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A different approach for conformable fractional biochemical reaction–diffusion models

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Abstract. This paper attempts to shed light on three biochemical reaction-diffusion models: conformable fractional Brusselator, conformable fractional Schnakenberg, and conformable fractional Gray-Scott. This is done using conformable residual power series (hence-form, CRPS) technique which has indeed, proved to be a useful tool for generating the solution. Interestingly, CRPS is an effective method of solving nonlinear fractional differential equations with greater accuracy and ease.

§1 Introduction

Brusselator model [1-5], Schnakenberg model [6-7], Gray-Scott model [8-10], were produced in biochemistry through various mathematical models, especially reaction-diffusion systems. In recent years, reaction-diffusion systems in biological and biochemical phenomena were formulated by fractional calculus [11-24]. The memory effect of the fractional operator gives the differential equations an increased expressive power [25-30]. Khalil et al. [31] introduce a new non-integer derivative called conformable fractional derivative which depends on the history of the previous time. The conformable fractional differential equation has successfully been fitted to various nonlinear fractional problems [32-34]. The exact and approximate solutions of conformable fractional derivatives can be given easily to understand physical phenomena arising in many scientific fields [35-37]. Conformable fractional derivative is a local fractional derivative which satisfies most of the properties of integer derivative and has clear physical interpretation. Residual power series (RPS) technique is a useful tool for generating the solution of fractional differential equations FDEs [38-39]. In this paper, we introduce new conformable fractional reaction-diffusion models arising in biochemical phenomena which can be expressed as follows:

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- The conformable fractional Brusselator model
$$\begin{cases} T_t^{\gamma} u = D_1 \nabla^2 u - (A+1)u + u^2 v + B, \\ T_t^{\gamma} v = D_2 \nabla^2 v + Au - u^2 v. \end{cases} \tag{1}$$
where u and v remain the concentrations of two reactants, and D_2 are positive diffusion coefficients, $\nabla^2 u$ and $\nabla^2 v$ are the diffusive terms, A and B are constants concentrations and T_t^{γ} is conformable fractional derivative.

- The conformable fractional Schnakenberg model
$$\begin{cases} T_t^{\gamma} u = D_1 \nabla^2 u - \delta(u - u^2 v - A), \\ T_t^{\gamma} v = D_2 \nabla^2 v - \delta(u^2 v - B). \end{cases} \tag{2}$$
where u and v remain the chemical species,
$$D_1$$
and D_2 are positive diffusion coefficients, $\nabla^2 u$ and $\nabla^2 v$ are the diffusive terms, δ is treated as dimensionless constant, A and B are positive parameters, and T_t^{γ} is conformable fractional derivative.

- The conformable fractional Gray-Scott model
$$\begin{cases} T_t^{\gamma} u = D_1 \nabla^2 u - A(1-u) - uv^2, \\ T_t^{\gamma} v = D_2 \nabla^2 v + (A+B)v + uv^2. \end{cases} \tag{3}$$
where u and v remain the concentration of activator and inhibitor, D_1 and D_2 are diffusion coefficients, $\nabla^2 u$ and $\nabla^2 v$ are the diffusive terms, A and B remain the dimensionless feed rate and dimensionless rate constant of the activator, and T_t^{γ} is conformable fractional derivative. The outline of this paper has the following sections: In section 2, a brief history of the fractional derivatives and conformable fractional derivative. In section 3, the key ideas of the conformable residual power series (CRPS) technique. In section 4, a new approximate results for the conformable fractional Brusselator model, conformable fractional Schnakenberg model, and conformable fractional Gray-Scott model are presented. Finally, a brief conclusion.

§2 Preliminaries

The most widely fractional derivatives used are the Riemann-Liouville and Caputo [40-41]. Recently, a new non integer operator conformable fractional operator to the nonlinear fractional problems are presented [32-34].

Definition 2.1

The conformable fractional operator T_t^{γ} of a function is denoted as:
$$T_t^{\gamma} u(x,y,t) = \lim_{\varepsilon \to 0} \frac{u(x,y,\varepsilon t^{1-\gamma}) - u(x,y,t)}{\varepsilon}, \quad 0 < \gamma \le 1, t > 0. \tag{4}$$

Theorem 2.1

Let $u(x,y,t)$,and $v(x,y,t)$ be γ-differentiable function at $(x,y,t) \in R \times (0,\infty)$,then

1. $T_t^\gamma(t^m) = mt^{m-\gamma}, m \in R$
2. $T_t^\gamma(au+bv) = aT_t^\gamma u + bT_t^\gamma v$
3. $T_t^\gamma(k) = 0$, k is a constant
4. $T_t^\gamma(u \cdot v) = uT_t^\gamma v + vT_t^\gamma u$
5. $T_t^\gamma(\frac{u}{v}) = \frac{vT_t^\gamma u - uT_t^\gamma v}{v^2}$
6. If u is differentiable with respect to t, then $T_t^\gamma u = t^{1-\gamma}\frac{\partial u}{\partial t}$.

Definition 2.2

The k-truncated series $u_k(x,y,t)$ of the CRPS method [35-36] take the following form:

$$u_k(x,y,t) = f(x,y) + \sum_{n=1}^{k} f_n(x,y)\frac{t^{n\gamma}}{\gamma^n n!}, \quad 0 < \gamma \leq 1, t > 0. \tag{5}$$

§3 Analysis of the CRPS technique

We outline the CRPS [38] technique to get approximate solutions of conformable fractional partial differential equations. Consider the fractional problem:

$$T_t^\gamma u(x,y,t) = \nabla^2 u(x,y,t) + N(u), \quad 0 < \gamma \leq 1, t > 0,$$

with initial condition

$$u(x,y,0) = f(x,y). \tag{6}$$

The k-truncated series take the following form:

$$u_k(x,y,t) = f(x,y) + \sum_{n=1}^{k} f_n(x,y)\frac{t^{n\gamma}}{\gamma^n n!}. \tag{7}$$

The approximate solutions $u_0(x,y,t)$ of CRPS method is

$$u_0(x,y,t) = f(x,y). \tag{8}$$

The k-residual function $Res_{uk}(x,y,t)$ is define as

$$Res_{uk}(x,y,t) = T_t^\gamma u_k(x,y,t) - \nabla^2 u_k(x,y,t - N(u_k(x,y,t). \tag{9}$$

To obtain the coefficients $f_n(x,y), n = 1,2,3,...,k$, solve the following equation

$$T_t^{(k-1)\gamma} Res_{uk}(x,y,0) \quad = \quad 0. \tag{10}$$

To determine$f_1(x,y)$, put (k =1) into Eq. (9) leading to:

$$Res_{u1}(x,y,t) = T_t^\gamma u_1(x,y,t) - \nabla^2 u_1(x,y,t - N(u_1(x,y,t), \tag{11}$$

where $u_1(x,y,t)$ is the 1st CRPS approximate solutions which take the form:

$$u_1(x,y,t) = f(x,y) + f_1(x,y)\frac{t^\gamma}{\gamma}. \tag{12}$$

Substitute Eq. (12) into Eq. (11), and using Eq. (10), we obtain the required coefficient $f_1(x,y)$.
To determine $f_2(x,y)$, put (k =2) into Eq. (9) leading to:

$$Res_u 2(x,y,t) = T_t^{\gamma} u_2(x,y,t) - \nabla^2 u_2(x,y,t) - N(u_2(x,y,t), \tag{13}$$

where $u_2(x,y,t)$ is the 2nd CRPS approximate solutions which take the form:

$$u_2(x,y,t) = f(x,y) + f_1(x,y)\frac{t^{\gamma}}{\gamma} + f_2(x,y)\frac{t^2\gamma}{2\gamma^2}.$$

Substitute Eq. (12) into Eq. (11), and using Eq. (10), we obtain the required coefficients$f_2(x,y)$. And so on.

§4 Applications

Here, we will use CRPS method to obtain new approximate solutions for three model under investigation (conformable fractional Brusselator model, conformable fractional Schnakenberg model, and conformable fractional Gray-Scott model)

Problem 4.1

Let us consider the new conformable fractional Brusselator model:

$$\begin{cases} T_t^{\gamma} u = D_1(u_{xx} + u_{yy}) - (A+1)u + u^2 v + B, \\ T_t^{\gamma} v = D_2(v_{xx} + v_{yy}) + Au - u^2 v, \qquad 0 < \gamma \leq 1, t > 0 \end{cases} \tag{14}$$

with the initial data [18]

$$\begin{cases} u(x,y,0) = e^{-x-y}, \\ v(x,y,0) = e^{x+y}. \end{cases} \tag{15}$$

The exact solution of the classical problem (4.1) when $\gamma \to 1$, take the form

$$\begin{cases} u(x,y,t) = e^{-x-y-0.5t}, \\ v(x,y,t) = e^{x+y+0.5t}. \end{cases} \tag{16}$$

The k-truncated series $u_k(x,y,t)$ and $v_k(x,y,t)$ take the following form:

$$\begin{cases} u_k(x,y,t) = f(x,y) + \sum_{n=1}^{k} f_n(x,y)\frac{t^{n\gamma}}{\gamma^n n!}, \\ v_k(x,y,t) = g(x,y) + \sum_{n=1}^{k} g_n(x,y)\frac{t^{n\gamma}}{\gamma^n n!}. \end{cases}$$

The approximate solutions $u_0(x,y,t)$ and $v_0(x,y,t)$ of CRPS method are

$$\begin{cases} u(x,y,0) \quad f(x,y), \\ v(x,y,0) \quad g(x,y). \end{cases} \tag{17}$$

The k-residual functions $Res_{uk}(x,y,t)$ and $Res_{vk}(x,y,t)$ are defined as

$$\begin{cases} Res_{uk}(x,y,t) = T_t^{\gamma} u_k - D_1((u_k)_{xx} + (u_k)_{yy}) + (A+1)u_k - u_k^2 v_k - B, \\ Res_{vk}(x,y,t) = T_t^{\gamma} v_k - D_2((v_k)_{xx} + (v_k)_{yy}) - Av_k + u_k^2 v_k. \end{cases} \tag{18}$$

To obtain the coefficients $f_n(x,y)$, and $g_n(x,y), n=1,2,3,...,k$, solve the following equations:

$$\begin{cases} T_t^{(k-1)\gamma} Res_{uk}(x,y,0)=0, \\ T_t^{(k-1)\gamma} Res_{vk}(x,y,0)=0. \end{cases} \tag{19}$$

To determine$f_1(x,y)$, and $g_1(x,y)$, put k =1 into Eq. (19) leading to:

$$\begin{cases} Res_{u1}(x,y,t)=T_t^{\gamma}u_1-D_1((u_1)_{xx}+(u_1)_{yy})+(A+1)u_1-u_1^2v_1-B, \\ Res_{v1}(x,y,t)=T_t^{\gamma}v_1-D_2((v_1)_{xx}+(v_1)_{yy})-Av_1+u_1^2v_1, \end{cases} \tag{20}$$

where

$$\begin{cases} u_1(x,y,t)=f(x,y)+f_1(x,y)\frac{t^\gamma}{\gamma}, \\ v_1(x,y,t)=g(x,y)+g_1(x,y)\frac{t^\gamma}{\gamma}. \end{cases} \tag{21}$$

Substituting Eq. (22) into Eq. (21), and using Eq. (20), we obtain the required coefficients $f_1(x,y), g_1(x,y)$ as:

$$\begin{cases} f_1(x,y,t)=D_1(f_{xx}+f_{yy})-(A+1)f+f^2g+B, \\ g_1(x,y,t)=D_2(g_{xx}+g_{yy})+Ag-f^2g. \end{cases} \tag{22}$$

The 1st CRPS approximate solutions take the form:

$$\begin{cases} u_1(x,y,t)=f(x,y)+\left(D_1(f_{xx}+f_{yy})-(A+1)f+f^2g+B\right)\frac{t^\gamma}{\gamma}, \\ v_1(x,y,t)=g(x,y)+\left(D_2(g_{xx}+g_{yy})+Ag-f^2g\right)\frac{t^\gamma}{\gamma}. \end{cases} \tag{23}$$

To determine $f_2(x,y)$, put (k =2) into Eq. (19) leading to:

$$\begin{cases} Res_{u2}(x,y,t)=T_t^{\gamma}u_2-D_1((u_2)_{xx}+(u_2)_{yy})+(A+1)u_2-u_2^2v_2-B, \\ Res_{v2}(x,y,t)=T_t^{\gamma}v_2-D_2((v_2)_{xx}+(v_2)_{yy})-Av_1+u_2^2v_2, \end{cases} \tag{24}$$

where

$$\begin{cases} u_2(x,y,t)=f(x,y)+f_1(x,y)\frac{t^\gamma}{\gamma}+f_2(x,y)\frac{t^{2\gamma}}{\gamma^2}, \\ v_2(x,y,t)=g(x,y)+g_1(x,y)\frac{t^\gamma}{\gamma}+g_2(x,y)\frac{t^{2\gamma}}{\gamma^2}. \end{cases} \tag{25}$$

Substituting Eq. (26) into Eq. (25), and using Eq. (20), we obtain the required coefficients $f_2(x,y), g_2(x,y)$ as:

$$\begin{cases} f_2(x,y,t)=D_1(f_{1_{xx}}+f_{1_{yy}})-(A+1)f_1+f^2g_1+2ff_1g+B, \\ g_2(x,y,t)=D_2(g_{1_{xx}}+g_{1_{yy}})+Af_1-f^2g_1-2ff_1g. \end{cases} \tag{26}$$

The 2nd CRPS approximate solutions take the form:

$$\begin{cases} u_2(x,y,t)=f(x,y)+\left(D_1(f_{xx}+f_{yy})-(A+1)f+f^2g+B\right)\frac{t^\gamma}{\gamma}+ \\ \qquad (D_1(f_{1_{xx}}+f_{1_{yy}})-(A+1)f_1+f^2g_1+2ff_1g+B)\frac{t^{2\gamma}}{\gamma^2}, \\ v_2(x,y,t)=g(x,y)+\left(D_2(g_{xx}+g_{yy})+Ag-f^2g\right)\frac{t^\gamma}{\gamma}+ \\ \qquad (D_2(g_{1_{xx}}+g_{1_{yy}})+Af_1-f^2g_1-2ff_1g)\frac{t^{2\gamma}}{\gamma^2}. \end{cases} \tag{27}$$

And so on. See Tables (1-2), and Figs. (1-4).

Table 1: Comparison between CRPS solution $u_2(x, y, t)$ (problem 4.1) with exact solution for $D_1 = D_2 = 0.25, A = 1, B = 0, \gamma = 1$.

(x,y)	(0.2,0.2)			(0.5,0.5)		
t	Exact	CRPS	$\lvert CRPS - Exact\rvert$	Exact	CRPS	$\lvert CRPS - Exact\rvert$
0.1	0.637628	0.637641	1.3792E-05	0.349937	0.349937	7.5693E-06
0.2	0.606530	0.606639	1.0898E-04	0.332871	0.332930	5.9810E-05
0.3	0.576949	0.577313	3.6332E-04	0.316636	0.316836	1.9939E-04
0.4	0.548811	0.549662	8.5080E-04	0.301194	0.301661	4.6692E-04
0.5	0.522045	0.523687	1.6417E-03	0.286504	0.287405	9.0101E-04
0.6	0.496585	0.499388	2.8031E-03	0.272531	0.274070	1.5383E-03
0.7	0.472366	0.476765	4.3985E-03	0.259240	0.261654	2.4139E-03
0.8	0.449329	0.455817	6.4886E-03	0.25015	0.25015	3.5610E-03
0.9	0.427414	0.436545	9.1309E-03	0.234570	0.239581	5.0111E-03

Table 2: Comparison between CRPS solution $v_2(x, y, t)$ (problem 4.1) with exact solution for $D_1 = D_2 = 0.25, A = 1, B = 0, \gamma = 1$.

(x,y)	(0.2,0.2)			(0.5,0.5)		
t	Exact	CRPS	$\lvert CRPS - Exact\rvert$	Exact	CRPS	$\lvert CRPS - Exact\rvert$
0.1	1.568312	1.568281	3.1472E-05	2.857651	2.857594	5.7345E-05
0.2	1.648721	1.648466	2.5497E-04	3.004166	3.003701	4.6460E-04
0.3	1.733253	1.732381	8.7158E-04	3.158193	3.156605	1.5881E-03
0.4	1.822119	1.820026	2.0926E-03	3.320117	3.316304	3.8130E-03
0.5	1.915541	1.911400	4.1404E-03	3.490343	3.482799	7.5443E-03
0.6	2.013753	2.006504	7.2484E-03	3.669297	3.656089	1.3207E-02
0.7	2.117000	2.105338	1.1662E-02	3.857426	3.836175	2.1250E-02
0.8	2.225541	2.207901	1.7640E-02	4.0552	4.023057	3.2142E-02
0.9	2.339647	2.314193	2.5453E-02	4.263115	4.216735	4.6379E-02

Table 3: The CRPS solution (problem 4.1) for $D_1 = D_2 = 0.25, A = 1, B = 0, x = y = 0.5$ at different values of fractional power γ.

t	$\gamma = 0.9$		$\gamma = 0.7$		$\gamma = 0.5$	
	u	v	u	v	u	v
0.1	0.3430496	2.915048	0.3191858	3.133294	0.2699397	3.713792
0.2	0.3229995	3.096205	0.2925668	3.420478	0.2401467	4.205763
0.3	0.305222	3.277327	0.272147	3.682698	0.2215655	4.614888
0.4	0.2891941	3.460926	0.2555364	3.932914	0.2087879	4.981131
0.5	0.2746596	3.648021	0.2416862	4.17626	0.1997193	5.319968
0.6	0.2614623	3.839124	0.2300077	4.415368	0.1932851	5.639338
0.7	0.2494953	4.034524	0.2201244	4.651788	0.1888472	5.943958
0.8	0.23868	4.234394	0.2117749	4.886509	0.1859898	6.2369
0.9	0.2289557	4.43884	0.2047675	5.120205	0.1844241	6.520297

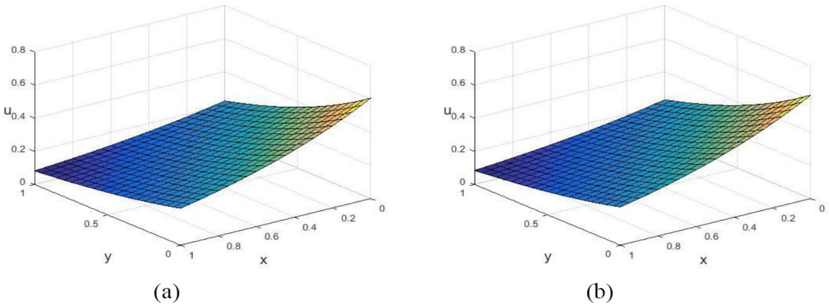


Figure 1: (a) Exact solution (classical case) (b) CRPS solution $u_2(x, y, t)$ (problem 4.1) for $D_1 = D_2 = 0.25, A = 1, B = 0, t = 1, \gamma = 1$.

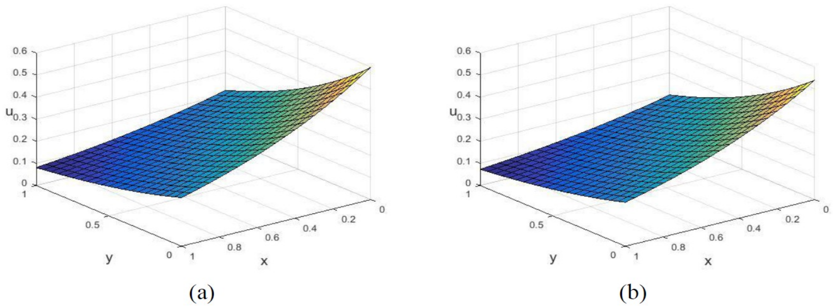


Figure 2: The CRPS solution $u_2(x, y, t)$ for problem 4.1 (a) $\gamma = 0.9$ (b) $\gamma = 0.7$.

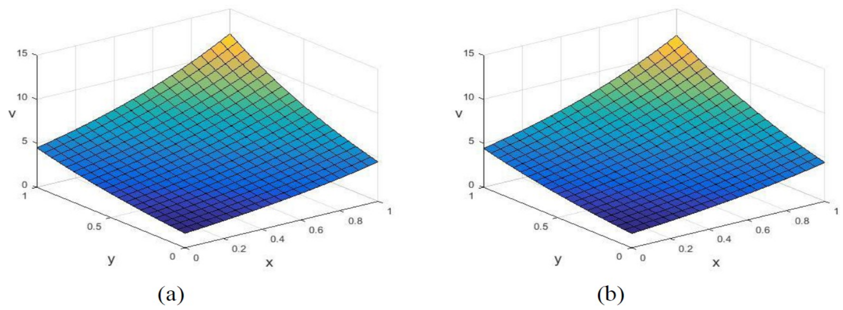
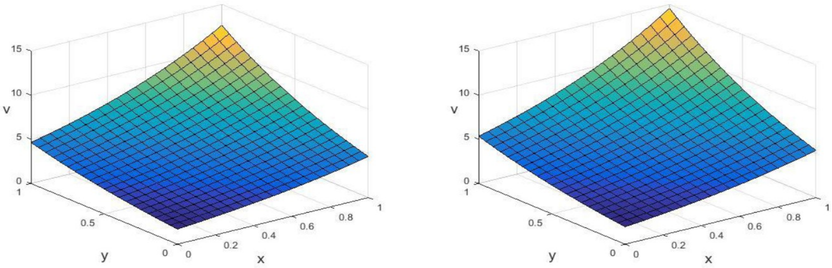


Figure 3: (a) Exact solution (classical case) (b) CRPS solution $v_2(x, y, t)$ (problem 4.1) for $D_1 = D_2 = 0.25, A = 1, B = 0, t = 1, \gamma = 1$.



(a) (b)

Figure 4: The CRPS solution $v_2(x, y, t)$ for problem 4.1 (a) $\gamma = 0.9$ (b) $\gamma = 0.7$.

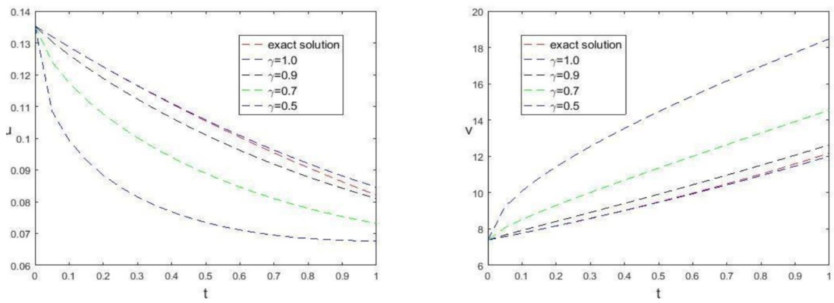


Figure 5: The CRPS solution$u_2(x, y, t), v_2(x, y, t)$ (problem 4.1) for $x = y = 1$ at different values of fractional power γ.

The comparisons between the CRPS solution with exact solution are shown in Figs. 1-4 and Tables 1-2, which proved the reliability of the method. The geometric behavior solutions of conformable fractional Brusselator model using CRPS method are shown in Fig. 5 and Table 3. The order of fractional derivative γ is an index of memory. Besides, the concentrations of two reactants $u(x, y, t)$ and $v(x, y, t)$ continuously depended on fractional power γ.

Problem 4.2

Let us consider the new conformable fractional Schnakenberg model:

$$\begin{cases} T_t^\gamma u = D_1(u_{xx} + u_{yy}) - \delta(u - u^2 v - A) \\ T_t^\gamma v = D_2(v_{xx} + v_{yy}) - \delta(u^2 v - B), \end{cases} \tag{28}$$

with the initial data [23]

$$\begin{cases} u(x, y, 0) = 1 - e^{-10(x-0.5)^2 + (y-0.5)^2}, \\ v(x, y, 0) = e^{-10(x-0.5)^2 + 2(y-0.5)^2}. \end{cases} \tag{29}$$

By repeating the above CRPS steps, we obtained the following results:

$$\begin{cases} f_1(x,y,t) = D_1(f_{xx}+f_{yy}) + \delta(A - f + f^2 g), \\ g_1(x,y,t) = D_2(g_{xx}+g_{yy}) + \delta(B - f^2 g). \end{cases} \tag{30}$$

$$\begin{cases} u_1(x,y,t) = f(x,y) + \left(D_1(f_{xx}+f_{yy}) + \delta(A - f + f^2 g)\right) \frac{t^\gamma}{\gamma} \\ v_1(x,y,t) = g(x,y) + \left(D_2(g_{xx}+g_{yy}) + \delta(B - f^2 g)\right) \frac{t^\gamma}{\gamma}. \end{cases} \tag{31}$$

$$\begin{cases} f_2(x,y,t) = D_1(f_{1_{xx}}+f_{1_{yy}}) + \delta(A - f_1 + f^2 g_1 + 2ff_1 g), \\ g_2(x,y,t) = D_2(g_{1_{xx}}+g_{1_{yy}}) + \delta(B - f^2 g_1 - 2ff_1 g). \end{cases} \tag{32}$$

$$\begin{cases} u_2(x,y,t) = f(x,y) + \left(D_1(f_{xx}+f_{yy}) + \delta(A - f + f^2 g)\right) \frac{t^\gamma}{\gamma} + \\ \qquad (D_1(f_{1_{xx}}+f_{1_{yy}}) + \delta(A - f_1 + f^2 g_1 + 2ff_1 g))\frac{t^{2\gamma}}{\gamma^2}, \\ v_2(x,y,t) = g(x,y) + \left(D_2(g_{xx}+g_{yy}) + \delta(B - f^2 g)\right) \frac{t^\gamma}{\gamma} + \\ \qquad (D_2(g_{1_{xx}}+g_{1_{yy}}) + \delta(B - f^2 g_1 - 2ff_1 g))\frac{t^{2\gamma}}{\gamma^2}. \end{cases} \tag{33}$$

And so on. See Figs. (6-8).

The geometric behavior solutions of conformable fractional Schnakenberg model using CRPS method are shown in Figs. 6-8. The order of fractional derivative γ is an index of memory. Besides, the concentration of activator and inhibitor u(x,y,t) and v(x,y,t) continuously depended on fractional power γ.

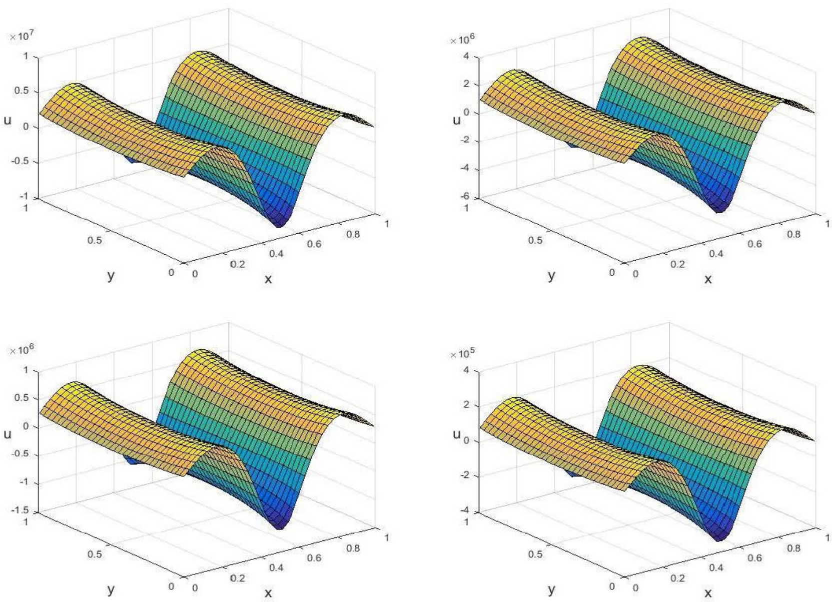


Figure 6: The CRPS solution $u_2(x,y,t)$ for problem 4.2 at $D_1 = 1, D_2 = 12, A = 0.1, B = 0.9, \delta = 3, t = 100$ (a) $\gamma = 1$ (b) $\gamma = 0.9$(c) $\gamma = 0.7$ (d) $\gamma = 0.5$.

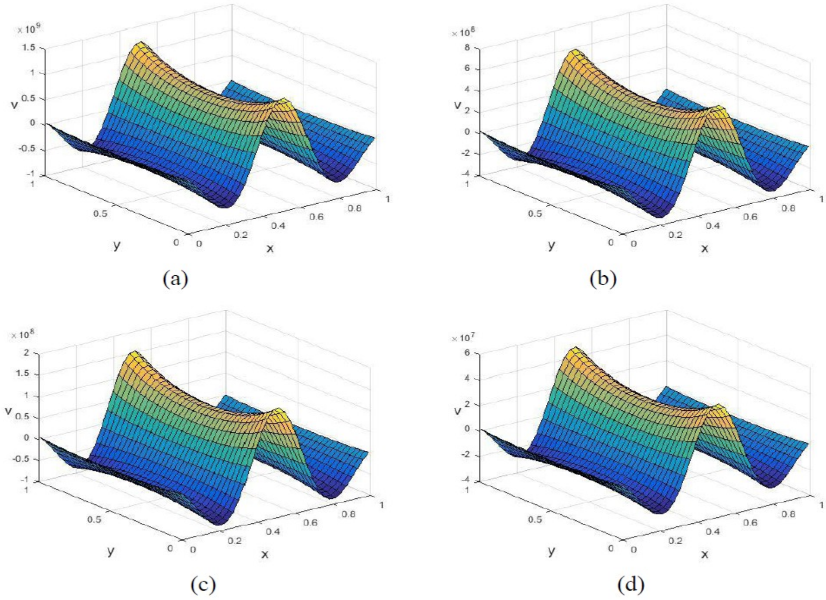


Figure 7: The CRPS solution $v_2(x, y, t)$ for problem 4.2 at $D_1 = 1, D_2 = 12, A = 0.1, B = 0.9, \delta = 3, t = 100$ (a) $\gamma = 1$ (b) $\gamma = 0.9$ (c) $\gamma = 0.7$ (d) $\gamma = 0.5$.

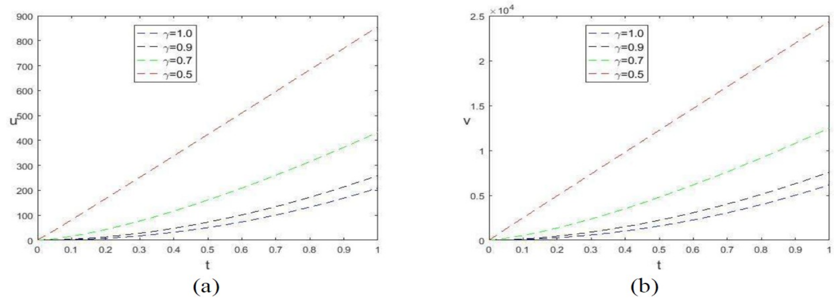


Figure 8: The CRPS solution $u_2(x, y, t), v_2(x, y, t)$ (problem 4.2) for $x = y = 1$ at different values of fractional power γ.

Problem 4.3

Let us consider the new conformable fractional Gray-Scott model:

$$\begin{cases} T_t^{\gamma} u = D_1(u_{xx} + u_{yy}) + A(1 - u) - uv^2, \\ T_t^{\gamma} v = D_2(v_{xx} + v_{yy}) - (A + B)v + uv^2, \end{cases} \tag{34}$$

with the initial data [23]

$$\begin{cases} u(x,y,0) = 1 - 0.5e^{-0.05(x^2+y^2)}, \\ v(x,y,0) = 0.25e^{-0.05(x^2+y^2)}. \end{cases} \tag{35}$$

By repeating the CRPS steps, we obtained the following results:

$$\begin{cases} f_1(x,y,t) = D_1(f_{xx}+f_{yy}) + A(1-f) - fg^2, \\ g_1(x,y,t) = D_2(g_{xx}+g_{yy}) - (A+B)g + fg^2. \end{cases} \tag{36}$$

$$\begin{cases} u_1(x,y,t) = f(x,y) + \left(D_1(f_{xx}+f_{yy}) + A(1-f) - fg^2\right)\frac{t^\gamma}{\gamma}, \\ v_1(x,y,t) = g(x,y) + \left(D_2(g_{xx}+g_{yy}) - (A+B)g + fg^2\right)\frac{t^\gamma}{\gamma}. \end{cases} \tag{37}$$

$$\begin{cases} f_2(x,y,t) = D_1(f_{1_{xx}}+f_{1_{yy}}) + A(1-f_1) - f_1g^2 - 2fgg_1, \\ g_2(x,y,t) = D_2(g_{1_{xx}}+g_{1_{yy}}) - (A+B)g_1 + f_1g^2 + 2fgg_1. \end{cases} \tag{38}$$

$$\begin{cases} u_2(x,y,t) = f(x,y) + \left(D_1(f_{xx}+f_{yy}) + A(1-f) - fg^2\right)\frac{t^\gamma}{\gamma} + \\ \qquad (D_1(f_{1_{xx}}+f_{1_{yy}}) + A(1-f_1) - f_1g^2 - 2fgg_1)\frac{t^{2\gamma}}{\gamma^2}, \\ v_2(x,y,t) = g(x,y) + \left(D_2(g_{xx}+g_{yy}) - (A+B)g + fg^2\right)\frac{t^\gamma}{\gamma} + \\ \qquad (D_2(g_{1_{xx}}+g_{1_{yy}}) - (A+B)g_1 + f_1g^2 + 2fgg_1)\frac{t^{2\gamma}}{\gamma^2}. \end{cases} \tag{39}$$

And so on. See Figs. 9-11.

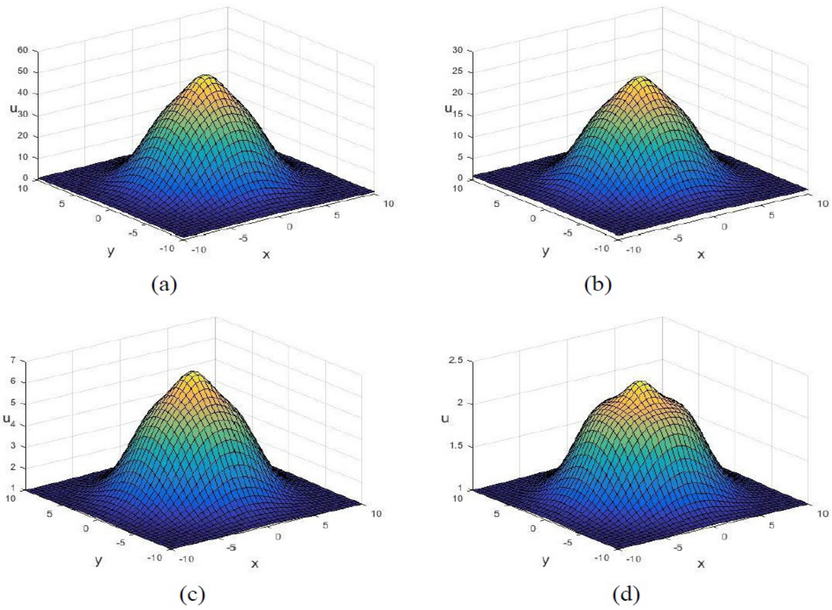


Figure 9: The CRPS solution $u_2(x,y,t)$ (problem 4.3) at $D_1 = 2(10)^{-5}, D_2 = 2D_1, A = 0.351, B = 0.33, t = 100$ (a) $\gamma = 1$ (b) $\gamma = 0.9$ (c) $\gamma = 0.7$ (d) $\gamma = 0.5$.

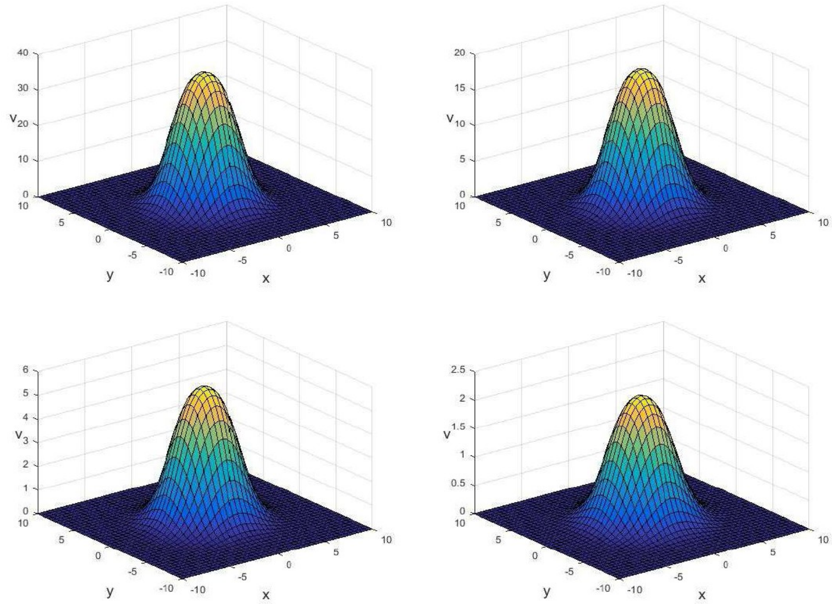


Figure 10: The CRPS solution $v_2(x, y, t)$ (problem 4.3) at $D_1 = 2(10)^{-5}, D_2 = 2D_1, A = 0.351, B = 0.33, t = 100$ (a) $\gamma = 1$ (b) $\gamma = 0.9$ (c) $\gamma = 0.7$ (d) $\gamma = 0.5$.

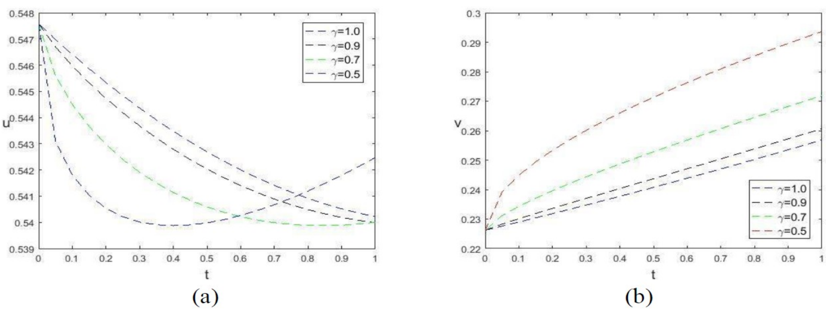


Figure 11: The CRPS solution $u_2(x, y, t), v_2(x, y, t)$ (problem 4.3) for $x = y = 1$ at different values of fractional power γ.

The geometric behavior solutions of conformable fractional Gray-Scott model using CRPS method are shown in Figs. 9-11. The order of fractional derivative γ is an index of memory. Besides, the concentration of activator and inhibitor $u(x, y, t)$ and $v(x, y, t)$ continuously depended on fractional power γ.

§5 Conclusions

The CRPS method has successfully been used to give new approximate solutions for Brusselator model, conformable fractional Schnakenberg model, and conformable fractional Gray-Scott model. It has more than adequately proved so effective and reliable a method for the purpose. The behavior of the solution seems to be extremely interesting in that it has proved a number of useful applications. The natural frequency of the solutions varies with the change of fractional power. It is noted that CRPS method is a very simple effective technique for solving time-fractional problems. Further, studies on the topic may still lead to greater conclusion and more interesting results.

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