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Optimal control of a quality supervision profit model for the electronic intermediary

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Abstract. Online customers execute transactions without inspecting products could expect to encounter risks of receiving products with unsatisfactory qualities, especially in food transactions. Thus quality supervision plays a key role in the establishment of trust as well as in the management of risk between online customers and sellers. Most papers in this field are in qualitative nature. In this paper, a quality supervision profit (QSP) model is formulated as a discrete-time optimal control problem. It is a quantitative approach, and it broadens the scope of current research in the area. The quality effort level of online sellers (QEL_s) and the quality supervision level of the electronic intermediary (QSL_m) are considered together with their corresponding profit in the proposed model. The aim is to optimize an overall profit. A case study arising from Suichang's food traceability system (FTS) of farm produce online transaction is carried out in details. The results reveal that QEL_s, QSL_m and the profit distribution coefficient have a strong influence upon the profits of both sides. Finally, some concluding remarks, including potential further research topics, are given.

§1 Introduction

The Internet provides a convenient way to set up online stores for conducting trading anywhere in the World. Suppliers can sell products and provide service on the Internet. E-commerce can be classified into four main categories: Business-to-Business (B2B), Business-to-Consumer (B2C), Consumer-to-Consumer (C2C), and Consumer-to-Business (C2B), depending on different providers and customers in online transactions [23].

In E-commerce, a customer can make a buying decisions of a product without physically checking the quality of the product. Thus, customers would tend to select only goods with high quality so as to reduce transaction risk. As the product qualities vary widely, the electronic intermediary, as the third party, is necessary to act for online sellers and customers. The electronic intermediary has incentive to prevent lower quality products from being offered so

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as to protect customers. This trust building process can lead to online transactions being concentrated on high quality products [9].

After a series of food safety incidents happened in the European Union, consumers lost their trust in public and private food companies. In order to recover consumers confidence, European Union has dramatically revised food laws and enacting regulation 178/2002 (European Parliament, 2002). From January 2005, traceability has become a mandatory requirement for food and feed supplies. Henson and Reardon [8] argued that this traceability system should become mandatory requirements for online sellers. Hatanaka et al. [6] found that electronic intermediary provides traceability for online sellers and hence the safety and quality of their products can be assured.

Traceability, as a guarantee of food quality and safety, can track food through all stages of production and distribution. That means movements can be traced one step backwards and forwards at any point in the supply chain. The electronic intermediary should establish polices and processes to ensure the qualities of products and services so as to reduce risks and help establish trust between online providers and customers. Previous research has pointed out that the electronic intermediary's quality supervision has played an important role in building customers' trust on sellers in E-commerce.

However, it became apparent that these qualitative study of evaluating quality supervision is inadequate to capture the essence of virtual operations in E-commerce. In this paper, a quality supervision profit (QSP for simplification) model of the electronic intermediary is constructed as a discrete-time optimal control problem aiming to provide a quantitative analysis on how the quality supervision level of the electronic intermediary (QSL_m for simplification), the quality effort level of online sellers (QEL_s for simplification) and the profit distribution coefficient could affect the profits of both the electronic intermediary and online sellers.

Optimal control has many successful real-world applications, such as train control problems [22], spacecraft pursuit-evasion problem [15], microbial batch culture process [4], and machine maintenance problem [17]. In particular, many multistage control and scheduling problems can be formulated naturally as appropriate discrete-time optimal control problems. There are many computational methods available in the literature for solving discrete-time optimal control problems, such as those in [2, 3, 7, 14, 19, 24].

In this paper, we give the gradient formulae derived in [18] to solve the discrete-time optimal control problem with canonical constraints. With these gradient formulae and how they are being calculated, this optimal control problem can be solved as a nonlinear optimization problem. Many existing optimization techniques, such as sequential quadratic programming method [18] and exact penalty method [10], can be used. In particular, the optimal control software DMISER (see [21]), is applicable.

This approach would provide us with insights in the evaluation of the quality supervision and the profit of the electronic intermediary in E-commerce. In this way, it becomes possible to determine whether or not it is valid to claim that customers in E-commerce can fully trust the online sellers, provided that the electronic intermediary has trust guarantee for customers. Thus, in our model, we plan to examine not only the QEL_s but also the QSL_m.

An analysis on Suichang's food traceability system (FTS for simplification) of farm produce online transaction in Zhejiang Province of China is carried out so as to verify the feasibility and efficiency of the QSP model. Applying the QSP model to Suichang's FTS, we obtain not only the QEL_s of the local farmers, but also the QSL_m of Suichang's FTS.

The rest of the paper is organized as follows. Following this introduction, a QSP model between the electronic intermediary and online sellers is formulated as a discrete-time optimal control problem in Sections 2. Then, a computational method is proposed in Section 3. In Section 4, a case study of Suichang's FTS is carried out in details. Some concluding remarks, including some future research topics for quality supervision, are made in Sections 5.

§2 The quality supervision profit (QSP) model

2.1 The electronic intermediary and quality supervision

To date, the online market is growing at an unprecedented pace and E-commerce trading volume is expanding rapidly. One obvious reason for this new trend is that online customers can compare multiple sellers, thus reducing procurement costs, and hence electronics market is more favourable than the traditional stores. Product quality plays a key role in winning customers and hence leading to enhance the competitiveness of the electronic market. When a customer makes a buying decision, an important selection criterion is the product quality of electronic market. Unfortunately, there are many fraud incidents and consumer victimization in electronic markets. According to China's National Consumers Association report [5], the most reported complaint is quality fraud which is 54.4% of the 300,346 total referred cases in 2010, 52.2% of the 256,713 cases in 2012, and 43.2% of the 327,564 cases in 2014. If the intermediary does not take responsibility for controlling online transactions between customers and sellers, E-commerce business model will be especially vulnerable to threats originating from Internet.

Therefore, the importance of quality supervision is evident in electronic intermediary. We can take advantage of network to improve product quality and reduce transaction costs. The electronic intermediary, as a social institution for E-commerce, can facilitate and govern the exchange according to institutional rules. Despite apparent risk, customers would purchase goods from unknown providers in electronic market, because they have trust on relatively well-known electronic intermediary for being able to supervise quality. The electronic intermediary minimizes transaction risks by establishing policies and rules to build trust between online sellers and customers.

In particular, electronic markets can develop their supervision through institutional mechanisms [11]. Firstly, customers will be assured that they are protected by a warranty guaranteed by a third party (e.g. VeriSign). Secondly, the electronic intermediary can focus on building a trusted brand through aggressive marketing programs. For example, customers find the well-known eBay and Amazon more trustworthy than unfamiliar markets. Finally, the more secure the electronic market appears, the more trustworthy the electronic intermediary is to customers.

On one hand, the QEL_s is associated with the belief that the population of online sellers at an electronic market is honest, dependable and reliable. It is perceived that electronic market institutional mechanism is an effective mean to impact the QEL_s [12]. On the other hand, the QSL_m is also influenced by the QEL_s [9].

Therefore, our study takes into consideration both the QEL_s and QSL_m, as they are central to customers' purchase behaviour in electronic markets. The more the customers have trust in the electronic intermediary, the more the customers will look to online sellers for their purchase, and hence bringing more overall profit to the electronic intermediary.

In the next subsection, we present a QSP model that shows the effects of quality supervision on online sellers behaviours in electronic markets from an optimal control perspective. The QSP model involves two parties, acting to serve customers for transactions: the electronic intermediary and online sellers. This model will enable us to have better insights of the correlation between QSL_m and QEL_s.

2.2 Model construction

An online seller signs a contract with an electronic intermediary. Then, the online seller can sell products on the trading platform of the electronic intermediary or certified by the electronic intermediary. In order to provide qualified products to customers, the online seller should improve product qualities. For the electronic intermediary, it should supervise and evaluate the product qualities, and making decision on whether to allow or refuse the online seller to sell products according to the evaluation results.

However, irrespective of whatever the quality effort level of online sellers (QEL_s) or the quality supervision level of the electronic intermediary (QSL_m), there is no 100% guarantee that products which online sellers eventually sell to customers would be qualified products. Let $u_s(k)$ be the qualified rate of the product quality provided by the online sellers, which represents the QEL_s, and let $u_m(k)$ be the unqualified rate of the product quality detected by the electronic intermediary when the unqualified products are provided by online sellers, which represents the QSL_m, where k is the time step, usually it can be taken as one day or one week, $k = 0, 1, \ldots, M-1$.

In our QSP model, the actions of the electronic intermediary are divided into three cases: case (1), when the products provided by online sellers are qualified, and the detection results made by the electronic intermediary show that these products are qualified, then online sellers are allowed to sell these products; case (2), when the products provided by online sellers are unqualified, and the detection results made by the electronic intermediary show that these products are unqualified, then online sellers are not allowed to sell these products and would be punished by a fine; case (3), when the products provided by online sellers are unqualified, but the detection results made by the electronic intermediary show that these products are qualified, then online sellers are allowed to sell these products. However, the unqualified products would be detected by customers and online sellers would be punished by a more serious fine.

The following four hypotheses are assumed to be satisfied, based on prior research findings presented above and economics theories.

H1. The market profit coefficients of the electronic intermediary corresponding to the three cases mentioned above are p_1, p_2, p_3, respectively. Furthermore, $p_1 > p_2 > p_3$.

H2. f_{sm_2} is the coefficient of fine to online sellers imposed by the electronic intermediary in case (2), f_{sm_3} is the coefficient of fine to online sellers imposed by the electronic intermediary in case (3). Furthermore, $f_{sm_3} > f_{sm_2}$.

H3. $C_s(u_s(k))$ is the quality effort cost function of online sellers, and $C_s^{'}(\cdot) > 0, C_s^{''}(\cdot) > 0$; $C_m(u_m(k))$ is the quality supervision cost function of the electronic intermediary, and $C_m^{'}(\cdot) > 0, C_m^{''}(\cdot) > 0$.

H4. An important premise for the electronic intermediary to obtain profit in E-commerce is online sellers' patronage. Thus, we consider the market profit of online sellers in the overall profit function of the electronic intermediary with a coefficient b, and $0 < b < 1$. The value of b describes the degree of consideration which the electronic intermediary gives to the profit of online sellers. The larger the b, the more degree of consideration which the electronic intermediary gives to the profit of online sellers. In this case, the more the electronic intermediary has to lose.

We now define three profit functions as follows.

The market profit function, $P_s(k)$, of online sellers is

$$P_s(k+1) = P_s(k) - (1-u_s(k)) \cdot u_m(k) \cdot f_{sm_2} - (1-u_s(k)) \cdot (1-u_m(k)) \cdot f_{sm_3} - C_s(u_s(k)), \quad (1)$$

where the second term means the fine in case (2), i.e. when the products provided by online sellers are unqualified(i.e. $1-u_s(k)$ for the online sellers), and the detection results made by the electronic intermediary show that these products are unqualified(i.e. $u_m(k)$ for the electronic intermediary), then online sellers are not allowed to sell these products and would be punished by a fine(i.e. f_{sm_2} for the coefficient of fine to online sellers); the third term means the fine in case (3), i.e. when the products provided by online sellers are unqualified(i.e. $1 - u_s(k)$ for the online sellers), but the detection results made by the electronic intermediary show that these products are qualified(i.e. $1 - u_m(k)$ for the electronic intermediary), then online sellers are allowed to sell these products, however, the unqualified products would be detected by customers and online sellers would be punished by a more serious fine(i.e. f_{sm_3} for the coefficient of fine to online sellers).

The overall profit function, $P_o(k)$, of the electronic intermediary is

$$P_o(k+1) = P_o(k) + P_m(k) + b \cdot P_s(k), \quad (2)$$

and the market profit function, $P_m(k)$, of the electronic intermediary is

$$\begin{aligned} P_m(k+1) = P_m(k) &+ u_s(k) \cdot p_1 + (1-u_s(k)) \cdot u_m(k) \cdot (p_2 + f_{sm_2}) \\ &+ (1-u_s(k)) \cdot (1-u_m(k)) \cdot (p_3 + f_{sm_3}) - C_m(u_m(k)), \end{aligned} \quad (3)$$

where the second term means the market profit in case (1), the third term means the market profit in case (2) and includes the corresponding fine to online sellers(i.e. $(1 - u_s(k)) \cdot u_m(k) \cdot f_{sm_2}$), and the fourth term means the market profit in case (3) and includes the corresponding fine to online sellers(i.e. $(1 - u_s(k)) \cdot (1 - u_m(k)) \cdot f_{sm_3}$).

The initial states of the profit functions are

$$P_s(0) = P_s^0, P_o(0) = P_o^0, P_m(0) = P_m^0. \quad (4)$$

Under asymmetric information, the QEL_s index $u_s(k)$ is a private information, while its interval distribution is known. Here, we suppose $a_1 \le u_s(k) \le a_2$. In other words, although the electronic intermediary could not observe the QEL_s, they could deduce the choice of online sellers according to some signals in $u_s(k)$. It is disadvantageous for the electronic intermediary to select $u_m(k)$ to be too large or too small. As in E-commerce, when the QSL_m index $u_m(k)$ is larger, the quality supervision cost function $C_m(u_m(k))$ increases in a rapid rate; when the QSL_m index $u_m(k)$ is smaller, customers' complaints will increase drastically, and this will reduce customers' confidence in the electronic intermediary, thus, the electronic intermediary should not choose $u_m(k)$ to be too small. Here we suppose $c_1 \le u_m(k) \le c_2$.

Our goal is to maximize the overall profit function, $P_o(M)$, of the electronic intermediary by suitable choice of the QEL_s index $u_s(k)$ and the QSL_m index $u_m(k)$. Thus, the QSP model can be formulated in the form of a discrete-time optimal control problem given below.

QSP Model Given system (1) − (4), find u_s and u_m such that the overall profit function of the electronic intermediary

$$P_o(M) \tag{5}$$

is maximized over $\mathcal{F}$, where $\mathcal{F}$ denotes the set of all u_s and u_m satisfying the following constraints:

$$a_1 \le u_s(k) \le a_2, c_1 \le u_m(k) \le c_2, \tag{6}$$

$$g_j(u) = \Phi_j(x(M|u)) + \sum_{k=0}^{M-1} \mathcal{L}_j(k, x(k|u), u(k)) \le 0, j = 1, \dots, N, \tag{7}$$

where $g_j, j = 1, \dots, N$, are constraint functions arising from various practical requirements, and the functions $\Phi_j, \mathcal{L}_j, j = 1, \dots, N$, are assumed to be continuously differentiable with respect to each of their arguments.

In order to reduce customers' complaints while increasing the overall profit $P_o(M)$, the electronic intermediary should improve the quality supervision level $u_m(k)$. This will urge online sellers to improve their quality effort level $u_s(k)$. Certainly, they also need to take into account their corresponding cost function.

§3 Optimal control computation

QSP model is a discrete-time optimal control problem which is a special case of a general discrete-time optimal control problem. Thus, we describe the general discrete-time optimal control problem in this section.

We consider the following system of difference equations:

$$x(k+1) = f(k, x(k), u(k)), k = 0, 1, \dots, M-1, \tag{8}$$

$$x(0) = x^0, \tag{9}$$

where $x \in \mathbb{R}^n, u \in \mathbb{R}^r$ are state and control vectors, respectively; $f(k, \cdot, \cdot) : \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}^n$ is a given continuously differentiable function; x^0 is the initial state.

Define

$$U = \{\nu = [v_1, \dots, v_r]^T \in \mathbb{R}^r : \alpha_i \le v_i \le \beta_i, i = 1, \dots, r\}, \tag{10}$$

where $\alpha_i, i = 1, \dots, r$, and $\beta_i, i = 1, \dots, r$, are given real numbers. Then U is a convex and compact subset in $\mathbb{R}^r$.

Suppose that u is a control sequence $\{u(k), k = 0, 1, \dots, M-1\}$ in U, then

$$u = (u(0), u(1), \dots, u(M-1)) = [(u(0))^T, (u(1))^T, \dots, (u(M-1))^T]^T \in \mathbb{R}^{Mr}. \tag{11}$$

We call u an **admissible control** and let $\mathcal{U}$ be the set of all such admissible controls. Then, for each control $u \in \mathcal{U}$, we denote that $x(k \mid u), k = 0, 1, \dots, M$ is the **solution** of the system (8) − (9).

A general discrete-time optimal control problem in canonical formulation may now be stated as follows (see [18]):

Problem (P). Given system (8) − (9), find a $u \in \mathbb{R}^{Mr}$ such that the cost function

$$g_0(u) = \Phi_0(x(M|u)) + \sum_{k=0}^{M-1} \mathcal{L}_0(k, x(k|u), u(k)) \tag{12}$$

is minimized subject to

$$g_j(u) = 0, j = 1, \dots, N_e, \tag{13}$$

$$g_j(u) \leq 0, j = N_e + 1, \dots, N, \tag{14}$$

$$\alpha_i \leq u_i(k) \leq \beta_i, i = 1, \dots, r, k = 0, 1, \dots, M-1, \tag{15}$$

where

$$g_j(u) = \Phi_j(x(M|u)) + \sum_{k=0}^{M-1} \mathcal{L}_j(k, x(k|u), u(k)), j = 1, \dots, N.$$

The functions $\Phi_j, \mathcal{L}_j, j = 0, 1, \dots, N$, are assumed to be continuously differentiable with respect to each of their arguments.

The equality constraints and inequality constraints appear in a similar form as the cost function, and these constraints are referred to as canonical constraints. In this formulation, the gradient formulae of constraint functions and cost function can be computed in a unified way. Obviously, QSP model is a special case of Problem (P). Thus, we propose the following gradient formulas based on the results in Chapter 11 of [18].

Theorem 3.1 Consider Problem (P). For each $j = 0, 1, \dots, N$, the gradient of $g_j(u)$ is

$$\frac{\partial g_j(u)}{\partial u} = \sum_{k=0}^{M-1} \frac{\partial H_j(k, x(k), u(k), \lambda_j(k+1))}{\partial u(k)}, \tag{16}$$

where H_j is the Hamiltonian sequence defined by

$$H_j(k, x(k), u(k), \lambda_j(k+1)) = \mathcal{L}_j(k, x(k), u(k)) + \lambda_j^T(k+1) \cdot f(k, x(k), u(k)), \tag{17}$$

where $\lambda_j(k) \in \mathbb{R}^n, k = M, M-1, \dots, 1$, are the costate sequence corresponding to the jth constraint function($j = 0$ denotes the cost function),

$$\begin{aligned} \lambda_j^T(k) &= \frac{\partial H_j(k, x(k), u(k), \lambda_j(k+1))}{\partial x(k)} \\ &= \frac{\partial \mathcal{L}_j(k, x(k), u(k))}{\partial x(k)} + \lambda_j^T(k+1) \cdot \frac{\partial f(k, x(k), u(k))}{\partial x(k)}, k = M-1, M-2, \dots, 1, \end{aligned} \tag{18}$$

and

$$\lambda_j^T(M) = \frac{\partial \Phi_j(x(M))}{\partial x(M)}. \tag{19}$$

We note that Problem (P) is a nonlinear constrained optimization problem in the control parameters. With these gradient formulae, Problem (P) can be solved effectively by any existing optimization technique, such as sequential quadratic programming method [18]. In particular, DMISER [21], a software packages for solving discrete-time optimal control problem, is applicable.

To solve QSP model using a gradient-based optimization method or software packages such as DMISER, we derive the required gradient formulae according to Theorem 3.1.

We consider the given system (1) − (4), where $x(k) = (P_s(k), P_o(k), P_m(k))^T$ is the state vector at time k, $u(k) = (u_s(k), u_m(k))^T$ is the control vector at time k, $x(0) = (P_s(0), P_o(0), P_m(0))^T$ is the initial state, $M > 0$ is a given terminal time, and $f = (f_1, f_2, f_3)^T$ is a continuously differentiable function given by (1) − (3). $g_0(u) = \Phi_0(x(M|u)) + \sum_{k=0}^{M-1} \mathcal{L}_0(k, x(k|u), u(k)) = P_o(M)$ defined by (5) is the cost function, where $\Phi_0(x(M|u)) = P_o(M)$ and $\mathcal{L}_0(k, x(k|u), u(k)) = 0$. $g_j(u), j = 1, \dots, N$, are constraint functions arising from various practical requirements satisfying (6) − (7).

Define the Hamiltonian function as in (17) − (19). To evaluate (16), it is necessary to first solve the state system (1) − (4) forward in time, and then solve the costate system (18) − (19)

backward in time. The more specific gradient formulae depend on the concrete representation of the quality effort cost function $C_s(u_s(k))$, the quality supervision cost function $C_m(u_m(k))$ in f, and constraint functions $g_j(u), j = 1, \ldots, N$.

§4 Case study-Suichang's food traceability system of farm produce online transation

The frequent food safety scandals in recent decades, such as the European BSE crisis of 1996 and the Belgian dioxin crisis of 1999, have aroused growing public concern in Europe about food safety [16]. The situation has been even more serious in China, as it witnessed a string of food safety scares – melamine in milk, Sudan 1 in sauces, gutter oil, vinegar dosed with industrial acid, etc. – which have spread panic among the Chinese people, followed by enormous damage to the public trust in food supply.

As a result, more and more customers demand that every stage of the food supply can be documented and tracked to ensure its safety. In January 2005, the European Union required that all food companies must trace their products from the initial suppliers to the final customers in all stages [1]. The Chinese government also issued in 2006 an official document to reinforce the supervision to food safety.

The food traceability system (FTS) has thus come into people's sight. According to ISO9000/BS5750, traceability is defined as a quality procedure ability to retrace steps and verify events which have taken place. Today FTS has been widely taken as a necessary guarantee to the quality control and safety enforcement of food industries. In the following, this paper will offer a case study of Suichang's FTS of farm produce online transaction based on the QSP model.

Suichang is a county located in a mountainous area in Zhejiang Proivince of China, where the local farmers mainly produce fresh fruits and vegetables such as edible lilies, green beans, bamboo shoots, strawberries, oranges, apples and so on. Their farm produce can be sold to other regions via an online sales-and-service platform offered by Zhejiang Suichang Electronic Commerce Co., LTD (hereafter referred to as ZSEC Co.). Since their products are usually difficult to keep fresh and tend to rot fast, the top priority of the online transaction is to guarantee the quality and safety of the farm produce. In this case, ZSEC Co. established an FTS in 2013.

With the help of the optimal control software DMISER, we employed a numerical sample study for Suichang's FTS in which the product quality is endogenously determined by the online sellers – the local farmers. This problem concerns the overall profit of Suichang's FTS which established by ZSEC Co.. We now define three profit functions as follows.

The market profit (hundred RMB), $P_s(k)$, of local farmers is

$$P_s(k+1) = P_s(k) - (1-u_s(k)) \cdot u_m(k) \cdot f_{sm_2} - (1-u_s(k)) \cdot (1-u_m(k)) \cdot f_{sm_3} - C_s(u_s(k)), \quad (20)$$

the overall profit (hundred RMB), $P_o(k)$, of Suichang's FTS is

$$P_o(k+1) = P_o(k) + P_m(k) + b \cdot P_s(k), \quad (21)$$

and the market profit (hundred RMB), $P_m(k)$, of Suichang's FTS is

$$\begin{aligned} P_m(k+1) = P_m(k) &+ u_s(k) \cdot p_1 + (1-u_s(k)) \cdot u_m(k) \cdot (p_2 + f_{sm_2}) \\ &+ (1-u_s(k)) \cdot (1-u_m(k)) \cdot (p_3 + f_{sm_3}) - C_m(u_m(k)), \end{aligned} \quad (22)$$

where the time step is taken as one day; $u_s(k)$ is the qualified rate of the product quality provided by the local farmers, which represents the QEL_s of the local farmers; $u_m(k)$ is the unqualified rate of the product quality detected by Suichang's FTS when the unqualified products are provided by the local farmers, which represents the QSL_m of Suichang's FTS; $p_1 = 3, p_2 = 2, p_3 = 1$ are the coefficients of the market profit of Suichang's FTS corresponding to the three cases mentioned in QSP model, which satisfies $p_1 > p_2 > p_3$; $f_{sm_2} = 1$ is the coefficient of fine to the local farmers imposed by Suichang's FTS in case (2), and $f_{sm_3} = 2$ is the coefficient of fine to the local farmers imposed by Suichang's FTS in case (3), which satisfies $f_{sm_3} > f_{sm_2}$; $0 < b < 1$ is the profit distribution coefficient due to the degree of consideration which Suichang's FTS gives to the profit of the local farmers; $C_s(u_s(k))$ is the quality effort cost function of the local farmers and $C_m(u_m(k))$ is the quality supervision cost function of Suichang's FTS, which are defined by the formulas

$$C_s(u_s(k)) = \frac{1}{2} \cdot k_1 \cdot u_s^2(k), C_m(u_m(k)) = \frac{1}{2} \cdot k_2 \cdot u_m^2(k),$$

where $k_1 = 0.8, k_2 = 0.9$ are the appropriate weighting factors.

The initial states of the model are

$$P_s(0) = 100, P_o(0) = 0, P_m(0) = 0, \tag{23}$$

where $P_s(0) = 100$ means that the local farmers will earn 10,000RMB when they sell the produce by themselves at the cost of more time and energy.

In this QSP model, we take the time-step period for one month, i.e. 30 days. To protect the profit of the local farmers, at the terminal time $M = 30$, the final market profit of the local farmers is constrained to be not less than 50% of the initial market profit, i.e., $P_s(M) \geq 50$, which can be written as follows

$$g_1 = P_s(M) - 50 \geq 0. \tag{24}$$

The bounds on controls are

$$0.1 \leq u_s(k) \leq 0.9, 0.1 \leq u_m(k) \leq 0.9 \tag{25}$$

for $k = 0, 1, \ldots, M - 1$.

Our optimal control problem is defined as follows: choose the suitable choice of the QEL_s index of the local farmers $u_s(k)$ and the QSL_m index of S-traceability system $u_m(k)$ to maximized $P_o(M)$ subject to the difference system $(20) - (23)$ and the constraints $(24) - (25)$.

We first solved this problem for $b = 0.05$ (i.e. Suichang's FTS gives little consideration to the profit of the local farmers). Using the optimal control software DMISER, we obtain an optimal overall profit of $P_o(M) = 69.93985$. The optimal controls, and the corresponding state trajectories, are shown in Fig. 1.

Remark 4.1 Fig. 1 shows that in order to reduce customers' complaints and enhance the overall profit $P_o(M)$, Suichang's FTS should raise the quality supervision level $u_m(k)$, so as to improve the local farmers' quality effort level $u_s(k)$.

To illustrate the effect of b on the optimal control policy, we solve the model for $b = 0.3$, and $b = 0.7$. Our numerical results are summarized in Table 1. The optimal controls and corresponding state trajectories with $b = 0.7$ are shown in Fig. 2.

Remark 4.2 (1) Fig. 2 clearly shows how increasing b "influences" the optimal control: the larger the b, the more degree of consideration which Suichang's FTS gives to the profit of the local farmers. In this case, the more Suichang's FTS has to lose. If b is large enough (e.g. b=0.7 in Table 1), the overall market profit is more than the initial market profit ($P_o(M) =$

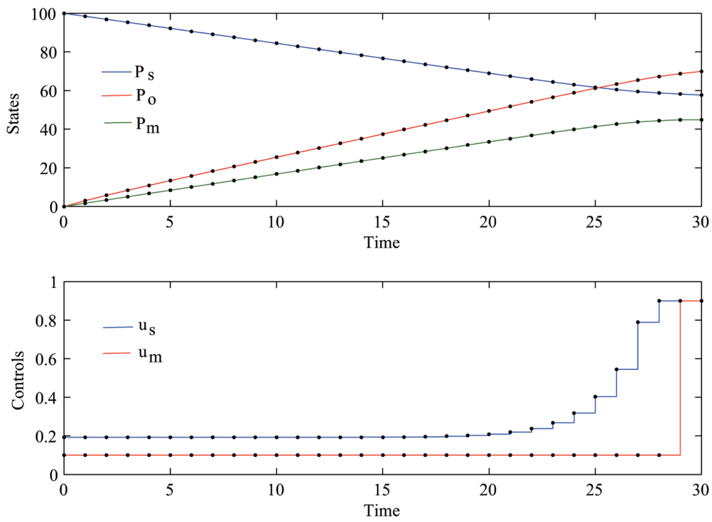

Figure 1. Numerical results for case analysis with $b = 0.05$.

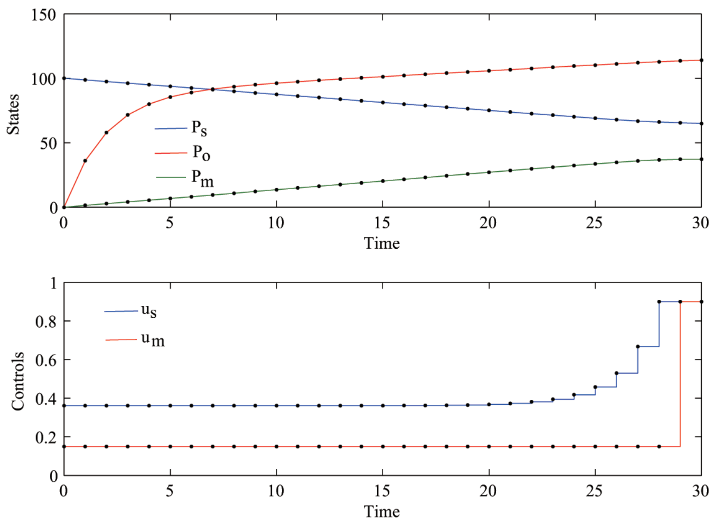

Figure 2. Numerical results for case analysis with $b = 0.7$.

$113.95945 > P_s(0) = 100$). That is to say, the quality supervision of Suichang's FTS has brought an increase in the overall market profit. The market profit of the local farmers and Suichang's FTS can both be adjustable in a certain range, which can be not only illustrated but also operated by the adjustment of coefficient b.

(2) To encourage the local farmers to provide products with high quality, ZSEC Co. must increase their profit as an incentive, as well as raise the quality supervision level; but this will

inevitably lead to an increase in the cost and a cutdown of its own profit. As a member of the market, however, the company will by nature pursue the maximization of its own profit above anything else. That is to say, coefficient b is not likely to be too large.

Table 1. Numerical results for case analysis.

b	$P_o(M)$	$P_s(M)$	$P_m(M)$
0.05	69.93985	57.65045	44.86671
0.3	99.42937	59.66500	40.88503
0.7	113.95945	64.95107	37.15143

§5 Conclusions

Once the online transaction standards are established, E-commerce will have more advantages than traditional markets, for it allows 24*7 transactions with a lower cost in the communication between the suppliers and the customers. Since the online customers can not check the products personally before the transaction, however, they might run the risk of receiving goods with poor quality. Thus emerged electronic intermediaries who have transformed the marketing and distribution channels. With the development of information technology and the standardization of business process, they efficiently facilitate the completion of online transactions between organizations. As food crises occurred frequently in the real world, the electronic intermediary for quality supervision has been an academic and practical hot spot recently. This paper is mainly devoted to using a quantitative method to evaluate quality supervision of the electronic intermediary.

The findings of the study can be summarized as follows. Firstly, we review the quality activities of sellers and intermediaries in the context of online transaction. We found that most of the past researches only made qualitative observations on how the electronic intermediary affects the seller's behavior and the customer's behavior in E-commerce. Our research attempts to explore the online transaction from a different perspective by dividing the E-commerce suppliers into two entities, namely the electronic intermediary and online sellers, and thus illustrate how these two elements affect the overall profit. Secondly, this research helps unveil the nature of interrelationship of the quality supervision level of the electronic intermediary (QSL_m) and the quality effort level of online sellers (QEL_s) in online markets. It will set forth related studies designed to explore practical strategies to ensure product quality more effectively. Thirdly, this research, for the first time, employs the discrete-time optimal control method to study the influence of the QSL_m as well as the QSL_s. It helps practitioners develop strategies to generate desired changes in customer behavior.

Finally, further studies are needed and we suggest the following topics.

(1) The mathematical model in this paper reveals that the soundness of profit results is significantly affected by quality supervision indicators, in our model represented by the QEL_s index $u_s(k)$, the QSL_m index $u_m(k)$, and the profit distribution coefficient b. The selection of more proper indicators is very important and calls for further study.

(2) Further researches and experiments are needed to verify whether the QSP model can be introduced into the quality supervision system of the entire E-service.

E-service quality is increasingly recognized as a key aspect of E-commerce. Parasuraman et al. [13] suggested that E-service quality is defined as how a Web site facilitates efficient and effective way to let customers select, purchase, and deliver goods to their homes. Poor E-service has difficulty in attracting, satisfying, and retaining customers. Without a quality supervision approach that guarantees quality from its systems and suppliers, a business will not be able to deliver the appropriate level of service quality to satisfy its customers. The existing research has mainly focused on a qualitative method to improve the E-service quality. So far little effort has been made to explore this issue from a quantitative perspective. This paper, by offering a new theoretical analysis and mathematical model, suggests the possibility to investigate the E-service quality from a different angle.

(3) We can adapt this QSP model to incorporate the new challenges by emphasizing the need and importance of online credibility evaluation in E-commerce settings. The credibility problem is also one of the main bottlenecks for E-commerce. To a certain extent, the credibility evaluation systems provided by E-commerce websites guarantee the safety of transactions on Internet, and promote customers' online shopping wishes. Thus, we can consider the credit score in the optimal function and the updated model includes two interrelated components of score: quality supervision and credibility evaluation.

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