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A new test on the conditional capital asset pricing model

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Abstract. Testing the validity of the conditional capital asset pricing model (CAPM) is a puzzle in the finance literature. Lewellen and Nagel[14] find that the variation in betas and in the equity premium would have to be implausibly large to explain important asset-pricing anomalies. Unfortunately, they do not provide a rigorous test statistic. Based on a simulation study, the method proposed in Lewellen and Nagel[14] tends to reject the null too frequently. We develop a new test procedure and derive its limiting distribution under the null hypothesis. Also, we provide a Bootstrap approach to the testing procedure to gain a good finite sample performance. Both simulations and empirical studies show that our test is necessary for making correct inferences with the conditional CAPM.

§1 Introduction

The capital asset pricing model (CAPM) is a cornerstone in both theoretical and empirical finance. It states a linear relationship between the excess return of a risky asset and the beta of that asset with respect to the market return. It is commonly assumed in the literature that the betas of the CAPM are constant over time. Recent empirical studies provide ample evidences against this assumption because the relative risk of a firm's cash flow varies over the business cycle and with the state of the economy. In other words, it is reasonable to believe that the CAPM holds under the condition of current information sets. This implies that the conditional CAPM should be preferred in real applications; see Jagannathan and Wang[12].

The validity of the conditional CAPM is empirically debatable in the finance literature. Theoretically, the conditional CAPM can hold perfectly from period to period so that it can deliver many successful asset pricing models in real applications. Following Hansen and Richard[8], a conditional model can deliver an unconditional testable restrictions given proper modeling of

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the conditioning information. Jagannathan and Wang[12] generalize the single factor model with time-varying loading to a multi-factor linear model and argue that the model can give a better explanation to the size effect. Lettau and Ludvigson[13] use a consumption-wealth ratio as a conditioning variable and find that the model performs better at explaining the cross-sectional return variation. Santos and Veronesi[21] choose the labor income to consumption ratio as a conditioning variable and yield a similar result. The success of the aforementioned models in real applications can be regarded as a further evidence to support using the conditional CAPM in real applications. However, He et al.[10] generalize the model in Harvey[9] and conclude that the conditional CAPM captures little variation in expected returns of momentum and book-to-market effects. Ghysels[7] shows that the conditional CAPM is often misspecified so that it leads to an even larger pricing error than the unconditional CAPM. Bali[3] proposes to study jointly the intertemporal and cross-sectional implications of the intertemporal CAPM (ICAPM). Essentially, he estimates the cross-sectional conditional risk-return relation with an intertemporal stability constraint on the relative risk aversion coefficient, and the estimated market risk premium is highly significant and positive. Recently, Ang and Kristensen[2] and Li and Yang[16] use nonparametric approach methods to estimate and test the conditional factor model. They suggest that the conditional CAPM fails to explain well-known asset-pricing anomalies. Li, Su and Xu[17] use a two-stage semiparametric approach to examine the performance of the conditional CAPM and find some mixed results. Other recent researches include Bali and Engle[4] employ the dynamic conditional correlation model and show that the risk premium of an ICAPM model is significantly positive. Bali, Engle and Tang[6] use a dynamic conditional correlation approach to model the conditional beta and find that conditional beta performs well in explaining the cross-section of daily stock returns. Moreover, Bali and Engle[5] conclude that the conditional CAPM can explain the value premium using the dynamic conditional correlation model.

Different from the convention in the literature that explores the significance of pricing errors, Lewellen and Nagel[14] (hereafter, LN) provide a new method to evaluate the conditional CAPM. Instead of modeling the conditional betas as functions of other instrumental variables, they assume the conditional betas keep stable in a short window and can be estimated by a simple regression in each window. They conclude that the covariance between the conditional betas and the equity premium would have to be implausibly large to explain important asset pricing anomalies like momentum and the value premium. They claim that the conditional CAPM performs nearly as poorly as the unconditional one.

However, there are some limitations for the method proposed by LN. Their conclusion is based on a simple comparison of the estimated values of two unobservable variables. Based on their method, the conditional CAPM should be rejected if the estimated values are far away from each other. Since the estimated values are statistics, it is not reliable to do a comparison by simply replacing the true values with their estimators. Although the magnitudes of those estimators are quite different as shown empirically by LN, a formal statistical test is definitely required. The main goal of this paper is to join the current debate about the validity of the conditional CAPM by revisiting the method proposed in LN. Based on the LN method, we propose a new test to check whether the covariance in betas and the equity premium is large enough to explain important asset pricing anomalies. In this paper, we contribute to complete the LN method by fixing the drawback and apply the new test on a larger data set including size, B/M sorted portfolios, as well as industrial portfolios.

To highlight the importance of the hypothesis testing procedure, we conduct some simulations; see Section 2.2 for the detailed settings and results. First, we assume that the data are generated from a conditional CAPM. Then, we follow the steps in LN to estimate the variables and do a comparison. Consequently, the frequency of inequality for those estimators (even in magnitude) is quite high. This suggests that even if the conditional CAPM holds, there is still a large chance that the two estimators are different in magnitude. Therefore, we suspect that one can make a valid inference on the conditional CAPM by following the LN method.

In this paper, we construct a statistic based on the difference between the estimated variables and derive its limiting distribution (asymptotic normality). To test its significance, we use the Bootstrap method to compute the empirical p-value. Simulations suggest that our test has a good size and indeed it is powerful in a finite sample performance. Empirically, to avoid the data-snooping problem mentioned in Lo and MacKinlay[18], we test a wide variety of portfolios: 6, 25, and 100 Fama-French portfolios; 5, 10 and 30 industrial portfolios; SMB, HML, and momentum (MOM) factors. All the data are downloaded from Kenneth French's web site. The findings are summarized in Table 1, which lists the numbers of the portfolios which can not

Table 1: Test results for comparison between our method and the LN method

This table reports the numbers of portfolios that fail to reject the conditional CAPM at the given significance level. LN do not provide a formal test statistic and they only argue that the ratio of covariance between equity premium and unconditional alpha should be close to 1. Therefore, we take it as a rejection if the value of this ratio does not belong to $[0.5, 2]$. The betas are assumed to be stable within one quarter window. The data are taken from Kenneth French's web site and the sample period is from July 1963 to December 2014.

	Our Test			LN
	1%	5%	10%	
Fama-French 6 portfolios	3	2	2	2
Fama-French 25 portfolios	9	8	6	2
Fama-French 100 portfolios	70	55	45	13
5 industry portfolios	4	4	3	1
10 industry portfolios	6	5	5	2
30 industry portfolios	21	17	14	6
SMB, HML, MOM factors	0	0	0	0

be rejected by our testing procedure or by the method in LN. We see from Table 1 that the numbers at different significance levels (from the second column to the forth column) by our method are always larger than those (the last column) by the LN method. Thus, there is a great chance that the conditional CAPM would be rejected by the LN method, while it is still accepted by our testing procedure. In other words, the LN method tends to over-reject the null hypothesis. This phenomenon is confirmed by our simulation study conducted in Section 2.2.

http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

Although the conditional CAPM fails to explain the SMB, HML and MOM anomalies, it can explain the individual portfolios reasonably well. The conditional CAPM can also explain most of the time series variation of industry portfolio returns.

The rest of this paper is organized as follows. Section 2 briefly reviews the LN method and illustrates its strengths and weaknesses based on a simple simulation study. Section 3 presents our new testing procedure and discusses the test statistic as well as gives its asymptotic distribution. In Section 4, some empirical studies on Fama-French portfolios and other common factors are reported. Section 5 is devoted to the robustness analysis and Section 6 concludes the paper. All the mathematical proofs are relegated to the Appendix.

§2 A direct comparison method

2.1 Revisit of the LN method

The validity of the conditional CAPM usually requires two conditions to hold at the same time. The first is that the pricing error is insignificant. This requirement has already been extensively discussed in the finance literature. The second, as advocated by LN, is that the unconditional pricing error is close to the covariance of the beta and the market equity premium. Let us focus on the second condition here. The conditional CAPM can be expressed as

$$R_{i,t} = \beta_{i,t} R_{M,t} + \varepsilon_{i,t}, \tag{1}$$

where $R_{i,t}$ is the excess return of risky asset i, $R_{M,t}$ is the excess return of the market portfolio, and the conditional beta, $\beta_{i,t}$, is given by

$$\beta_{i,t} = \mathrm{Cov}(R_{i,t}, R_{M,t}|I_{t-1})/\mathrm{Var}(R_{M,t}|I_{t-1}),$$

where I_{t-1} is the information set up to time $t-1$. We use γ_t and σ_t to denote the conditional equity premium and the standard error of the market factor, and γ and σ_M to denote the corresponding unconditional values. Further, β_i^u is the corresponding unconditional beta. We also define the unconditional pricing error (or unconditional alpha) as $\alpha_i^u \equiv E[R_{i,t}] - \beta_i^u \gamma$. LN show that if the conditional CAPM holds, then the following equation holds

$$\alpha^u = \left[1 - \frac{\gamma^2}{\sigma_M^2}\right] \mathrm{Cov}(\beta_t, \gamma_t) - \frac{\gamma}{\sigma_M^2} \mathrm{Cov}\left[\beta_t, (\gamma_t - \gamma)^2\right] - \frac{\gamma}{\sigma_M^2} \mathrm{Cov}(\beta_t, \sigma_t^2), \tag{2}$$

which is the same as (3) of LN. Note that in the above equation, the subscript i is dropped because this equation holds for every asset.

Based on the empirical analysis, LN argue that compared with the first term on the right-hand side of (2), the last two terms on the right-hand side of (2) are negligible. Since γ^2/σ_M^2 is usually very small, whether (2) holds or not is equivalent to whether $\alpha^u \approx \mathrm{Cov}(\beta_t, \gamma_t)$. Therefore, we only need to estimate both α^u and $\mathrm{Cov}(\beta_t, \gamma_t)$, respectively. To this end, LN suggest splitting the whole time series into short windows and estimating the conditional moments of each window. Then, they compare the left-hand side with the right-hand side to see if they are close enough.

However, this procedure has a drawback. Both estimators for the left-hand side and the right-hand side in (2) are random variables. The direct comparison, even in magnitude, of two estimated values is not reliable since they have variations even in large sample. The test size of their procedure can be severely distorted. On the other hand, to make an inference, LN suggest comparing the estimated value of ratio $\delta = \mathrm{Cov}(\beta_t, \gamma_t)/\alpha^u$ to one or not. However, α^u is commonly small so the estimate for δ can be very volatile. Therefore, the estimated value for δ might not be reliable. This drawback, unfortunately, has not been addressed in the finance literature and it is illustrated clearly by the following simulation.

2.2 An illustrative example

Here we conduct a simple simulation to argue that there exists the aforementioned drawback that the LN method tends to over-rejection. In our simulation setting, the conditional CAPM is a true data generating process (DGP). Then, we follow the steps in LN to compare two sides of (2). Suppose the whole sample contains T daily observations. We divide the whole sample into H short windows so that there are w observations in each window, where $T = w\,H$. The h-th window is denoted by $W_h = [hw+1, (h+1)w]$, from time $hw+1$ to $(h+1)w$. Following the assumptions in LN, we keep the conditional β_t constant in each short window, while it varies across the windows. Then, we generate β_h by $\beta_h = \beta + \eta_h$, where $h = 1, 2, \ldots, H$, and η_h is the cross-window disturbance. So, for the beta observed at time t, β_t is fixed for any $t \in W_h$ and $\beta \equiv E[\beta_t]$.

Next, the excess return of market portfolio is generated by $R_{M,t} = \gamma + u_h + v_t$ for any $t \in W_h$, where u_h is the cross-window disturbance and v_t is the daily disturbance. Finally, we generate the excess return for all individual stocks based on the conditional CAPM as $R_t = \beta_t R_{M,t} + \varepsilon_t$, where ε_t is the noise. We calibrate $\gamma = 0.00183$ from the time series mean of market excess return from July 1963 to December 2009. We set the idiosyncratic standard deviation as $\sigma_\varepsilon = 0.0083$, which is estimated from the Fama-French 25 size and B/M (book-to-market) portfolios. The simulated unconditional mean β is evenly spanned from 0.5 to 2 across 10 assets. For the purpose of robustness check, we consider the following 6 cases:

C1. Cross-window disturbances in conditional betas and excess market returns u_h and η_h follow a multivariate normal distribution, with $\sigma_\eta = 0.7071$, $\sigma_u = 0.0012$, and the correlation coefficients are randomly draw from $U[-1, 1]$. We generate $v_t \sim$ i.i.d. $N(0, \sigma_v^2)$ with $\sigma_v = 0.0098$ which is estimated from the real market excess return.

C2. Data are generated with the same parameter set as in C1, except the correlation coefficients are set to 0.

C3. Cross-window disturbances in conditional betas and excess market returns u_h and η_h are generated independently, while u_h is generated from i.i.d. random numbers from student-t distribution with degrees of freedom (df) of 4, then multiply by a constant of 0.00084 to scale it to have the same standard deviation as in C1. The within window disturbances η_h and v_t are generated from the same parameters as in C1.

C4. We set $u_h = 0$ and all the other parameters are the same as in C1.

C5. We set $u_h = 0$ and generate v_t by drawing random numbers from i.i.d. student-t with degrees of freedom of 4, then multiply by 0.0069 to scale it to have the same standard deviation as in C1.

C6. We set $R_{M,t}$ using the real market data extracted from French's web site. We generate β and ε from the same parameters set as in C1.

Last, we simulate 10 portfolios with different βs as in each of the above description and follow the procedure in LN to estimate the two sides of (2). We repeat the simulation 1000 times and compute the results. According to LN, (2) reduces to

$$\alpha^u \approx \mathrm{Cov}(\beta_t, \gamma_t) \tag{3}$$

and $\delta = \mathrm{Cov}(\beta_t, \gamma_t)/\alpha^u$ is approximately equal to 1. Table 2 summarizes the simulation results for $\hat{\delta}$. The results in Table 2 suggest that even when the true DGP is the conditional CAPM,

Table 2: Results of direct comparison

This table reports the mean, median and standard deviation (std) of the estimated value for the ratio δ among 1000 replications, where γ_t and η_t are generated from 6 different distributions as specified in C1 - C6. Parameters σ_u^2, σ_η^2, and σ_v^2 are calculated from real data. The window size is quarterly, namely 63 trading days in the simulation. The whole sample is 40 years, close to our real data. We report the unconditional regression results over the whole sample time series. Then, we estimate the conditional α, β and equity premium, and we report the results of $\hat{\delta}$ for each portfolio. Note that $\hat{\delta}$ should be very close to 1 if the LN method is valid.

Model	Portfolio	1	2	3	4	5	6	7	8	9	10
	Mean	1.012	0.985	1.019	1.012	0.971	1.011	1.011	1.030	1.013	1.010
C1	Median	1.010	1.008	1.010	1.010	1.003	1.010	1.011	1.008	1.013	1.010
	Std	0.068	1.384	0.249	0.084	1.678	0.022	0.032	0.426	0.074	0.022
	Mean	0.165	0.466	1.330	1.617	0.804	0.478	0.457	-0.873	-9.556	0.756
C2	Median	0.625	0.565	0.585	0.617	0.551	0.564	0.560	0.582	0.542	0.529
	Std	23.211	11.016	17.570	22.031	12.546	11.768	6.224	48.246	347.047	36.340
	Mean	0.846	0.963	1.517	1.016	3.446	-0.061	0.867	0.717	-0.036	0.932
C3	Median	0.986	0.985	0.966	0.993	0.967	1.008	0.986	1.002	0.974	0.979
	Std	4.051	3.461	27.749	3.349	73.193	27.274	5.905	9.672	30.841	10.707
	Mean	0.788	0.868	-0.592	1.537	0.098	-2.202	-0.530	-0.292	2.377	0.593
C4	Median	0.450	0.386	0.416	0.381	0.407	0.458	0.421	0.403	0.437	0.454
	Std	6.267	11.149	33.092	50.496	10.276	67.790	24.025	12.633	30.158	11.740
	Mean	-0.153	1.323	0.588	1.118	-27.976	0.216	0.789	-22.515	-0.386	0.860
C5	Median	0.386	0.399	0.425	0.382	0.430	0.394	0.430	0.411	0.410	0.403
	Std.	27.347	37.035	7.918	31.488	859.730	33.903	18.869	718.871	18.887	14.356
	Mean	0.521	-0.144	-0.166	1.673	-4.191	0.118	0.185	0.321	61.193	0.346
C6	Median	0.439	0.423	0.479	0.406	0.412	0.384	0.370	0.447	0.402	0.477
	Std	16.514	26.119	14.347	44.339	140.918	7.851	15.327	6.385	1937.700	36.760

there is still a large chance the two sides of (2) are not close to each other. In other words, δ is far away from 1. Moreover, we find that δ has a large variance for all the DGP except case C1. Thus, the inequality of the estimated values of (2) can not be regarded as a concrete evidence against the conditional CAPM. This implies that the testing procedure in LN tends to over-reject the true model. Therefore, we need to conduct a formal statistical testing procedure in order to make rigorous statistical inferences.

§3 A new test based on bootstrap method

3.1 Testing procedure

One of main difficulties in testing conditional asset pricing models is how to deal with the true conditional information. Here, for simplicity, we do not impose any exogenous macro variables. Instead, we focus on the time series patterns of the asset returns and equity premium. The model is assumed to be $R_t = \beta_t R_{M,t} + \varepsilon_t$, where R_t denotes excess return, $R_{M,t}$ is the excess market return, and β_t is the factor loading. Here, we use t to denote the observation of each day, and h to denote the index of each window. Moreover, we assume that $R_{M,t}$ and β_t follow

$$R_{M,t} = \gamma + u_t + v_t \quad \text{and} \quad \beta_t = \beta + \eta_t,$$

where $u_t = u_h$ and $\eta_t = \eta_h$ for any $t \in W_h$. This means u_t and η_t are fixed within each window. Also, η_t and u_t are correlated, the variables u_t, v_t, ε_t are mutually independent, and η_t, v_t, ε_t are also mutually independent.

We assume that the factor loading keeps constant within each window while varies between windows. The decomposition of $R_{M,t}$ is for calculation convenience and can be relaxed. Within the h-th window, we define

$$\bar{v}_h \equiv \frac{1}{w}\sum_{t\in W_h} v_t, \qquad \bar{\varepsilon}_h \equiv \frac{1}{w}\sum_{t\in W_h} \varepsilon_t, \qquad \bar{R}_{M,h} \equiv \frac{1}{w}\sum_{t\in W_h} R_{M,t} = \gamma + u_h + \bar{v}_h,$$

and

$$\bar{R}_h \equiv \frac{1}{w}\sum_{t\in W_h} R_t = \beta^h(\gamma + u_h + \bar{v}_h) + \bar{\varepsilon}_h,$$

where $W_h \equiv \{j : j \in [w(h-1)+1, wh]\}$. Then, we run the regression within-window to get the conditional alpha and beta for each window. They can be expressed as

$$\hat{\beta}_h = \mathrm{Cov}_h(R_t, R_{M,t})/\mathrm{Var}_h(R_{M,t}),$$

where $\mathrm{Cov}_h(.)$ and $\mathrm{Var}_h(.)$ denote the estimates of covariance and variance for the sample in the h-th window, W_h, respectively. Then, $\hat{\alpha}_h = \bar{R}_h - \hat{\beta}_h \bar{R}_{M,h}$ and the estimate of γ_h is $\hat{\gamma}_h = \bar{R}_{M,h}$.

Next, we estimate the unconditional model. The factor loading in the unconditional model is

$$\hat{\beta}^u = \mathrm{Cov}_w(R_t, R_{M,t})/\mathrm{Var}_w(R_{M,t}),$$

where $\mathrm{Cov}_w(.)$ and $\mathrm{Var}_w(.)$ denote the estimation over the whole sample. Also, the unconditional alpha is $\hat{\alpha}^u = \bar{R} - \hat{\beta}^u \bar{R}_M$. Thus, the test statistic is formulated as

$$Q_T \equiv \hat{\alpha}^u - \left[1 - \frac{\hat{\gamma}^2}{\hat{\sigma}_M^2}\right]\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h) + \frac{\hat{\gamma}}{\hat{\sigma}_M^2}\widehat{\mathrm{Cov}}\left[\hat{\beta}_h, (\hat{\gamma}_h - \hat{\gamma})^2\right] + \frac{\hat{\gamma}}{\hat{\sigma}_M^2}\widehat{\mathrm{Cov}}(\hat{\beta}_h, s_{v,h}^2), \tag{4}$$

where $s_{v,h}^2$ is the sample analog of the variance of $v_t \in W_h$. By assuming that $H = T/w \to \infty$, when $T \to \infty$, the asymptotic normality of Q_T can be established as

$$\sqrt{T/w}\, Q_T/\sigma_Q \ \xrightarrow{d}\ N(0,1), \tag{5}$$

where σ_Q is the asymptotic standard deviation of $\sqrt{T/w}\, Q_T$. The detailed proof of (5) is provided in the Appendix.

It is clear that under the null hypothesis that the conditional CAPM is the true DGP, Q_T

should be statistically insignificant. Since the asymptotic variance of Q_T is complicated and contains many unobserved parameters, the p-value implied by the asymptotic distribution is not easily tabulated, in particular, when the sample size is small. To circumvent this difficulty and to make computing simple, we propose using a direct Bootstrap method to obtain the p-value of Q_T by following the Bootstrap procedure proposed by Horowitz[11] and MacKinnon[19], described as below.

S1. Run a two-step regression to get the statistic Q_T and all the other estimators.

S2. Calculate the re-centered η_h^* and u_h^* for between-window and v_t^* and ε_t^* for within-window. Then, resample η_h^* and u_h^* pairwise, and resample v_t^* and ε_t^* pairwise within-window.

S3. Impose the constraint that the conditional alpha is zero and generate R_t^*. Then, compute Q_{Tj}^*.

S4. Repeat S2 and S3 for a large number of the Bootstrap replications, say $B = 499$ times. Then, use Q_T and Q_{Tj}^* to calculate the p-value $\hat{p}(Q_T)$. According to the equal-tail Bootstrap p-value equation, the p-value is computed based on

$$p\text{-value} = 2\min\left(\frac{1}{B}\sum_{j=1}^{B}\mathcal{I}(Q_{Tj}^* \le Q_T),\ \ \frac{1}{B}\sum_{j=1}^{B}\mathcal{I}(Q_{Tj}^* > Q_T)\right),$$

where $\mathcal{I}(A)$ is the indicator function of event A.

3.2 Size and power based on simulations

To illustrate the finite sample performance of the proposed test, we conduct some simulations and report the empirical size and power of the test. The simulation procedure is as follows. First, we calculate the parameters including the mean and variance from real data taken from French's web site. Then we simulate excess market return and beta using the parameters calculated from the real data. According to $R_{M,t} = \gamma + u_h + v_t$ for any $t \in W_h$, we choose $\gamma = 0.000183$ and simulate u_h and v_t from i.i.d normal distribution with mean zero and a variance of $\sigma_u = 0.0012$ and $\sigma_v = 0.0098$, respectively. We simulate β_h from $\beta_h = \beta + \eta_h$, where β is randomly generated from $U(0, 2)$, and η_h is generated from $N(0, 0.5)$. All the above parameters are estimated from real data. Next, we simulate excess returns using the excess market return and beta. Last, we apply our test to the simulated data. We repeat these steps 1000 times, and calculate the rejection frequency. The results are reported in Table 3. We find that the empirical sizes are close to the nominal levels when the time period is above 40 quarters, i.e., 10 years. This concludes that the proposed test can deliver the right test size.

As for the power of our test, we also validate it through simulations. We add pricing errors α^c to the portfolio returns of each day while the other settings are kept the same as in the simulation calculating empirical size. Table 4 shows that when the pricing errors are set to

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According to Paparoditis and Politis[20], estimating the residuals under imposing the test restrictions may cause a loss of power. Our simulation results also show the loss of power if we impose the restriction before estimating. So, we impose the restriction after we calculate the residuals.

Table 3: Size of the our test.

This table shows the empirical size of our test. Excess market returns and betas are simulated based on the parameters estimated from real data. The window size is $w = 63$ trading days. The whole sample period contains H quarters. so that $T = 63\,H$. We calculate the ratio of rejections in the simulations to obtain empirical size.

	1%	5%	10%
$H = 40$	0.013	0.051	0.101
$H = 80$	0.015	0.062	0.112
$H = 160$	0.012	0.064	0.126
$H = 320$	0.013	0.066	0.121

6.3% annually, our test has a power of 0.557 as the sample size goes to 160 quarters or 40 years. The power goes to 0.986 when the pricing error goes to 12.6% annually. This implies that the proposed test is indeed quite powerful and it can be used in real applications.

Table 4: Power of the our test

This table reports the empirical power under nominal size 1% and 5%. The sample period T is in quarters and α^c denotes the anualized pricing error .

	$H = 40$	$H = 80$	$H = 160$	$H = 320$
$\alpha^c = 6.3\%$				
1%	0.058	0.142	0.288	0.607
5%	0.188	0.321	0.557	0.820
$\alpha^c = 12.6\%$				
1%	0.272	0.565	0.905	0.997
5%	0.517	0.832	0.986	1.000
$\alpha^c = 25.2\%$				
1%	0.896	0.999	1.000	1.000
5%	0.981	0.999	1.000	1.000

§4 Empirical analysis

In this section, we use the proposed test to analyze U.S. equity data and report our empirical findings. Also, we compare our conclusions with those in the literature.

4.1 Description of data

We use the daily data from Kenneth French's web site. The sample period is from July 1963 to December 2014. The market return in the model is the value-weighted return on all NYSE, AMEX and NASDAQ stocks from CRSP. The risk-free rate is represented by the one-month Treasury bill rate.

We implement our test on a wide variety of portfolios: 6, 25, and 100 Fama-French size-B/M portfolios; 5 and 10 industry portfolios; SMB, HML, and MOM factors. Extending the test to several portfolios rather than only to the conventional SMB, HML, and MOM factors has at least two advantages. The first is to avoid the data-snooping problem pointed out by Lo and MacKinlay[18]. Another is that it enhances robustness; see Lewellen, Nagel and Shanken[15]. The Fama-French portfolios are sorted by their size (S) and book-to-market ratio (B/M). The industry portfolios are constructed based on the four-digit standard industry classification (SIC) code. The industry portfolios can be divided into 5, 10 or 30 industries respectively according to different criteria. The SMB, HML and MOM portfolios are the factor portfolios mimicking size effect, book-to-market effect and momentum strategies, respectively. We assume the betas keep stable within a quarterly window.

4.2 Main findings

4.2.1 Fama-French portfolios

To compare our results with those in Lewellen and Nagel[14], we conduct our test on the Fama-French 25 portfolios. We set the window size equal to one quarter and follow the LN method to calculate $\hat{\alpha}^u$ and $\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$. Afterwards, we compute the estimators for the two sides of equation (2) and for Q_T. Then we use the Bootstrap procedure to obtain the corresponding p-value for testing the null hypothesis.

Table 5 reports the results. The first column indicates the order of the portfolios. The portfolios are sorted by size (S) and book-to-market ratio (B/M). The unconditional pricing error, $\hat{\alpha}^u$, with its standard error and t-statistic are reported in the next three columns. The covariance between the conditional betas and equity premiums, $\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$, and its ratio over the pricing error, $\hat{\delta} = \widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)/\hat{\alpha}^u$, are shown in the fifth and the sixth columns, respectively. The last two columns summarize the test statistic Q_T and its corresponding p-values.

From Table 5, we see that under the 5% significance level, there are 8 of the 25 portfolios for which the null hypothesis can not be rejected, while by looking at the ratios of $\mathrm{Cov}(\beta_h, \gamma_h)$ over α^u, there are only 2 portfolios that lie in the interval $[0.5, 2]$. That means that for portfolios S2/B2, S4/B1, S5/B2, S5/B3, S5/B4 and S5/B5, comparison of the ratios of $\mathrm{Cov}(\beta_h, \gamma_h)$ over α^u leads to the conclusion that the conditional CAPM does not explain their time series pricing errors, while it is just opposite according to our test.

We also consider the Fama-French 6 portfolios and the Fama-French 100 portfolios. The results are shown in Tables 6 and 7, respectively.

For the Fama-French 6 portfolios, the results are consistent with LN. Two out of six portfolios are not be rejected. However, for the Fama-French 100 portfolios, 55 portfolios pass our test at the 5% significant level, while the method in LN suggests that only 13 portfolios are supported by the conditional CAPM. Moreover, it is very interesting to see that the portfolios passing the LN criterion have a high p-value in our test. That means that the LN method rejects the

For details of industry classification, please refer to Professor Kenneth French's web site.

We also report the monthly window results in the appendix. The choice of window size has minor impact on the results. Theoretically, an optimal window size could be developed according to certain criterion. Here we just use quarterly window for its economic significance. We thank the anonymous referee for pointing out the importance of window size selection.

Table 5: Estimation and testing results for Fama-French 25 portfolios

This table summarizes the estimation and testing results for the Fama-French 25 Portfolios. The first column indicates the order of the portfolios. The unconditional pricing error, $\hat{\alpha}^u$, with its standard error and t-statistic are reported in the next three columns. The covariances between the conditional betas and equity premium, $\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$, and the corresponding ratio over the pricing errors are shown in the fifth and the sixth columns. The last two columns summarize the statistic for our test and its p-value.

Portfolio	$\hat{\alpha}^u$	$se.(\hat{\alpha}^u)$	t-stat.	$\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$	$\hat{\delta}$	Q_T	$p(\hat{Q}_T)$
S1/B1	-0.0211	0.0070	-3.0013	-0.0072	0.3396	-0.0147	0.0000
S1/B2	0.0090	0.0059	1.5123	-0.0026	-0.2878	0.0110	0.0000
S1/B3	0.0142	0.0052	2.7058	0.0003	0.0218	0.0134	0.0000
S1/B4	0.0236	0.0051	4.6199	0.0007	0.0305	0.0229	0.0000
S1/B5	0.0285	0.0055	5.2032	0.0013	0.0451	0.0271	0.0000
S2/B1	-0.0090	0.0059	-1.5273	-0.0061	0.6710	-0.0043	0.2111
S2/B2	0.0064	0.0048	1.3122	-0.0002	-0.0376	0.0063	0.0503
S2/B3	0.0187	0.0047	4.0051	0.0001	0.0066	0.0189	0.0000
S2/B4	0.0196	0.0047	4.1547	0.0016	0.0820	0.0187	0.0000
S2/B5	0.0207	0.0058	3.5635	0.0015	0.0722	0.0196	0.0000
S3/B1	-0.0065	0.0051	-1.2735	-0.0053	0.8276	-0.0016	0.8744
S3/B2	0.0107	0.0040	2.6974	-0.0007	-0.0611	0.0113	0.0000
S3/B3	0.0138	0.0040	3.4518	-0.0008	-0.0576	0.0149	0.0000
S3/B4	0.0187	0.0043	4.3719	0.0017	0.0905	0.0173	0.0000
S3/B5	0.0253	0.0051	4.9610	0.0009	0.0341	0.0244	0.0000
S4/B1	0.0003	0.0040	0.0733	-0.0026	-8.9447	0.0026	0.0704
S4/B2	0.0019	0.0033	0.5884	-0.0008	-0.4145	0.0032	0.0201
S4/B3	0.0091	0.0038	2.4289	0.0004	0.0425	0.0094	0.0000
S4/B4	0.0171	0.0041	4.2047	0.0022	0.1280	0.0146	0.0000
S4/B5	0.0158	0.0053	2.9896	0.0003	0.0180	0.0152	0.0000
S5/B1	-0.0025	0.0030	-0.8450	0.0005	-0.2103	-0.0042	0.0000
S5/B2	0.0013	0.0030	0.4284	0.0023	1.8292	-0.0015	0.1809
S5/B3	0.0003	0.0038	0.0825	0.0015	4.7965	-0.0008	0.6935
S5/B4	0.0049	0.0044	1.1331	0.0021	0.4218	0.0031	0.2915
S5/B5	0.0063	0.0056	1.1201	0.0040	0.6367	0.0016	0.5628

Table 6: Estimation and testing results for Fama-French 6 portfolios

This table summarizes the estimation and testing results for the Fama-French 6 Portfolios. The first column indicates the order of the portfolios. They are sorted by size (S) and book-to-market ratio (B/M). The unconditional pricing error, $\hat{\alpha}^u$, with its standard error and t-statistic are reported in the next three columns. The covariance between the conditional betas and equity premium, $\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$, and its ratio over the pricing error are shown in the fifth and the sixth columns. The last two columns summarize the statistic for our test and its p-value.

Portfolio	$\hat{\alpha}^u$	$se.(\hat{\alpha}^u)$	t-stat.	$\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$	$\hat{\delta}$	Q_T	$p(\hat{Q}_T)$
S1/B1	-0.0067	0.0052	-1.2961	-0.0051	0.7614	-0.0024	0.2613
S1/B2	0.0169	0.0042	4.0390	0.0000	0.0029	0.0169	0.0000
S1/B3	0.0237	0.0047	4.9988	0.0016	0.0662	0.0223	0.0000
S2/B1	-0.0020	0.0021	-0.9337	0.0001	-0.0585	-0.0030	0.0000
S2/B2	0.0023	0.0026	0.8888	0.0018	0.7491	0.0009	0.3719
S2/B3	0.0097	0.0042	2.3040	0.0027	0.2796	0.0072	0.0101

Table 7: Estimation and testing results for Fama-French 100 portfolios

This table summarizes the estimation and testing results for the Fama-French 100 Portfolios. We only report the results of the portfolios which have different testing results for the LN procedure and for our testing procedure. The first column indicates the order of the portfolios. They are sorted by size (S) and book-to-market ratio (B/M). The unconditional pricing error, $\hat{\alpha}^u$, with its standard error and t-statistic are reported in the next three columns. The covariance between the conditional beta and risk premium, $\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$, and its ratio over the pricing error are shown in the fifth and the sixth columns. The last two columns summarize the statistic for our test and its p-value.

Portfolio	$\hat{\alpha}^u$	$se.(\hat{\alpha}^u)$	t-stat.	$\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$	$\hat{\delta}$	Q_T	$p(\hat{Q}_T)$
S1/B2	-0.0134	0.0076	-1.7746	-0.0072	0.5369	-0.0075	0.5427
S1/B3	0.0019	0.0073	0.2662	-0.0053	-2.7304	0.0064	0.4221
S1/B4	0.0136	0.0069	1.9750	-0.0034	-0.2530	0.0158	0.0603
S1/B5	0.0079	0.0064	1.2268	-0.0031	-0.3991	0.0099	0.0603
S2/B2	-0.0099	0.0074	-1.3321	-0.0069	0.6903	-0.0037	0.4322
S2/B3	-0.0026	0.0071	-0.3590	-0.0028	1.0955	-0.0004	0.8844
S2/B4	-0.0035	0.0067	-0.5219	-0.0029	0.8259	-0.0007	0.9045
S2/B5	0.0056	0.0064	0.8834	-0.0014	-0.2451	0.0076	0.2010
S3/B2	-0.0004	0.0075	-0.0508	-0.0060	15.7271	0.0053	0.6834
S3/B3	0.0034	0.0067	0.5096	-0.0000	-0.0132	0.0036	0.3015
S3/B4	0.0014	0.0065	0.2222	-0.0001	-0.0820	0.0013	0.8241
S3/B5	0.0077	0.0062	1.2372	-0.0011	-0.1456	0.0093	0.0704
S3/B7	0.0087	0.0061	1.4126	0.0014	0.1608	0.0083	0.1206
S4/B1	-0.0162	0.0076	-2.1493	-0.0078	0.4785	-0.0112	0.1106
S4/B2	-0.0043	0.0068	-0.6284	-0.0012	0.2832	-0.0042	0.5327
S4/B3	-0.0054	0.0063	-0.8573	-0.0023	0.4214	-0.0038	0.3417
S4/B4	0.0051	0.0061	0.8333	-0.0014	-0.2728	0.0068	0.3719
S4/B10	0.0005	0.0090	0.0567	-0.0017	-3.3608	0.0046	0.6332
S5/B1	-0.0118	0.0071	-1.6560	-0.0065	0.5506	-0.0075	0.2111
S5/B2	-0.0043	0.0062	-0.6929	-0.0029	0.6605	-0.0012	0.7638
S5/B3	0.0062	0.0060	1.0310	-0.0009	-0.1423	0.0074	0.2312
S5/B5	0.0055	0.0056	0.9705	-0.0016	-0.2896	0.0074	0.3417
S5/B7	0.0134	0.0061	2.2057	0.0036	0.2660	0.0107	0.0603
S6/B2	0.0002	0.0057	0.0298	-0.0043	-24.8443	0.0053	0.2714
S6/B3	0.0039	0.0055	0.7167	-0.0008	-0.1981	0.0050	0.2010
S6/B4	0.0064	0.0053	1.2147	-0.0012	-0.1856	0.0071	0.0603
S7/B1	-0.0021	0.0061	-0.3359	-0.0029	1.4226	-0.0014	0.6533
S7/B2	0.0025	0.0052	0.4826	-0.0028	-1.1134	0.0050	0.0804
S7/B3	0.0021	0.0048	0.4418	-0.0014	-0.6381	0.0040	0.1508
S7/B4	-0.0013	0.0050	-0.2532	-0.0023	1.7970	0.0023	0.4422
S7/B5	0.0025	0.0052	0.4738	-0.0004	-0.1526	0.0045	0.3116
S7/B8	0.0114	0.0059	1.9244	0.0002	0.0170	0.0117	0.0503
S8/B1	-0.0039	0.0056	-0.6898	-0.0014	0.3667	-0.0038	0.3719
S8/B2	-0.0018	0.0045	-0.4046	-0.0019	1.0171	0.0001	0.8643
S8/B3	0.0008	0.0048	0.1736	-0.0020	-2.4011	0.0032	0.1005
S8/B4	-0.0067	0.0051	-1.3027	-0.0010	0.1489	-0.0045	0.0804
S8/B6	0.0037	0.0054	0.6935	0.0001	0.0285	0.0048	0.3015
S8/B9	0.0092	0.0068	1.3430	-0.0006	-0.0678	0.0103	0.1106
S9/B1	-0.0061	0.0050	-1.2290	-0.0010	0.1646	-0.0058	0.1709
S9/B2	-0.0019	0.0042	-0.4484	-0.0018	0.9571	0.0006	0.9447
S9/B3	-0.0001	0.0042	-0.0130	-0.0021	38.1143	0.0027	0.3920
S9/B4	0.0051	0.0046	1.1115	0.0014	0.2769	0.0042	0.1608
S9/B5	0.0064	0.0047	1.3621	0.0013	0.2038	0.0059	0.2211
S9/B6	0.0049	0.0048	1.0140	0.0007	0.1476	0.0036	0.4523
S9/B8	0.0084	0.0050	1.6633	0.0011	0.1350	0.0069	0.2412
S9/B9	0.0118	0.0061	1.9190	0.0005	0.0457	0.0106	0.0503
S10/B2	-0.0001	0.0039	-0.0139	0.0013	-23.9112	-0.0022	0.5628
S10/B3	0.0027	0.0044	0.6234	0.0027	0.9909	-0.0010	0.7437
S10/B4	-0.0016	0.0048	-0.3425	0.0009	-0.5559	-0.0014	0.5829
S10/B5	-0.0091	0.0054	-1.6862	-0.0008	0.0833	-0.0071	0.0603
S10/B6	-0.0012	0.0053	-0.2267	0.0027	-2.2538	-0.0036	0.3819
S10/B7	0.0011	0.0058	0.1893	0.0010	0.9064	0.0002	0.8241
S10/B8	-0.0055	0.0072	-0.7656	-0.0015	0.2683	-0.0012	0.7739
S10/B9	0.0042	0.0087	0.4846	0.0032	0.7708	0.0021	0.7638
S10/B10	-0.0002	0.0113	-0.0170	0.0037	-19.4032	-0.0056	0.6332

conditional CAPM too often, as revealed by our simulation results.

4.2.2 SMB, HML, MOM portfolios

We also apply the test to the SMB, HML, and MOM factors portfolios. The first column in Table 8 reports the unconditional CAPM regression results. The p-value of the unconditional

Table 8: Estimation and testing results for Fama-French factors

This table summarizes the estimation and testing results for the Fama-French three factor portfolios. The first column indicates the portfolio. The unconditional pricing error, $\hat{\alpha}^u$, with the standard error and t-statistic are reported in the next three columns. The covariance between the conditional beta and equity premium, $\widehat{\text{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$, and its ratio over the pricing error are shown in the fifth and the sixth columns, respectively. The last two columns summarize the statistic for our test and its p-value.

Portfolio	$\hat{\alpha}^u$	$se(\hat{\alpha}^u)$	t-stat.	p-value	$\widehat{\text{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$	$\hat{\delta}$	Q_T	$p(\hat{Q}_T)$
SMB	0.0079	0.0046	1.7051	0.0913	-0.0026	-0.3285	0.0105	0.0000
HML	0.0217	0.0044	4.8833	0.0000	0.0046	0.2121	0.0181	0.0000
MOM	0.0333	0.0070	4.7845	0.0000	-0.0000	-0.0003	0.0292	0.0000

alpha for the SMB factor is 0.0913, which means the unconditional alpha is insignificant at 5% level, while both of the other two unconditional alphas are significantly not equal to zero. The findings are consistent with several empirical studies in the literature, such as LN and Ang and Chen[1]. The size effect is not statistically significant either, while neither HML nor MOM can be explained by the original unconditional CAPM.

To test the conditional CAPM statistically, we use the Bootstrap procedure as proposed above. The last two columns in Table 8 report the statistic and its p-value, respectively. The results somewhat support the findings in LN. Clearly, the variation of the conditional beta is too small to explain the pricing error. The unconditional alpha of the SMB factor is about 0.0079% daily, which is not statistically significant, while the numbers increase to 0.0217% and 0.0333% for the HML and MOM factors, and both are significantly not zero at 5% level. This provides a strong evidence to support value effect and momentum effect. The p-values in Table 8 are all less than 5%. This result is consistent with the finding in LN that the conditional CAPM fails to explain the SMB, HML, and MOM factors.

4.2.3 Industry portfolios

We also want to check whether the conditional CAPM can explain the returns of industry portfolios. The results are reported in Tables 9 and 10, respectively, from which we see that, for most industry portfolios, the unconditional alphas and covariances between beta and the market premium are not close to each other. Most of the ratios lie between -30% and 30%. That means that for those portfolios, the estimated covariance can not explain the unconditional alpha. But according to our Bootstrap test Q_T, we can not reject the conditional CAPM for more than half of these portfolios. Another empirical finding worth noting is that several industry portfolios have insignificant unconditional pricing errors while have significant Q_T.

Industry portfolios are considered not very helpful in asset pricing test since the simple CAPM seemly explain the time-series variation quit well in the literature. But out test suggest that even the conditional CAPM under our assumptions can not explain the time-series variation pattern of several industry portfolios. We should look into the time-series variation of return and covariance more carefully.

Table 9: Estimation and testing results for 5 industry portfolios

This table summarizes the estimation and testing results for 5 industry portfolios. The first column indicates the portfolios. The unconditional pricing error, $\hat{\alpha}^u$, with its standard error and t-statistic are reported in the next three columns. The covariance between the conditional betas and equity premium, $\widehat{\text{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$, and its ratio over the pricing error are shown in the fifth and sixth columns. The last two columns summarize the statistic for our test and its p-value.

Industry	$\hat{\alpha}^u$	$se.(\hat{\alpha}^u)$	t-stat.	$\widehat{\text{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$	$\hat{\delta}$	Q_T	$p(\hat{Q}_T)$
Cnsmr	0.0066	0.0035	1.9034	0.0025	0.3751	0.0024	0.1709
Manuf	0.0046	0.0035	1.2905	0.0003	0.0695	0.0043	0.0000
HiTec	-0.0032	0.0047	-0.6788	-0.0001	0.0364	-0.0032	0.4221
Hlth	0.0113	0.0059	1.9174	0.0011	0.1010	0.0081	0.0503
Other	-0.0008	0.0038	-0.2174	-0.0006	0.7614	0.0007	0.5528

Table 10: Estimation and testing results for 10 industry portfolios

This table summarizes the estimation results for 10 industry portfolios. The first column indicates the portfolios. The unconditional pricing error, $\hat{\alpha}^u$, with its standard error and t-statistic are reported in the next three columns. The covariance between the conditional betas and equity premium, $\widehat{\text{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$, and its ratio over the pricing error are shown in the fifth and sixth columns. The last two columns summarize the statistic for our test and its p-value.

Industry	$\hat{\alpha}^u$	$se(\hat{\alpha}^u)$	t-stat.	$\widehat{\text{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$	$\hat{\delta}$	Q_T	$p(\hat{Q}_T)$
NoDur	0.0143	0.0044	3.2791	0.0031	0.2137	0.0098	0.0000
Durbl	-0.0057	0.0066	-0.8622	0.0045	-0.7934	-0.0110	0.0000
Manuf	0.0030	0.0032	0.9095	0.0012	0.3979	0.0012	0.4925
Enrgy	0.0103	0.0081	1.2706	-0.0001	-0.0065	0.0109	0.0000
HiTec	-0.0024	0.0064	-0.3692	-0.0003	0.1310	-0.0035	0.5126
Telcm	0.0019	0.0060	0.3196	0.0013	0.6764	0.0021	0.3618
Shops	0.0066	0.0048	1.3801	0.0008	0.1280	0.0047	0.3417
Hlth	0.0113	0.0059	1.9174	0.0011	0.1010	0.0081	0.0402
Utils	0.0064	0.0057	1.1272	-0.0011	-0.1709	0.0083	0.0000
Other	-0.0008	0.0038	-0.2174	-0.0006	0.7614	0.0007	0.4925

§5 Robustness analysis

To check the robustness of the testing results based on our testing procedure, we also perform the proposed test on two subsamples, one from July 1963 to December 1989, another from January 1990 to December 2014. The summary of the two subsample results is reported in Table 12.

Table 11: Estimation and testing results for 10 industry portfolios

This table summarizes the estimation results for 10 industry portfolios. The first column indicates the portfolios. The unconditional pricing error, $\hat{\alpha}^u$, with its standard error and t-statistic are reported in the next three columns. The covariance between the conditional betas and equity premium, $\widehat{\text{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$, and its ratio over the pricing error are shown in the fifth and sixth columns. The last two columns summarize the statistic for our test and its p-value.

	$\hat{\alpha}^u$	$se(\hat{\alpha}^u)$	t-stat.	$\widehat{\text{Cov}}(\hat{\beta}^\tau, \hat{\gamma}^\tau)$	δ	$\hat{Q}_T$	$p(\hat{Q}_T)$
Food	0.0145	0.0053	2.7379	0.0018	0.1273	0.0116	0.0000
Beer	0.0189	0.0081	2.3498	0.0047	0.2491	0.0119	0.0000
Smoke	0.0308	0.0106	2.9122	0.0055	0.1787	0.0227	0.0000
Games	0.0042	0.0081	0.5184	-0.0033	-0.7952	0.0060	0.0402
Books	0.0019	0.0061	0.3181	0.0028	1.4686	0.0004	0.7437
Hshld	0.0051	0.0063	0.8080	0.0017	0.3428	0.0006	0.5829
Clths	0.0066	0.0071	0.9340	-0.0005	-0.0755	0.0077	0.0201
Hlth	0.0113	0.0059	1.9174	0.0011	0.1010	0.0081	0.0704
Chems	0.0020	0.0062	0.3261	0.0025	1.2296	-0.0004	0.6935
Txtls	0.0057	0.0087	0.6597	0.0014	0.2475	0.0054	0.2613
Cnstr	-0.0005	0.0060	-0.0827	0.0022	-4.4542	-0.0021	0.1809
Steel	-0.0135	0.0088	-1.5435	-0.0012	0.0881	-0.0101	0.0000
FabPr	-0.0010	0.0055	-0.1834	0.0005	-0.4658	-0.0004	0.7839
ElcEq	0.0100	0.0065	1.5378	0.0020	0.2017	0.0077	0.2412
Autos	-0.0070	0.0081	-0.8730	0.0044	-0.6237	-0.0121	0.0000
Carry	0.0099	0.0073	1.3490	-0.0016	-0.1623	0.0099	0.0000
Mines	0.0042	0.0123	0.3432	-0.0023	-0.5548	0.0082	0.0905
Coal	0.0109	0.0164	0.6643	-0.0028	-0.2533	0.0179	0.0201
Oil	0.0106	0.0082	1.2888	-0.0002	-0.0143	0.0111	0.0000
Util	0.0064	0.0057	1.1272	-0.0011	-0.1709	0.0083	0.0000
Telcm	0.0019	0.0060	0.3196	0.0013	0.6764	0.0021	0.3317
Servs	0.0032	0.0056	0.5743	-0.0032	-0.9732	0.0057	0.1206
BusEq	-0.0036	0.0074	-0.4867	-0.0006	0.1586	-0.0042	0.4121
Paper	0.0035	0.0053	0.6641	0.0019	0.5393	0.0012	0.9648
Trans	0.0019	0.0063	0.3084	-0.0004	-0.2087	0.0003	0.6935
Whlsl	0.0072	0.0054	1.3392	-0.0018	-0.2430	0.0073	0.0302
Rtail	0.0074	0.0058	1.2667	0.0015	0.2029	0.0050	0.4020
Meals	0.0134	0.0077	1.7499	-0.0037	-0.2740	0.0145	0.0000
Fin	0.0007	0.0052	0.1405	0.0008	1.0487	0.0023	0.0704
Other	-0.0076	0.0062	-1.2377	-0.0045	0.5849	-0.0031	0.6231

The results in Table 12 for subsamples show clearly that the conditional CAPM has much more power for the second subsample. We fail to reject the conditional CAPM hypothesis for 19 out of 25 size and book-to-market sorted portfolios. To look into the individual portfolios, we also report the detailed test results on Fama-French 25 portfolios in Table 13 for the first subsample and Table 14 for the second subsample, respectively. All other results are also available upon request. The test results on individual portfolios show clearly the pattern of rejection. From both Tables 13 and 14, one can see clearly that most portfolios well explained by the conditional CAPM for the first subsample are still well explained for the later subsample. Meanwhile, the conditional CAPM model gains the explanation power over some other portfolios for the second subsample. This finding is consistent with the phenomenon documented in the literature that anomalies tend to fade away after they are studied.

Table 12: Test results comparison between our method and the LN method for two subsamples

This table reports the numbers of portfolios that fail to reject the conditional CAPM at the given significance level for subsamples. The betas are assumed to be stable within one quarter window. The data are taken from Kenneth French's web site.

	Our Test			LN	Our Test			LN
	1%	5%	10%		1%	5%	10%	
	Jul. 1963 - Dec. 1989				Dec. 1989 - Dec. 2014			
Fama-French 6 portfolios	2	2	2	0	5	3	3	2
Fama-French 25 portfolios	9	8	7	5	19	17	16	4
Fama-French 100 portfolios	68	57	48	13	91	76	69	12
5 industry portfolios	5	5	2	1	5	3	3	0
10 industry portfolios	8	4	4	0	9	8	7	1
30 industry portfolios	27	20	16	2	29	27	23	5
SMB, HML, MOM factors	0	0	0	0	0	0	0	0

§6 Conclusion

In order to discuss the validity of the conditional CAPM, LN propose a comparison of the covariance of beta and the risk premium with the unconditional price error. Their method is intuitive but not statistically applicable. We propose a statistic based on their argument and provide a direct Bootstrap method to test the validity of the conditional CAPM.

Based on our simulations, we find that even when the true DGP follows the conditional CAPM, there is a large chance that the method in LN tends to over-reject the model, which highlights the importance of our testing procedure. Indeed, our test has good size and power in a finite sample. After applying our method to several popular data sets, we find that for many portfolios, we can not find an evidence against the conditional CAPM from the perspective of LN. Test results on a wide range of size, book-to-market sorted and industrial portfolios also show the conditional CAPM can provide a better explanation to real applications.

Table 13: Estimation and testing results for Fama-French 25 portfolios for the first subsample

This table summarizes the estimation and testing results for the Fama-French 25 Portfolios. The first column indicates the order of the portfolios. The unconditional pricing error, $\hat{\alpha}^u$, with its standard error and t-statistic are reported in the next three columns. The covariances between the conditional betas and equity premium, $\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$, and the corresponding ratio over the pricing errors are shown in the fifth and the sixth columns. The last two columns summarize the statistic for our test and its p-value.

Portfolio	$\hat{\alpha}^u$	$se.(\hat{\alpha}^u)$	t-stat.	$\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$	$\hat{\delta}$	Q_T	$p(\hat{Q}_T)$
S1/B1	-0.0124	0.0091	-1.3686	-0.0115	0.9266	-0.0020	0.7839
S1/B2	0.0102	0.0077	1.3246	-0.0063	-0.6127	0.0153	0.0000
S1/B3	0.0156	0.0070	2.2255	-0.0034	-0.2176	0.0178	0.0000
S1/B4	0.0257	0.0067	3.8204	-0.0024	-0.0915	0.0269	0.0000
S1/B5	0.0330	0.0069	4.7680	-0.0007	-0.0227	0.0324	0.0000
S2/B1	-0.0081	0.0075	-1.0712	-0.0101	1.2582	0.0012	0.9749
S2/B2	0.0106	0.0064	1.6748	-0.0024	-0.2222	0.0122	0.0000
S2/B3	0.0224	0.0060	3.7419	-0.0030	-0.1360	0.0245	0.0000
S2/B4	0.0266	0.0056	4.7379	-0.0007	-0.0255	0.0265	0.0000
S2/B5	0.0323	0.0067	4.8460	-0.0006	-0.0176	0.0315	0.0000
S3/B1	-0.0063	0.0062	-1.0174	-0.0076	1.2048	0.0008	0.6633
S3/B2	0.0125	0.0053	2.3528	-0.0020	-0.1566	0.0141	0.0000
S3/B3	0.0160	0.0051	3.1222	-0.0042	-0.2615	0.0195	0.0000
S3/B4	0.0249	0.0049	5.0382	-0.0005	-0.0191	0.0245	0.0000
S3/B5	0.0271	0.0062	4.3520	0.0000	0.0001	0.0263	0.0000
S4/B1	-0.0045	0.0045	-1.0097	-0.0037	0.8145	-0.0006	0.8543
S4/B2	-0.0027	0.0042	-0.6467	-0.0042	1.5571	0.0020	0.3417
S4/B3	0.0132	0.0043	3.0418	-0.0028	-0.2092	0.0159	0.0000
S4/B4	0.0207	0.0048	4.3381	0.0019	0.0941	0.0177	0.0000
S4/B5	0.0235	0.0062	3.7687	-0.0004	-0.0151	0.0227	0.0000
S5/B1	-0.0070	0.0042	-1.6896	0.0023	-0.3203	-0.0093	0.0000
S5/B2	-0.0038	0.0037	-1.0244	0.0023	-0.5976	-0.0055	0.0201
S5/B3	-0.0011	0.0047	-0.2429	0.0007	-0.5810	-0.0016	0.6131
S5/B4	0.0095	0.0049	1.9272	0.0023	0.2408	0.0071	0.0704
S5/B5	0.0097	0.0064	1.5050	0.0043	0.4479	0.0045	0.3015

Table 14: Estimation and testing results for Fama-French 25 portfolios for the second subsample

This table summarizes the estimation and testing results for the Fama-French 25 Portfolios. The first column indicates the order of the portfolios. The unconditional pricing error, $\hat{\alpha}^u$, with its standard error and t-statistic are reported in the next three columns. The covariances between the conditional betas and equity premium, $\widehat{\text{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$, and the corresponding ratio over the pricing errors are shown in the fifth and the sixth columns. The last two columns summarize the statistic for our test and its p-value.

Portfolio	$\hat{\alpha}^u$	$se.(\hat{\alpha}^u)$	t-stat.	$\widehat{\text{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$	$\hat{\delta}$	Q_T	$p(\hat{Q}_T)$
S1/B1	-0.0290	0.0108	-2.6735	-0.0025	0.0871	-0.0265	0.0000
S1/B2	0.0091	0.0092	0.9881	0.0012	0.1328	0.0078	0.5025
S1/B3	0.0139	0.0079	1.7516	0.0043	0.3088	0.0097	0.2111
S1/B4	0.0226	0.0079	2.8810	0.0039	0.1740	0.0195	0.0000
S1/B5	0.0249	0.0086	2.9040	0.0034	0.1374	0.0225	0.0000
S2/B1	-0.0095	0.0092	-1.0334	-0.0018	0.1940	-0.0096	0.0603
S2/B2	0.0027	0.0074	0.3667	0.0014	0.5107	0.0008	0.5427
S2/B3	0.0158	0.0073	2.1632	0.0026	0.1632	0.0137	0.0000
S2/B4	0.0131	0.0077	1.7143	0.0028	0.2150	0.0110	0.1106
S2/B5	0.0093	0.0096	0.9766	0.0037	0.3913	0.0072	0.5427
S3/B1	-0.0060	0.0081	-0.7384	-0.0024	0.4066	-0.0043	0.3719
S3/B2	0.0096	0.0059	1.6270	0.0006	0.0620	0.0087	0.1407
S3/B3	0.0126	0.0062	2.0265	0.0024	0.1910	0.0104	0.0101
S3/B4	0.0125	0.0070	1.7911	0.0028	0.2234	0.0105	0.0101
S3/B5	0.0234	0.0082	2.8639	0.0025	0.1076	0.0217	0.0000
S4/B1	0.0053	0.0068	0.7820	-0.0015	-0.2740	0.0054	0.1407
S4/B2	0.0072	0.0051	1.3899	0.0029	0.4002	0.0044	0.1206
S4/B3	0.0058	0.0062	0.9388	0.0032	0.5583	0.0032	0.6432
S4/B4	0.0133	0.0066	2.0032	0.0029	0.2175	0.0111	0.0000
S4/B5	0.0075	0.0087	0.8674	0.0033	0.4409	0.0058	0.2412
S5/B1	0.0020	0.0041	0.4766	-0.0008	-0.3813	0.0014	0.3015
S5/B2	0.0069	0.0046	1.4986	0.0029	0.4301	0.0027	0.2412
S5/B3	0.0015	0.0060	0.2462	0.0028	1.8958	-0.0006	0.7035
S5/B4	-0.0007	0.0073	-0.0916	0.0034	-5.0340	-0.0030	0.6332
S5/B5	0.0027	0.0094	0.2880	0.0045	1.6506	-0.0023	0.6332

Appendix

A: Mathematical derivations

Here we provide the detailed proof of the asymptotic property of Q_T. The model specification is given in Section 3.1. To derive the asymptotic property of Q_T, we need the following assumptions.

A1. The disturbances $\eta_h \sim i.i.d.(0, \sigma_\eta^2)$ and $\varepsilon_t \sim i.i.d.(0, \sigma_\varepsilon^2)$ where $\sigma_\eta^2 < \infty$ and $\sigma_\varepsilon^2 < \infty$.

A2. The cross-window disturbance u_h follows independent distribution $(0, \sigma_{u_h}^2)$, where $\mathrm{Var}(u_h) = \sigma_{u_h}^2 < \infty$ and $\mathrm{Cov}(\eta_h, u_h) = \sigma_{\eta u_h}$. Note we allow conditional heteroskedasticity here. Denote $E[\sigma_{\eta u_h}] = \sigma_{\eta u}$ and $E[\sigma_{u_h}^2] = \sigma_u^2$. Also, assume that $E[u_h^4] < \infty$ and $\bar{\sigma}_{u_h^2}^2 = \frac{1}{H}\sum_{h=1}^{H}(E[u_h^4] - \sigma_{u_h}^4) < \infty$. Assume the Lindeberg condition holds for u_h.

A3. v_t is i.i.d. in each window W_h with variance $\sigma_{v_h}^2$ and $\mathrm{Cov}(\eta^h, \sigma_{v_h}^2) = \sigma_{\eta v_h^2}$. Denote $E[\sigma_{v_h}^2] = \sigma_v^2$ and $E[\sigma_{\eta v_h^2}] = \sigma_{\eta v^2}$. Also, assume that $E[v_t^4] < \infty$ and $\sigma_{v_h}^2$ also satisfy the Lindeberg condition.

Run OLS over the time series to get conditional α and β for each window h:

$$\begin{aligned}
\hat{\beta}_h &= \frac{\sum_{t \in W_h}[(R_t - \bar{R}_h)(R_{Mt} - \bar{R}_{M,h})]}{\sum_{t \in W_h}[(R_{Mt} - \bar{R}_{M,h})^2]} \\
&= \frac{\sum_{t \in W_h}[((\beta_t R_{Mt} + \varepsilon_t) - (\beta_t \bar{R}_{M,h} + \bar{\varepsilon}_h))(R_{Mt} - \bar{R}_{M,h})]}{\sum_{t \in W_h}[(R_{Mt} - \bar{R}_{M,h})^2]} \\
&= \beta_h + \frac{\sum_{t \in W_h}[(R_{Mt} - \bar{R}_{M,h})(\varepsilon_t - \bar{\varepsilon}_h)]}{\sum_{t \in W_h}[(R_{Mt} - \bar{R}_{M,h})^2}] \\
&= \beta_h + \frac{\sum_{t \in W_h}[(v_t - \bar{v}_h)(\varepsilon_t - \bar{\varepsilon}_h)]}{\sum_{t \in W_h}[(v_t - \bar{v}_h)^2]} = \beta_h + e_{\beta_h} = \beta + \eta_h + e_{\beta_h},
\end{aligned}$$

where $e_{\beta_h} = \sum_{t \in W_h}[(v_t - \bar{v}_h)(\varepsilon_t - \bar{\varepsilon}_h)] / \sum_{t \in W_h}[(v_t - \bar{v}_h)^2] \sim N(0, \sigma_\epsilon^2 / w\sigma_v^2)$. Run unconditional regression to obtain

$$\begin{aligned}
\hat{\beta}^u &= \frac{\sum_{t=1}^{T}(R_t - \bar{R})(R_{Mt} - \bar{R}_M)}{\sum_{t=1}^{T}(R_{Mt} - \bar{R}_M)^2} \\
&= \frac{\sum_{t=1}^{T}((\beta + \eta_t)R_{Mt} + \varepsilon_t - (\beta \bar{R}_M + \frac{1}{T}\sum_{t=1}^{T}(\eta_t(\gamma + u_t + v_t)) + \bar{\varepsilon}))(R_{Mt} - \bar{R}_M)}{\sum_{t=1}^{T}(R_{Mt} - \bar{R}_M)^2} \\
&= \beta + \frac{\sum_{t=1}^{T}(\eta_t R_{Mt} - \frac{1}{T}\sum_{t=1}^{T}\eta_t R_{Mt} + \varepsilon_t - \bar{\varepsilon})(R_{Mt} - \bar{R}_M)}{\sum_{t=1}^{T}(R_{Mt} - \bar{R}_M)^2} \\
&= \beta + \frac{\sum_{t=1}^{T}(\eta_t R_{Mt} - \frac{1}{T}\sum_{t=1}^{T}\eta_t R_{Mt})(R_{Mt} - \bar{R}_M)}{\sum_{t=1}^{T}(R_{Mt} - \bar{R}_M)^2} + \frac{\sum_{t=1}^{T}(\varepsilon_t - \bar{\varepsilon})(R_{Mt} - \bar{R}_M)}{\sum_{t=1}^{T}(R_{Mt} - \bar{R}_M)^2} \\
&= \beta + \underbrace{\frac{\widehat{\mathrm{Cov}}(\eta_t R_{Mt}, R_{Mt})}{S_M^2}}_{e_{\eta T}} + \underbrace{\frac{\widehat{\mathrm{Cov}}(\varepsilon_t, R_{Mt})}{S_M^2}}_{e_{\epsilon T}} = \beta + e_{\eta T} + e_{\varepsilon T},
\end{aligned}$$

where the definitions of $e_{\eta T}$ and $e_{\varepsilon T}$ are obvious. Hence,

$$\begin{aligned}\hat{\alpha}^u &= \bar{R} - \hat{\beta}^u \bar{R}_M = \beta \bar{R}_M + \frac{1}{T}\sum_{t=1}^{T}(\eta_t(\gamma + u_t + v_t)) + \bar{\varepsilon} - \hat{\beta}^u \bar{R}_M \\ &= \frac{1}{T}\sum_{t=1}^{T}\eta_t u_t + \frac{1}{T}\sum_{t=1}^{T}\eta_t v_t + \bar{\eta}\gamma + \bar{\varepsilon} - \bar{R}_M(e_{\eta T} + e_{\varepsilon T}) \\ &= \frac{1}{H}\sum_{h=1}^{H}\eta_h u_h + \frac{1}{H}\sum_{h=1}^{H}\eta_h \bar{v}_h + \bar{\eta}\gamma + \bar{\varepsilon} - \bar{R}_M(e_{\eta T} + e_{\varepsilon T}).\end{aligned}$$

Since

$$S_M^2 = \frac{1}{T}\sum_{t=1}^{T}(R_{Mt} - \frac{1}{T}\sum_{t=1}^{T}R_{Mt})^2 = \frac{1}{T}\sum_{t=1}^{T}(u_t + v_t - \frac{1}{T}\sum_{t=1}^{T}(u_t + v_t))^2 \xrightarrow{p} \sigma_u^2 + \sigma_v^2$$

with the convergence rate of $\sqrt{H}$. This conclusion follows from the Lindeberg-Feller central limit theorem. Also,

$$\begin{aligned}e_{\eta T} &= \frac{\hat{\mathrm{Cov}}(\eta_t R_{Mt}, R_{Mt})}{S_M^2} \\ &= \frac{1}{T}\frac{1}{S_M^2}\sum_{t=1}^{T}\left[\eta_t R_{Mt} - \frac{1}{T}\sum_{t=1}^{T}(\eta_t R_{Mt})\right]\left[R_{Mt} - \frac{1}{T}\sum_{t=1}^{T}R_{Mt}\right] \\ &= \frac{1}{T}\frac{1}{S_M^2}\sum_{t=1}^{T}\left[\gamma\eta_t u_t + \eta_t u_t^2 - (\gamma\bar{\eta} + \overline{\eta u})(u_t - \bar{u}) - \gamma\eta_t\bar{u} + \eta_t u_t \bar{u} + \gamma\eta_t v_t^2\right] + P(v_t) \\ &= \frac{1}{H}\frac{1}{S_M^2}\sum_{h=1}^{H}\left[\gamma\eta_h u_h + \eta_h {u_h}^2 + \gamma\eta_h s_{v_h}^2 - (\gamma\bar{\eta} + \overline{\eta u})(u_t - \bar{u}) - \gamma\eta_h\bar{u} + \eta_h u_h \bar{u}\right] + P(v_t),\end{aligned}$$

where $\overline{\eta u} \equiv \frac{1}{T}\sum_{t=1}^{T}(\eta_t u_t)$, and $P(v_t)$ is a polynomial of v_t. It is easy to show that $P(v_t) \xrightarrow{p} 0$ with the convergence rate of $\sqrt{T}$. Then,

$$e_{\eta T} = \frac{1}{H}\frac{1}{S_M^2}\sum_{h=1}^{H}(\gamma\eta_h u_h + \eta_h {u_h}^2 - \overline{\eta u}u_h + \gamma\eta_h s_{v_h}^2) + O(\frac{1}{T}),$$

where $s_{v_h}^2 \equiv \frac{1}{w}\sum_{t\in W_h}(R_{Mt} - \bar{R}_{M,h})^2 \xrightarrow{p} \sigma_{v_h}^2$ with convergence rate $\sqrt{w}$. Using the same arguments, we can easily show that $e_{\varepsilon T} \xrightarrow{p} 0$ with the convergence rate of $\sqrt{T}$. Now, we can calculate $E[\hat{\alpha}^u]$ as follows

$$\begin{aligned}E[\hat{\alpha}^u] &= E\left[\frac{1}{H}\sum_{h=1}^{H}\eta_h u_h + \frac{1}{H}\sum_{h=1}^{H}\eta_h \bar{v}_h + \bar{\eta}\gamma + \bar{\varepsilon} - \bar{R}_M(e_{\eta T} + e_{\varepsilon T})\right] \\ &= E\left[\frac{1}{H}\sum_{h=1}^{H}\eta_h u_h - \bar{R}_M e_{\eta T}\right] \\ &= (1 - \frac{\gamma^2}{\sigma_u^2 + \sigma_v^2})\sigma_{\eta u} + \frac{\gamma}{\sigma_u^2 + \sigma_v^2}\sigma_{\eta u^2} + \frac{\gamma}{\sigma_u^2 + \sigma_v^2}\sigma_{\eta v^2} + O(\frac{1}{H}).\end{aligned}$$

Next, we consider the asymptotic property of $\hat{\alpha}^u$. To this end, we consider

$$\begin{aligned}
\sqrt{H}\hat{\alpha}^u &= \frac{1}{\sqrt{H}}\left[\sum_{h=1}^{H}(\eta_h u_h) + \gamma\sum_{h=1}^{H}\eta_h - H\bar{R}_M e_{\eta T} + \sum_{h=1}^{H}(\eta_h \bar{v}_h) + \frac{1}{w}\sum_{t=1}^{T}\varepsilon_t - H\bar{R}_M e_{\varepsilon T}\right] \\
&= \frac{1}{\sqrt{H}}\left[\sum_{h=1}^{H}(\eta_h u_h) + \gamma\sum_{h=1}^{H}\eta_h - H\bar{R}_M e_{\eta T} + \sum_{h=1}^{H}(\eta_h \bar{v}_h) + \frac{1}{w}\sum_{t=1}^{T}\varepsilon_t - H\bar{R}_M e_{\varepsilon T}\right] \\
&= \frac{1}{\sqrt{H}}\left[\sum_{h=1}^{H}(\eta_h u_h) + \gamma\sum_{h=1}^{H}\eta_h - \frac{\bar{R}_M}{S_M^2}\sum_{h=1}^{H}(\gamma\eta_h u_h + \eta_h {u_h}^2 - \overline{\eta u}u_h + \gamma\eta_h s_{v_h}^2)\right. \\
&\quad \left. + \sum_{h=1}^{H}(\eta_h \bar{v}_h) + \frac{1}{w}\sum_{t=1}^{T}\varepsilon_t - H\bar{R}_M e_{\varepsilon T}\right].
\end{aligned}$$

According to Slutsky's theorem, we can treat $\bar{R}_M$ and S_M^2 as constant in this equation. Then, by combining this with Assumptions A1 - A3, the Lindeberg-Feller central limit theorem holds true as

$$\sqrt{H}\,\frac{\hat{\alpha^u} - E[\hat{\alpha}^u]}{\sigma_\alpha} \xrightarrow{d} N(0,1),$$

where $\sigma_\alpha^2 \equiv \lim_{H\to\infty}\mathrm{Var}(\sqrt{H}\hat{\alpha}^u)$. By recalling the definition of Q_T in equation (4),

$$Q_T \equiv \hat{\alpha}^u - \left[1 - \frac{\hat{\gamma}^2}{\hat{\sigma}_M^2}\right]\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h) + \frac{\hat{\gamma}}{\hat{\sigma}_M^2}\widehat{\mathrm{Cov}}\left[\hat{\beta}_h, (\hat{\gamma}_h - \hat{\gamma})^2\right] + \frac{\hat{\gamma}}{\hat{\sigma}_M^2}\widehat{\mathrm{Cov}}(\hat{\beta}_h, s_{v_h}^2),$$

we see easily that Q_T is actually $\hat{\alpha}^u - \hat{E}[\hat{\alpha}^u]$, where $\hat{E}[\hat{\alpha}^u]$ is the sample analog of $E[\hat{\alpha}^u]$. Additionally, sample variance and covariance estimators are consistent estimators. Then, $\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$, $\widehat{\mathrm{Cov}}\left[\hat{\beta}_h, (\hat{\gamma}_h - \hat{\gamma})^2\right]$ and $\widehat{\mathrm{Cov}}(\hat{\beta}_h, s_{v_h}^2)$ are consistent estimators of the true values when $H \to \infty$. Meanwhile, $\hat{\gamma}, \hat{\sigma}_M^2$ also converges to the true value in probability. Then, using Slutsky's theorem, we have Q_H as asymptotically normal with the convergence rate of H. This proves equation (5).

B: Alternative stable window size

To see how the choice of the size of the rolling window affects the testing results, we repeat our procedures above by using the monthly window (21 trading days) instead of the quarterly window. The following tables (Tables 15 and 16) report the test results when betas are assumed stable with the monthly window. From both Tables 15 and 16, one can see clearly that the results for the monthly window are similar to those for the quarterly window although there are some differences.

Table 15: Test results comparison between our method and the LN method for the monthly window size

This table reports the numbers of portfolios that fail to reject the conditional CAPM at the given significance level. LN do not provide a formal test statistic and they only argue that the ratio of covariance between equity premium and unconditional alpha should be close to 1. Therefore, we take it as a rejection if the value of this ratio does not belong to $[0.5, 2]$. The betas are assumed to be stable within one month window. The data are taken from Kenneth French's web site and the sample period is from July 1963 to December 2014.

	Our Test			LN
	1%	5%	10%	
Fama-French 6 portfolios	3	3	2	2
Fama-French 25 portfolios	7	5	4	4
Fama-French 100 portfolios	61	43	35	17
5 industry portfolios	3	2	2	1
10 industry portfolios	6	4	4	2
30 industry portfolios	18	14	13	6
SMB, HML, MOM factors	0	0	0	0

Table 16: Estimation and testing results for Fama-French 25 portfolios for the monthly window size

This table summarizes the estimation and testing results for the Fama-French 25 Portfolios. The first column indicates the order of the portfolios. The unconditional pricing error, $\hat{\alpha}^u$, with its standard error and t-statistic are reported in the next three columns. The covariances between the conditional betas and equity premium, $\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$, and the corresponding ratio over the pricing errors are shown in the fifth and the sixth columns. The last two columns summarize the statistic for our test and its p-value.

Portfolio	$\hat{\alpha}^u$	$se.(\hat{\alpha}^u)$	t-stat.	$\widehat{\mathrm{Cov}}(\hat{\beta}_h, \hat{\gamma}_h)$	$\hat{\delta}$	Q_T	$p(\hat{Q}_T)$
S1/B1	-0.0210	0.0070	-2.9849	-0.0154	0.7357	-0.0057	0.0704
S1/B2	0.0092	0.0059	1.5540	-0.0074	-0.7984	0.0164	0.0000
S1/B3	0.0142	0.0052	2.7114	-0.0029	-0.2019	0.0168	0.0000
S1/B4	0.0237	0.0051	4.6290	-0.0015	-0.0633	0.0254	0.0000
S1/B5	0.0283	0.0055	5.1777	-0.0007	-0.0257	0.0291	0.0000
S2/B1	-0.0089	0.0059	-1.5095	-0.0131	1.4676	0.0032	0.1106
S2/B2	0.0065	0.0048	1.3421	-0.0029	-0.4501	0.0093	0.0201
S2/B3	0.0188	0.0047	4.0179	-0.0012	-0.0639	0.0204	0.0000
S2/B4	0.0197	0.0047	4.1626	0.0007	0.0342	0.0198	0.0000
S2/B5	0.0204	0.0058	3.5070	-0.0008	-0.0404	0.0218	0.0000
S3/B1	-0.0063	0.0051	-1.2545	-0.0087	1.3641	0.0021	0.0101
S3/B2	0.0109	0.0039	2.7549	-0.0027	-0.2529	0.0139	0.0000
S3/B3	0.0136	0.0040	3.4092	-0.0017	-0.1243	0.0157	0.0000
S3/B4	0.0186	0.0043	4.3589	0.0006	0.0305	0.0185	0.0000
S3/B5	0.0250	0.0051	4.9033	0.0000	0.0002	0.0250	0.0000
S4/B1	0.0004	0.0040	0.0932	-0.0041	-11.0465	0.0044	0.0000
S4/B2	0.0021	0.0033	0.6313	-0.0018	-0.8500	0.0045	0.0000
S4/B3	0.0092	0.0037	2.4448	-0.0012	-0.1327	0.0113	0.0000
S4/B4	0.0169	0.0041	4.1619	0.0011	0.0655	0.0156	0.0000
S4/B5	0.0154	0.0053	2.9244	-0.0007	-0.0482	0.0159	0.0000
S5/B1	-0.0026	0.0030	-0.8624	0.0026	-1.0072	-0.0066	0.0000
S5/B2	0.0014	0.0030	0.4821	0.0027	1.9099	-0.0018	0.0000
S5/B3	0.0004	0.0038	0.1084	0.0020	4.9088	-0.0012	0.6633
S5/B4	0.0047	0.0044	1.0739	0.0016	0.3438	0.0033	0.5628
S5/B5	0.0063	0.0056	1.1158	0.0030	0.4722	0.0026	0.4523

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