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A popular reaction-diffusion model fractional Fitzhugh-Nagumo equation: analytical and numerical treatment

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Abstract. The main objective of this article is to obtain the new analytical and numerical solutions of fractional Fitzhugh-Nagumo equation which arises as a model of reaction-diffusion systems, transmission of nerve impulses, circuit theory, biology and the area of population genetics. For this aim conformable derivative with fractional order which is a well behaved, understandable and applicable definition is used as a tool. The analytical solutions were got by utilizing the fact that the conformable fractional derivative provided the chain rule. By the help of this feature which is not provided by other popular fractional derivatives, nonlinear fractional partial differential equation is turned into an integer order differential equation. The numerical solutions which is obtained with the aid of residual power series method are compared with the analytical results that obtained by performing sub equation method. This comparison is made both with the help of three-dimensional graphical representations and tables for different values of the γ.

§1 Introduction

The subject of nonlinear dynamical systems has settled the mind of many people in the last decade, due to their wider occurrence in daily life. This subject involves many areas in the fields of science, engineering and technology. For instance, the topic includes dynamics, non-equilibrium processes in physics, complex matter and networks, computational biology, fluctuations and random processes, self-organization, social phenomena, fractal geometry, media with self-similar properties; technology and other interesting subject close related to nonlinear dynamical system [25–29]. To comprehend and estimate the forthcoming behavior of these complex problems, scientists trust mathematical models, which depend most of the time on mathematical tools known as differential and integral operators.

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One can find in the existing literature two groups of differential and integral operators involving classical and non-conventional. In addition, with many proofs in published research papers the classical are more appropriate for classical mechanics and suppose to describe only those processes known as memoryless, which implies, these differential operators forecasts the future with no memory. This is a highly misleading, as many real world problems do not follow Markovian processes, rather they mostly follow a non-Markovian scenario. To solve this problem, the second group of differential operators has been suggested and they can be identified as differential operators with arbitrary order called fractional derivatives. These derivatives have some advantages over the traditional integer order Newtonian concept derivative. For instance fractional calculus can easily predict the future of the event by using the historical dependence of the evolution of system analysis and taking the global correlation into consideration, but integer calculus is not convenient to represent this process. Theoretical model results considered with integer calculus often fail to coincide with the experimental results. On the contrary, fractional calculus models overcome this critical defect of integer calculus. In addition fractional calculus has a clearer physical significance and a simpler expression when describing complicated physical mechanics problems, compared with the integer order model. Because of these benefits it has gained a great demand and significance in the past few decades in several fields of science and engineering. Efficient analytical and numerical methods have been improved but still need specific attention.

Recently a well behaved, feasible and comprehensible fractional derivative and integral definition called "conformable fractional derivative and integral" are introduced by Khalil et al. [1].

Definition Let the function f be defined as $f : [0,\infty) \to \mathbb{R}$ be a function. The γ^{th} order *"conformable fractional derivative"* (CFD) [1] of f can be declared as,

$$D_\gamma(f)(t) = \lim_{\varepsilon\to 0} \frac{f(t+\varepsilon t^{1-\gamma}) - f(t)}{\varepsilon},$$

for all $t > 0, \gamma \in (0,1)$. Then let f be a γ-differentiable function in some $(0,a), a > 0$ and $\lim_{t\to 0^+} f^{(\gamma)}(t)$ exists and so $f^{(\gamma)}(0) = \lim_{t\to 0^+} f^{(\gamma)}(t)$.

The *"conformable fractional integral"* of the function f starting from $a \geq 0$ is expressed as:

$$I_\gamma^a(f)(t) = \int_a^t \frac{f(x)}{x^{1-\gamma}} dx$$

where the integral denotes the Riemann improper integral, and $\gamma \in (0,1]$. The properties that satisfied by the CFD are given as follows [1].

Theorem 1.1. *Let $\gamma \in (0,1]$ and assume that f, g are γ-differentiable at point $t > 0$. Then*

1. $D_\gamma(cf + dg) = cD_\gamma(f) + cD_\gamma(g)$ *where* $c, f \in \mathbb{R}$.

2. $D_\gamma(t^p) = pt^{p-\gamma}$ *where* $p \in \mathbb{R}$.

3. *Let λ be a constant then* $D_\gamma(\lambda) = 0$.

4. $D_\gamma(fg) = fD_\gamma(g) + gD_\gamma(f)$.

5. $D_\gamma\left(\frac{f}{g}\right)=\frac{gD_\gamma(f)-fD_\gamma(g)}{g^2}$.

From the theorem it is understood that CFD satisfies the basic conditions [2] to be a fractional derivative operator. To take the advantage of these situation many numerical and analytical methods are applied to fractional partial differential equations involving conformable derivative [3–7]. One of the most important advantages of conformable fractional derivative is that it provides the chain rule [8], so the analytical solution of the nonlinear fractional partial differential equations can be produced. This is one of the most important features not provided by the other fractional derivatives.

In this article new analytical and numerical solutions of conformable fractional Fitzhugh-Nagumo equation

$$D_t^\gamma u - D_x^2 u = u(u-\alpha)(1-u) \tag{1}$$

is usually used to model the transmission of nerve impulses [9,10]; also used in circuit theory, biology and the area of population genetics [11] and an important nonlinear reactiondiffusion equation. Fitzhugh-Nagumo equation became a favorite model for reaction-diffusion systems which simulate propagation of waves in excitable media, such as heart tissue or nerve fiber. To the best of our knowledge all the obtained results are seen firstly in the literature. To get the exact and numerical solutions we employed sub equation method and residual power series method respectively.

The rest of article is organized as follows. In second chapter brief description of implemented methods are given. In the third chapter the exact and approximate analytical solutions of conformable fractional Fitzhugh-Nagumo equation are given. Also some graphical representations and comparative tables for numerical results are expressed.

§2 Brief Description of Implemented Methods

2.1 Sub Equation Method

Sub equation method [19] built on the Riccati equation

$$\varphi'(\xi) = \sigma + (\varphi(\xi))^2 . \tag{2}$$

First of all consider the following nonlinear time fractional partial differential equation

$$P\left(u, D_t^\gamma u, D_x u, D_x^2 u, ...\right) = 0 \tag{3}$$

where $D_t^\gamma u$ indicates conformable derivative of function $u(x,t)$ with fractional order. With the aid of the fractional wave transformation [15]

$$u(x,t) = U(\xi),\ \xi = kx + w\frac{t^\gamma}{\gamma} \tag{4}$$

where k, w are arbitrary constants are going to be evaluated and the chain rule [8], Eq. (3) changes into integer order nonlinear ODE

$$G(U(\xi), U'(\xi), U''(\xi), ...) = 0. \tag{5}$$

Assuming Eq. (5) has the solution as follows

$$U(\xi)=\sum_{i=0}^{N}a_i\varphi^i(\xi),\ a_N\neq 0, \tag{6}$$

where a_i $(0\leq i\leq N)$ are constants are going to examined and N is going to be evaluated by balancing principle [18] in equation (5) and $\varphi(\xi)$ is the solution of Riccati equation (2). Some of the solutions of the equation (2) can be given below.

$$\varphi(\xi)=\begin{cases}-\sqrt{-\sigma}\tanh\left(\sqrt{-\sigma}\xi\right), & \sigma<0\\ -\sqrt{-\sigma}\coth\left(\sqrt{-\sigma}\xi\right), & \sigma<0\\ \sqrt{\sigma}\tan\left(\sqrt{\sigma}\xi\right), & \sigma>0\\ -\sqrt{\sigma}\cot\left(\sqrt{\sigma}\xi\right), & \sigma>0\\ -\frac{1}{\xi+\varpi},\ \varpi\ is\ a\ cons., & \sigma=0\end{cases} \tag{7}$$

Gathering all the obtained data sets we acquire a polynomial including $\varphi(\xi)$. Equating all the coefficients of $\varphi^i(\xi)$ $(i=0,1,...,N)$ to zero led to a algebraic system in k,w,a_i $(i=0,1,...,N)$. Evaluating the solution of these algebraic system we get the values of w,k,a_i $(i=0,1,...,N)$. Using all the obtained values in the formulas (7) we get the analytical results for equation (3).

2.2 Residual Power Series Method

To illustrate the basics of RPSM [13,14], handle the following fractional partial differential equation [17]:

$$T_\gamma u(x,t)+N[x]u(x,t)+L[x]u(x,t)=c(x,t), \tag{8}$$

where $n-1<n\gamma\leq n$, $x\in\mathbb{R}$, $t>0$ and given with the initial condition

$$f_0(x)=u(x,0)=f(x). \tag{9}$$

Here, $L[x]$ symbolizes a linear, $N[x]$ denotes a nonlinear operator and $c(x,t)$ are continuous functions.

The RPSM method based upon evaluating the solution of the equation (8) with the initial condition (9) by expanding a fractional power series around $t=0$.

$$f_{(n-1)}(x)=T_t^{(n-1)\gamma}u(x,0)=h(x) \tag{10}$$

The expanded form of the approximate solution is shown as

$$u(x,t)=f(x)+\sum_{n=1}^{\infty}f_n(x)\frac{t^{n\gamma}}{\gamma^n n!}. \tag{11}$$

Then, the k. truncated series of $u(x,t)$, called $u_k(x,t)$ can be rewritten as follows:

$$u_k(x,t)=f(x)+\sum_{n=1}^{k}f_n(x)\frac{t^{n\gamma}}{\gamma^n n!}. \tag{12}$$

Since the $1st$ approximate solution $u_1(x,t)$ is

$$u_1(x,t)=f(x)+f_1(x)\frac{t^\gamma}{\gamma^n} \tag{13}$$

then $u_k(x,t)$ might be rearranged as

$$u_k(x,t) = f(x) + f_1(x)\frac{t^\gamma}{\gamma^n} + \sum_{n=2}^{k} f_n(x)\frac{t^{n\gamma}}{\gamma^n n!},\ k = 2,3,4,... \tag{14}$$

for $0 < \gamma \le 1$, $0 \le t < \mathbb{R}^{\frac{1}{v}}$, $x \in I$.

Initially we express the residual function and the $k-th$ residual function

$$Resu(x,t) = T_\gamma u(x,t) + N[x]u(x,t) + L[x]u(x,t) - c(x,t), \tag{15}$$

$$Resu_k(x,t) = T_\gamma u_k(x,t) + N[x]u_k(x,t) + L[x]u_k(x,t) - g(x,t),\ k = 1,2,3,... \tag{16}$$

respectively. Obviously, $Res(x,t) = 0$ and $\lim\limits_{k\to\infty} Resu_k(x,t) = Resu(x,t)$ for each $x \in I$ and $t \ge 0$. Indeed this bring about $\frac{\partial^{(n-1)\gamma}}{\partial t^{(n-1)\gamma}} Resu_k(x,t) = 0$ for $n = 1,2,3,...,k$. [12, 16]. Solving the equation $\frac{\partial^{(n-1)\gamma}}{\partial t^{(n-1)\gamma}} Resu_k(x,0) = 0$ concludes the required $f_n(x)$ coefficients. So the $u_n(x,t)$ solutions can be acquired by this way. Also some different versions of the RPS newly studied in various studies [20–24].

§3 Solutions of the Fitzhugh-Nagumo Equation

3.1 Analytical Solutions of the Fitzhugh-Nagumo Equation

Regard the time fractional Fitzhugh-Nagumo Equation (1). Performing the chain [8] and wave transform (4) the Eq. (1), we acquire integer order nonlinear differential equation as follows.

$$wU'(\xi) - k^2U''(\xi) - U(\xi)(1-U(\xi))(U(\xi)-\alpha) = 0. \tag{17}$$

Suppose that the solution of Eq. (17) is given in terms of $\varphi(\xi)$ where is the exact solutions of equation (2) as follows.

$$U(\xi) = \sum_{i=0}^{N} a_i\varphi^i(\xi),\ a_N \ne 0. \tag{18}$$

Employing the balancing principle [18], we have $N = 1$. Evaluating all the obtained data in Eq. (17), an algebraic equation system arises with respect to k, w, a_0, a_1. By solving this system solution set is obtained as follows

$$a_0 = \frac{\alpha+1}{2}, a_1 = \frac{\sqrt{(\alpha-1)^2}}{2\sqrt{-\sigma}}, w = -\frac{(\alpha+1)\sqrt{(\alpha-1)^2}}{4\sqrt{-\sigma}}, k = \pm\frac{\sqrt{(\alpha-1)^2}}{2\sqrt{-2\sigma}}. \tag{19}$$

When $\sigma < 0$, using (7) and (4) the traveling wave solutions of Eq. (1) can be deducted

$$u_{1,2}(x,t) = \frac{\alpha+1}{2} - \frac{1}{2}(\alpha-1)\tanh\left(\frac{\left(1-\alpha^2\right)t^\gamma}{4\gamma} \pm \frac{(\alpha-1)x}{2\sqrt{2}}\right),\ \text{for } \alpha > 1,$$

$$u_{3,4}(x,t) = \frac{\alpha+1}{2} - \frac{1}{2}(\alpha-1)\coth\left(\frac{\left(1-\alpha^2\right)t^\gamma}{4\gamma} \pm \frac{(\alpha-1)x}{2\sqrt{2}}\right),\ \text{for } \alpha > 1,$$

$$u_{5,6}(x,t)=\frac{\alpha+1}{2}-\frac{1}{2}(1-\alpha)\tanh\left(\frac{\left(\alpha^2-1\right)t^\gamma}{4\gamma}\pm\frac{(\alpha-1)x}{2\sqrt{2}}\right),\ \text{for } \alpha<1,$$

$$u_{7,8}(x,t)=\frac{\alpha+1}{2}-\frac{1}{2}(1-\alpha)\coth\left(\frac{\left(\alpha^2-1\right)t^\gamma}{4\gamma}\pm\frac{(\alpha-1)x}{2\sqrt{2}}\right),\ \text{for } \alpha<1.$$

Similar solutions are obtained with the above solutions when $\sigma>0$.

3.2 Approximate Solutions of the Fitzhugh-Nagumo Equation

Consider the nonlinear time-fractional F-N equation

$$D_t^\gamma u-D_x^2u=u(u-\alpha)(1-u) \tag{20}$$

where $u=u(x,t)$, $t\geq 0$, $0<\alpha\leq 1$. The initial condition obtained from the exact solution is

$$u(x,0)=\frac{1}{2}\left(\alpha+(\alpha-1)\tanh\left(\frac{(\alpha-1)x}{2\sqrt{2}}\right)+1\right) \tag{21}$$

For residual power series

$$u(x,t)=f(x)+\sum_{n=1}^{\infty}f_n(x)\frac{t^{n\gamma}}{\gamma^n n!} \tag{22}$$

and k. truncated series of $u(x,t)$

$$u_k(x,t)=f(x)+\sum_{n=1}^{k}f_n(x)\frac{t^{n\gamma}}{\gamma^n n!},\ k=1,2,3,... \tag{23}$$

Therefore, the k-th residual functions of time-fractional F-N equation is:

$$Resu_k(x,t)=t^{1-\gamma}\left(u_k\right)_t(x,t)-u_k(x,t)\left(1-u_k(x,t)\right)\left(u_k(x,t)-\alpha\right)-\left(u_k\right)_{xx}(x,t). \tag{24}$$

To determine the coefficient $f_1(x)$, in $u_1(x,t)$, we should replace the $1st$ truncated series $u_1(x,t)=f(x)+f_1(x,y)\frac{t^\gamma}{\gamma}$ into the first truncated residual function as

$$\begin{aligned} Resu_1(x,t) &= f_1(x)-f''(x)-\left(-\frac{f_1(x)t^\gamma}{\gamma}-f(x)+1\right)\left(\frac{f_1(x)t^\gamma}{\gamma}+f(x)\right)\\ &\times\left(-\alpha+\frac{f_1(x)t^\gamma}{\gamma}+f(x)\right)-\frac{t^\gamma f_1''(x)}{\gamma}. \end{aligned} \tag{25}$$

Now for the substitution of $t=0$ through the equation $Resu_1(x,t)$ to obtain

$$f_1(x)=\left(f''\right)^2-\alpha f(x)-f(x)^3+f(x)^2. \tag{26}$$

Thus, we acquire the $1st$ RPS approximate results for time-fractional F-N equation as

$$u_1(x,t)=\frac{t^\gamma\left(\left(f''\right)^2-\alpha f(x)-f(x)^3+f(x)^2\right)}{\gamma}+f(x). \tag{27}$$

Again, to determine the second unknown coefficient $f_2(x)$, we replace the $2nd$ truncated series solution $u_2(x,t)=f(x)+f_1(x)\frac{t^\gamma}{\gamma}+f_2(x)\frac{t^{2\gamma}}{2\gamma^2}$ into the $2nd$ truncated residual function and obtain

$$\begin{aligned} Resu_2(x,t) &= -f''(x) - \frac{t^{2\gamma} f_2''(x)}{2\gamma^2} + t^{1-\gamma}\left(f_1(x)t^{\gamma-1} + \frac{f_2(x)t^{2\gamma-1}}{\gamma}\right) - \frac{t^{\gamma} f_1''(x)}{\gamma} \\ &+ \left(\frac{f_2(x)t^{2\gamma}}{2\gamma^2} + \frac{f_1(x)t^{\gamma}}{\gamma} + f(x)\right)\left(-\left(-\frac{f_2(x)t^{2\gamma}}{2\gamma^2} - \frac{f_1(x)t^{\gamma}}{\gamma} - f(x) + 1\right)\right) \\ &\times \left(-\alpha + \frac{f_2(x)t^{2\gamma}}{2\gamma^2} + \frac{f_1(x)t^{\gamma}}{\gamma} + f(x)\right) \end{aligned} \tag{28}$$

Now, applying T_α on both sides of $Resu_2(x,t)$ and equating to 0 for $t = 0$ gives:

$$f_2(x) = 2\alpha f_1(x)f(x) - \alpha f_1(x) + f_1''(x) - 3f_1(x)f(x)^2 + 2f_1(x)f(x). \tag{29}$$

So the $2nd$ RPS approximate solutions of time-fractional F-N equation is:

$$\begin{aligned} u_2(x,t) &= f(x) + \frac{f_1(x)t^{\gamma}}{\gamma} \\ &+ \frac{t^{2\gamma}\left(2\alpha f_1(x)f(x) - \alpha f_1(x) + f_1''(x) - 3f_1(x)f(x)^2 + 2f_1(x)f(x)\right)}{2\gamma^2}. \end{aligned} \tag{30}$$

Similarly, we rerun the same technique for $n = 3$ and 4 to get the below expressed results.

$$\begin{aligned} f_3(x) &= 2\alpha f_2(x)f(x) + 2\alpha f_1(x)^2 - \alpha f_2(x) + f_2''(x) - 3f_2(x)f(x)^2 \\ &\quad -6f_1(x)^2 f(x) + 2f_2(x)f(x) + 2f_1(x)^2, \end{aligned} \tag{31}$$

$$\begin{aligned} u_3(x,t) &= f(x) + \frac{f_1(x)t^{\gamma}}{\gamma} + \frac{f_2(x)t^{2\gamma}}{2\gamma^2} + \frac{t^{3\gamma}\left(2\alpha f_2(x)f(x) + 2\alpha f_1(x)^2 - \alpha f_2(x)\right)}{6\gamma^3} \\ &+ \frac{t^{3\gamma}\left(+f_2''(x) - 3f_2(x)f(x)^2 - 6f_1(x)^2 f(x) + 2f_2(x)f(x) + 2f_1(x)^2\right)}{6\gamma^3}, \end{aligned} \tag{32}$$

$$\begin{aligned} f_4(x) &= 6\alpha f_2(x)f_1(x) - \alpha f_3(x) + 2\alpha f(x)f_3(x) + f_3''(x) - 6f_1(x)^3 \\ &\quad -18f(x)f_2(x)f_1(x) + 6f_2(x)f_1(x) - 3f(x)^2 f_3(x) + 2f(x)f_3(x), \end{aligned} \tag{33}$$

$$\begin{aligned} u_4(x,t) &= f(x) + \frac{f_2(x)t^{2\gamma}}{2\gamma^2} + \frac{f_1(x)t^{\gamma}}{\gamma} + \frac{f_3(x)t^{3\gamma}}{6\gamma^3} + \frac{t^{4\gamma} f(x)f_3(x)}{12\gamma^4} \\ &+ \frac{t^{4\gamma}\left(f_3''(x) - 6f_1(x)^3 - 18f(x)f_2(x)f_1(x) + 6f_2(x)f_1(x) - 3f(x)^2 f_3(x)\right)}{24\gamma^4} \\ &+ \frac{t^{4\gamma}\left(6\alpha f_2(x)f_1(x) - \alpha f_3(x) + 2\alpha f(x)f_3(x)\right)}{24\gamma^4}. \end{aligned} \tag{34}$$

In Table 1, the fourth-order RPSM solutions of time fractional F-N equation are compared with the analytical solution

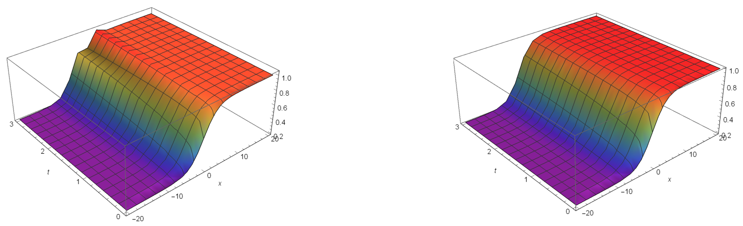
$$u(x,t) = \frac{1}{2}\left(\alpha + (\alpha - 1)\tanh\left(\frac{(\alpha - 1)x}{2\sqrt{2}}\right) + 1\right). \tag{35}$$

As it can clearly seen from the table and 3D graphical illustrations, both of the solutions are in good agreement. The absolute errors are in admissible norms.

Also, Tablo 1 states that, as γ increases the absolute error between numerical and analytical solution decreases. This means while $\gamma \longrightarrow 1$ the solutions becomes more accurate. In the other words, the numerical results and the analytical results will be compatible if we use integer order derivation for this problem. The graphical representations confirm that the absolute errors arising in the approximate solutions are reasonable.

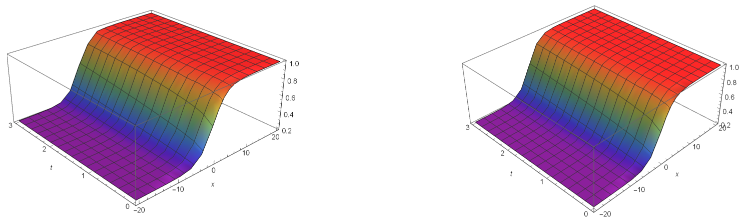
Table 1. RPSM approximate results and comparison with the exact solutions by absolute errors for $\alpha = 0.2$ and $t = 0.1$.

	$\gamma = 0.25$			$\gamma = 0.50$			$\gamma = 0.75$		
x	*RPSM*	*Exact*	*Abs. Error*	*RPSM*	*Exact*	*Abs. Error*	*RPSM*	*Exact*	*Abs. Error*
0.0	0.79496	0.79715	2.18743E-3	0.66025	0.66025	4.25769E-6	0.62274	0.62273	3.17819E-8
0.1	0.80351	0.80559	2.08224E-3	0.67126	0.67126	4.17753E-6	0.63399	0.63399	3.14221E-8
0.2	0.81185	0.81380	1.95180E-3	0.68215	0.68216	4.04180E-6	0.64520	0.64520	3.06396E-8
0.3	0.81996	0.82176	1.79890E-3	0.69293	0.69293	3.85363E-6	0.65633	0.65633	2.94526E-8
0.4	0.82784	0.82947	1.62673E-3	0.70356	0.70356	3.61726E-6	0.66737	0.66737	2.78882E-8
0.5	0.83550	0.83694	1.43882E-3	0.71403	0.71404	3.33795E-6	0.67831	0.67831	2.59815E-8
0.6	0.84292	0.84416	1.23889E-3	0.72434	0.72434	3.02181E-6	0.68912	0.68912	2.37750E-8
0.7	0.85010	0.85114	1.03079E-3	0.73447	0.73447	2.67554E-6	0.69981	0.69981	2.13162E-8
0.8	0.85705	0.85787	8.18338E-4	0.74441	0.74441	2.30629E-6	0.71034	0.71034	1.86570E-8
0.9	0.86375	0.86436	6.05275E-4	0.75415	0.75415	1.92138E-6	0.72071	0.72071	1.58513E-8
1.0	0.87022	0.87061	3.95126E-4	0.76367	0.76367	1.52814E-6	0.73090	0.73090	1.29540E-8



(a) RPSM solution (b) Exact solution

Figure 1. RPSM solution and exact solution for $\alpha = 0.2$ and $\gamma = 0.25$.

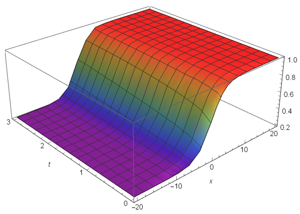


(a) RPSM solution (b) Exact solution

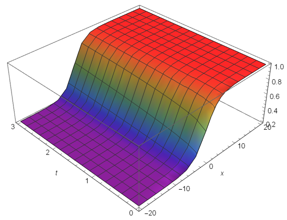
Figure 2. RPSM solution and exact solution for $\alpha = 0.2$ and $\gamma = 0.50$.

§4 Conclusion

The paper indicated the new approximate and analytical solutions of a popular reaction-diffusion model called Fitzhugh-Nagumo equation. All the obtained results show that acquired solutions are compatible. To show compatibleness of the acquired results, 3D graphical illustrations and tables are given for the different values of γ. It is clearly understood that both sub equation and residual power series method are efficient, accurate and reliable tools for obtaining the exact and numerical solutions of nonlinear models arising in mathematical physics.



(a) RPSM solution



(b) Exact solution

Figure 3. RPSM solution and exact solution for $\alpha = 0.2$ and $\gamma = 0.75$.

We hope that our work will be very useful in better understanding the solution structures of the fractional models arising in as a model of reaction-diffusion systems, transmission of nerve impulses, circuit theory, biology and the area of population genetics. We believe our manuscript is very timely and will interest the broad range of scientists who study on these areas. At the same time, this study will help scientists working in related fields to develop a new view point.

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