-expansion and improved auxiliary equation methods. In this paper, we obtain a family of solitons and solitary waves results containing unidentified parameters. The constructed exact solutions show diverse kinds of solitons and solitary waves when specific values are given to unidentified parameters, such as bright and dark solitons, combined with bright soliton, solitary wave, etc. The authors used other techniques such as trial equation approach [17], modified simple equation methodology [25, 26], F-expansion scheme [44] to construct bright-dark solitons and periodic solitary waves. Thus, many constructed results are novel and not exist in the previous study.

The three-dimensional and 2-dimensional plots for some achieved outcomes of this model are demonstrated. The physical structures of some results are depicted out by giving appropriate values to the parameters. The Figures 1 show the exact waves in dissimilar forms, Figure(1-A) and Figure(1-B) shows bright soliton in 3D and 2D from the solution(9), Figure(1-C) and

(1-D) shows kink soliton from solutions (10).In Figures 2, the Figure(2-A) and Figure(2-B) demonstrate the 3D and 2D periodic solitary wave from solution (13). where Figure(2-c) and Figure(2-D) illustrate dark soliton in 3D and 2D from solution (18).

In Figures 3, the solution achieved from the auxiliary method the Figure(3-A), Figure(3-B) shows the bright solitary wave from solution (30), and (3-C) and (3-D) demonstrate periodic traveling solitary waves of solution of(32)in 3D and contours plot. In Figure(4-A) and (4-B) demonstrate combined bright and dark soliton in 3D and 2D of solution (40). In Figure (4-c) and (4-D) shows traveling wave in 3D and 2D of solution (43).

§4 Conclusion

We have fruitfully employed the improved F-expansion and improved auxiliary equation techniques for constructing solitons, solitary waves, Jacobi elliptic function and other solutions of Sasa-satsuma dynamical equation. This dynamical equation portrays the propagation in optical fibers of femto-second pulses, and also analyzes the connection and propagation of the pulses of ultra-short in the femtosecond or sub pico-second system. The achieved exact solitons and other solutions of a different kind such as dark-bright solitons, periodic soliton, and solitary waves have been derived that have key applications in engineering, physics, and applied mathematics. We have also shown some obtained results graphically, by giving appropriate values to the parameters which facilitate to recognize the physical phenomena of this nonlinear model. All the achieved results are novel and do not exist in literature. This has been shown that the proposed techniques are concise, effective and can be employed on other NLPDE.

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[1]Mathematics department, Faculty of science, Taibah University, Al-Madinah Al-Munawarah, Saudi Arabia.

[2]Faculty of Science, Jiangsu University, Zhenjiang 212013, China.

Email: Aly742001@yahoo.com