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Schrödinger格点系统在加权空间中的随机吸引子与随机指数吸引子

张思佳, 周盛凡*

(浙江师范大学 数学科学学院, 浙江金华 321004)

摘 要: 该文主要研究带可乘白噪声和弱阻尼的非自治Schrödinger格点系统在无穷序列加权空间(元素的各分量在其范数的权重不完全相同)中的随机吸引子和随机指数吸引子的存在性. 文中的研究表明在一定条件下原来无穷维系统的解的极限行为可以用有限个参数来描述, 即原来无穷维的系统最终可退化为有限维系统.

关键词: Schrödinger格点系统; 加权空间; 随机吸引子; 随机指数吸引子

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§1 引 言

吸引子是描述随时间变化的动力系统的状态的长期渐近行为的重要工具. 自治、非自治与随机动力系统的渐近行为可以用全局吸引子、拉回吸引子与随机吸引子来描述[1-3]. 但无穷维动力系统的这些吸引子可能是无限维的, 而且它们吸引其它轨道的速率有可能是比较慢的, 不便于观察. 为了改进以上不足, Eden、Zelik等人引进了具有有限维数与指数率吸引轨道的指数吸引子、拉回指数吸引子、随机指数吸引子等概念[4-7]. 无穷多个常微分方程组成的格点系统的各类吸引子自文献[1]问世以来受到了许多学者的关注和研究. 格点系统可以作为有无穷多个节点的动态网络系统的数学模型.

本文考虑带可乘白噪声和弱阻尼的非自治Schrödinger格点系统

$$\begin{cases} \mathrm{i}\mathrm{d}u_k + [(Au)_k + \mathrm{i}\lambda_k u_k + f(|u_k|^2)u_k]\mathrm{d}t = g_k(t)\mathrm{d}t + \mathrm{i}(a+b\mathrm{i})u_k \circ \mathrm{d}W, \quad t > \tau, \tau \in \mathbf{R}, \\ u_k(\tau) = u_{k\tau}, k \in \mathbf{Z}, a, b \in \mathbf{R}, \end{cases} \tag{1}$$

其中"i"是虚数单位; 对任意$k \in \mathbf{Z}$, $u_k = u_k(t)$, $g_k(t) \in \mathbf{C}$; 实参数$\lambda_k > 0$表示弱阻尼; $u = (u_k)_{k\in\mathbf{Z}}$; $f \in C^1(\mathbf{R}_+, \mathbf{R})$; $W(t)$是概率空间$(\Omega, \mathcal{F}, \mathbb{P})$上的双边实值Wiener过程, $\Omega = \{\omega \in C(\mathbf{R}, \mathbf{R}) : \omega(0) = 0\}$, $\mathcal{F}$是Ω上由紧开拓扑生成的Borel σ-代数, $\mathbb{P}$是$(\Omega, \mathcal{F})$上的Wiener测度;

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* 通讯作者, E-mail: shengfanzhou@163.com
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"$\circ$"表示Stratonovich意义下的随机项; A是线性耦合算子. 在特殊情况下, 如当$(Au)_n = 2u_n - u_{n+1} - u_{n-1}$时, (1)可看作是带初值的连续的随机Schrödinger方程

$$\mathrm{i}\mathrm{d}u + [\Delta u + \mathrm{i}\lambda(x)u + f(|u|^2)u]\mathrm{d}t = g(x,t)\mathrm{d}t + \mathrm{i}(a+b\mathrm{i})u \circ \mathrm{d}W, u(\tau,x) = u_\tau(x), x \in \mathbf{R}$$

关于空间变量x的有限差分形式的模拟[8-9]. Schrödinger方程是量子力学的基本方程之一, 用于描述微观粒子运动状态随时间变化的情况, 它被广泛应用于原子、分子、固体物理、核物理和化学等多个领域.

系统(1)的相空间是无穷序列空间. 当无穷序列空间中的元素的各分量在其范数的权重不完全相同时, 就是无穷序列加权空间; 权重满足不同条件时, 对应的加权空间的大小可能会有所不同[8]. 设正值函数$\rho: \mathbf{Z} \to \mathbf{R}^+$, $\forall k \in \mathbf{Z}$, 记$\rho(k) = \rho_k$, 无穷序列加权空间l^2_ρ定义为

$$l^2_\rho = \{u | u = (u_k)_{k\in\mathbf{Z}}, u_k \in \mathbf{C}, \sum_{k\in\mathbf{Z}} \rho_k |u_k|^2 < \infty\},$$

其内积和范数定义为

$$(u,v)_\rho = \sum_{k\in\mathbf{Z}} \rho_k u_k \overline{v_k}, \|u\|^2_\rho = (u,u)_\rho = \sum_{k\in\mathbf{Z}} \rho_k |u_k|^2, u = (u_k)_{k\in\mathbf{Z}}, v = (v_k)_{k\in\mathbf{Z}} \in l^2_\rho.$$

当$\rho(\cdot) \equiv 1$时, l^2_ρ为通常的无加权的无穷序列空间l^2(元素范数各分量权重相同), 其内积和范数分别记为$(\cdot,\cdot)$, $\|\cdot\|$. l^2上的对称线性耦合算子在加权空间l^2_ρ中一般不再是对称的. 在无穷序列加权空间中考虑格点系统的动力学行为具有一定的实际和理论意义.

对于$g_k(t) = g_k$(与t无关) 且$a = b = 0$的自治Schrödinger格点系统(1), Karachalios等[8]研究其在l^2及加权空间l^2_ρ中的全局吸引子的存在性, 但其中的权重函数ρ的限制条件要比本文中的条件(见下面的(A4))强得多. 近年来, 对于随机Schrödinger格点系统已有不少研究结果, 李扬荣等[10]在2024年研究了具有自治与随机变量为系数的Schrödinger格点系统在l^2中的全局吸引子、数值吸引子、随机吸引子关于外力项与耗散系数的上、下半连续性; 陈章等[11-12]分别在2022年与2024年研究了具有时滞的非自治随机Schrödinger格点系统的解确定的Markov转移半群的不变测度的存在性和小噪声解的大偏差原理. 对于带可乘白噪声的非自治随机Schrödinger格点系统(1), 崔红珍等[13]、江旭莹等[14]、王碧祥等[15]分别证明了其在l^2中随机吸引子与随机指数吸引子的存在性. 本文将在[8, 13-14]的基础上, 研究系统(1)在加权空间l^2_ρ中的随机吸引子和随机指数吸引子的存在性. 由于随机指数吸引子要求有有限维数且指数率吸引轨道, 如其存在, 则随机吸引子必存在且含于随机指数吸引子中, 从而随机吸引子也是有限维的, 此时原来无穷维系统的解的极限行为可以用有限个参数来描述, 即原来无穷维的系统最终可退化为有限维系统来处理.

§2给出连续余圈(或连续非自治随机动力系统)、随机吸引子与随机指数吸引子的概念. §3和§4分别考虑系统(1)在加权空间l^2_ρ中的随机吸引子和随机指数吸引子的存在性.

§2 基本概念

本节介绍文中涉及到的几个基本概念, 详见文[6, 16]. 设$(\Omega, \mathcal{F}, \mathbb{P})$为概率空间, $(X, \|\cdot\|_X)$是可分的Hilbert空间, $\mathcal{B}$为X上的Borel σ-代数. 对于$E, F \subset X$, 定义

$$d_h(E,F) = \sup_{u\in E} \inf_{v\in F} \|u - v\|_X$$

为E到F的Hausdorff半距离. 文中将$\omega \in \Omega$等同于a.e. $\omega \in \Omega$.

定义2.1[16] 设$(\Omega, \mathcal{F}, \mathbb{P})$上的映射簇$\theta_t: \Omega \to \Omega$, $\omega \to \theta_t\omega$, $t \in \mathbf{R}$满足条件: (1) θ_0是Ω上

的恒等映射, 即$\theta_0 = \mathrm{I}$; (2) 对任意的$s,\ t \in \mathbf{R}$, $\theta_{t+s} = \theta_t \circ \theta_s$; (3) $(t,\omega) \mapsto \theta_t\omega$是可测的; (4) 对$\forall A \in \mathcal{F}$, $t \in \mathbf{R}$, $\mathbb{P}(A) = \mathbb{P}(\theta_t A)$; 则称$(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbf{R}})$为度量动力系统. 如果对$\forall t \in \mathbf{R}$, 均有$\theta_t A = A$, $\theta = (\theta_t)_{t\in\mathbf{R}}$, 则$A$是$\theta$不变的. 若对任意$\theta$不变集$A \in \mathcal{F}$, 都有$\mathbb{P}(A) = 0$或$\mathbb{P}(A) = 1$, 那么称$(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbf{R}})$是遍历的.

定义2.2[16] 称映射$\Phi : \mathbf{R}^+ \times \mathbf{R} \times \Omega \times X \to X$为由$\mathbf{R}$和$(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbf{R}})$驱动的取值于$X$中的连续余圈(连续非自治随机动力系统), 若对$\forall\tau \in \mathbf{R}$, $\omega \in \Omega$, $t, s \in \mathbf{R}^+$, 满足下述条件: (1) $\Phi(0, \tau, \omega, \cdot)$是空间$X$上的恒等映射; (2) $\Phi(\cdot, \tau, \cdot, \cdot) : \mathbf{R}^+ \times \mathbf{R} \times \Omega \times X \to X$是$(\mathcal{B}(\mathbf{R}^+) \times \mathcal{F} \times \mathcal{B}(X), \mathcal{B}(X))$可测的; (3) $\Phi(t+s, \tau, \omega, \cdot) = \Phi(t, \tau+s, \theta_s\omega, \Phi(s, \tau, \omega, \cdot))$; (4) $\Phi(t, \tau, \omega, \cdot) : X \to X$是连续的. 记为$\Phi = \{\Phi(t, \tau, \omega)\}_{t\geq 0, \tau\in\mathbf{R}, \omega\in\Omega}$.

定义2.3[16] 称双参数集值映射$B : \mathbf{R} \times \Omega \to 2^X \setminus \emptyset$, $(\tau, \omega) \to B(\tau, \omega)$为$X$中的(非自治)随机集, 如果对任意$\tau \in \mathbf{R}$, $x \in X$, 映射$\omega \to d(x, B(\tau, \omega))$是可测的. 称随机集是有界的(或闭的, 紧的), 如果对任意$\tau \in \mathbf{R}$, $B(\tau, \cdot)$是有界的(或闭的, 紧的). 称集族$D = \{D(\tau, \omega) : \tau \in \mathbf{R}, \omega \in \Omega\}$关于$\{\theta_t\}_{t\in\mathbf{R}}$是缓增的, 若对任意的$\varepsilon > 0$, $\omega \in \Omega$, $\tau \in \mathbf{R}$, 满足

$$\lim_{t\to\infty} \mathrm{e}^{-\varepsilon|t|} \|D(\tau + t, \theta_t\omega)\|_X = \lim_{t\to\infty} \mathrm{e}^{-\varepsilon|t|} \sup_{x\in D(\tau+t, \theta_t\omega)} \|x\|_X = 0.$$

记$\mathcal{D} = \mathcal{D}(X)$表示$X$中所有非空缓增子集族的集合.

定义2.4[16] 称集族$K_0 = \{K_0(\tau, \omega) : \tau \in \mathbf{R}, \omega \in \Omega\} \in \mathcal{D}$为连续余圈$\Phi$的$\mathcal{D}$-可测拉回吸收集, 若对任意$\tau \in \mathbf{R}$, $\omega \in \Omega$, $K \in \mathcal{D}$, 满足: (1) $K_0(\tau, \omega)$关于Ω是$\mathcal{F}$可测, 且在X中是随机集; (2) 存在$\mathrm{T} = \mathrm{T}(K, \tau, \omega)$, 当$t \geq \mathrm{T}$时, 有$\Phi(t, \tau - t, \theta_{-t}\omega, K(\tau - t, \theta_{-t}\omega)) \subseteq K_0(\tau, \omega)$.

定义2.5(随机吸引子)[16] 称随机集$\mathcal{K} = \{\mathcal{K}(\tau, \omega)\}_{\tau\in\mathbf{R}, \omega\in\Omega}$为连续余圈$\Phi$的$\mathcal{D}$-拉回随机吸引子, 若对$\forall\tau \in \mathbf{R}$, $\omega \in \Omega$,有: (1) $\mathcal{K}(\tau, \omega)$是$X$中的紧集, 关于$\omega$可测; (2) $\mathcal{K}$是不变的, 即$\forall t \geq 0$, $\Phi(t, \tau, \omega, \mathcal{K}(\tau, \omega)) = \mathcal{K}(t + \tau, \theta_t\omega)$; (3) $\mathcal{K}$拉回吸引$\mathcal{D}$中的所有集合, 即

$$\lim_{t\to\infty} d_h(\Phi(t, \tau - t, \theta_{-t}\omega, K(\tau - t, \theta_{-t}\omega)), \mathcal{K}(\tau, \omega)) = 0, \forall K \in \mathcal{D}(X).$$

定义2.6(随机指数吸引子)[6] 称随机集$\mathcal{H} = \{\mathcal{H}(\tau, \omega)\}_{\tau\in\mathbf{R}, \omega\in\Omega}$为$\mathbf{R}$和$(\Omega, \mathcal{F}, \mathbb{P}, (\theta_t)_{t\in\mathbf{R}})$驱动的连续余圈$\Phi$的$\mathcal{D}$-随机指数吸引子, 若对任意$\tau \in \mathbf{R}$, $\omega \in \Omega$, 有: (1) $\mathcal{H}(\tau, \omega)$是$X$中的紧集, 关于$\omega$可测; (2) 存在随机变量$\zeta_\omega < \infty$, 使得$\sup_{\tau\in\mathbf{R}} \dim_f \mathcal{H}(\tau, \omega) \leq \zeta_\omega$, 其中$\dim_f \mathcal{H}(\tau, \omega)$表示$\mathcal{H}(\tau, \omega)$的分形维数; (3) $\Phi(t, \tau - t, \theta_{-t}\omega, \mathcal{H}(\tau - t, \theta_{-t}\omega)) \subseteq \mathcal{H}(\tau, \omega)$, $t \geq 0$; (4) 存在常数$\vartheta > 0$使得对任意$K \in \mathcal{D}$, 存在随机变量$t_K(\tau, \omega) \geq 0$和$\xi(\tau, \omega, \|K\|_X) > 0$满足

$$d_h(\Phi(t, \tau - t, \theta_{-t}\omega, K(\tau - t, \theta_{-t}\omega)), \mathcal{H}(\tau, \omega)) \leq \xi(\tau, \omega, \|K\|_X)\mathrm{e}^{-\vartheta t}, t \geq t_K(\tau, \omega).$$

§3 Schrödinger格点系统在加权空间中的随机吸引子

本节考虑带可乘白噪声的非自治Schrödinger格点系统(1)在加权空间l^2_ρ中的随机吸引子的存在性. 在Ω上定义映射族

$$\theta_t\omega(\cdot) = \omega(\cdot + t) - \omega(t), t \in \mathbf{R}, \omega \in \Omega,$$

则$(\Omega, \mathcal{F}, \mathbb{P}, \{\theta_t\}_{t\in\mathbf{R}})$是遍历度量动力系统[17].

3.1 假设

对系统(1), 做如下假设.

(A1) $A=E^*E=EE^*$是线性耦合算子, E的定义为

$$(Eu)_k=\sum_{l=-m_0}^{m_0} d_l u_{k+l},\ \forall u=(u_k)_{k\in\mathbf{Z}},\ |d_l|\le a_0,\, m_0\in\mathbf{N},\, k\in\mathbf{Z},$$

a_0是正常数, E^*是E在l^2空间中的共轭;

(A2) $\forall k\in\mathbf{Z}$, $0<\underline{\lambda}\le\lambda_k\le\widehat{\lambda}<+\infty$, $\underline{\lambda}$, $\widehat{\lambda}$是两个正常数;

(A3) $f\in C^1(\mathbf{R}^+,\mathbf{R})$, $f(0)=0$, 并且存在常数f_0, $p\ge 0$使得$f'(|s|)\le f_0(1+|s|^p)$对$\forall s\in\mathbf{R}$都成立;

(A4) 存在正常数$\underline{\rho}$, $\widehat{\rho}$, b_1, b_0使得$\forall k\in\mathbf{Z}$, $0<\underline{\rho}\le\rho_k\le\widehat{\rho}<+\infty$, $|\rho_{k\pm1}-\rho_k|\le b_1\rho_k$, $\rho_{k\pm1}\le b_0\rho_k$, 其中

$$0\le b_1\le\frac{\underline{\lambda}}{2a_0b_2b_3(a_0^2+1)(2m_0+1)^2}, b_2=1+b_0+b_0^2+\cdots+b_0^{m_0-1}, b_3=b_2+b_0^{m_0};$$

(A5) $g(t)=(g_k(t))_{k\in\mathbf{Z}}\in C_b(\mathbf{R},l_\rho^2)$, 且$\forall\varepsilon>0$, $\exists\mathrm{I}_{(\varepsilon)}\in\mathbf{N}$, 使得$\sup_{t\in\mathbf{R}}\sum\limits_{|k|>\mathrm{I}_{(\varepsilon)}}\rho_k|g_k(t)|^2<\varepsilon$, 其中

$$C_b(\mathbf{R},l_\rho^2)=\{\mu\in C(\mathbf{R},l_\rho^2)|\|\mu\|_\rho^2=\sup_{t\in\mathbf{R}}\|\mu(t)\|_\rho^2=\sup_{t\in\mathbf{R}}\sum_{k\in\mathbf{Z}}\rho_k|\mu_k(t)|^2<+\infty\};$$

(A6) $|a|\le\frac{\underline{\lambda}\sqrt{\pi}}{32}$.

附注3.1 l^2等同于$\rho_k\equiv 1(\forall k\in\mathbf{Z})$的加权空间, 条件(A4)中的$b_1=0$, $b_0=1$, 即条件(A4)自然满足, 此时无需加上条件(A4), 见文[8, 13-14].

3.2 方程转化

系统(1)可以写成等价的向量形式

$$\begin{cases}\mathrm{i}\mathrm{d}u+[Au+\mathrm{i}\lambda u+f(|u|^2)u]\mathrm{d}t=g(t)\mathrm{d}t+\mathrm{i}(a+b\mathrm{i})u\circ\mathrm{d}W, & t>\tau,\tau\in\mathbf{R},\\ u(\tau)=u^\tau=(u_{k\tau})_{k\in\mathbf{Z}}, & a,b\in\mathbf{R},\end{cases}\tag{2}$$

式中$u=(u_k)_{k\in\mathbf{Z}}$, $\lambda u=(\lambda_k u_k)_{k\in\mathbf{Z}}$, $f(|u|^2)u=(f(|u_k|^2)u_k)_{k\in\mathbf{Z}}$, $g(t)=(g_k(t))_{k\in\mathbf{Z}}$, $u\circ\mathrm{d}W=(u_k\circ\mathrm{d}W)_{k\in\mathbf{Z}}$.

称连续随机过程$u(t)=u(t,\omega)=u(t,\tau,\omega,u(\tau))(t\ge\tau)$是(2)的解, 若对$\forall t>\tau$, $\tau\in\mathbf{R}$, $\omega\in\Omega$, $u(t)$满足随机积分方程

$$u(t)=u(t,\tau,\omega,u(\tau))=u(\tau)+\int_\tau^t F(u(s))\mathrm{d}s+(a+b\mathrm{i})\int_\tau^t u\circ\mathrm{d}W(s),\tag{3}$$

其中$F(u(t))=\mathrm{i}Au-\lambda u+\mathrm{i}f(|u|^2)u-\mathrm{i}g(t)$, $\int_\tau^t u\circ\mathrm{d}W(s)$为Stratonovich意义下的随机积分.

对任意的$\omega\in\Omega$, $t\in\mathbf{R}$, 取O-U平稳过程$z(\theta_t\omega)=-\int_{-\infty}^0\mathrm{e}^s(\theta_t\omega)(s)\mathrm{d}s$, 它满足方程$\mathrm{d}z(\theta_t\omega)+$

$z(\theta_t\omega)\mathrm{d}t=\mathrm{d}W(t)$, 其中$W(t)=W(t,\omega)=\omega(t)$. 由文[6, 18]知, $z(\theta_t\omega)$关于t连续且满足

$$\begin{cases}\lim_{t\to\infty}\mathrm{e}^{-\epsilon|t|}|z(\theta_t\omega)|=\lim\limits_{t\to\pm\infty}\frac{|z(\theta_t\omega)|}{t}=\lim\limits_{t\to\pm\infty}\frac{\int_0^t z(\theta_s\omega)\mathrm{d}s}{t}=0,\forall\epsilon>0,\\ \lim\limits_{t\to\pm\infty}\frac{\int_0^t|z(\theta_s\omega)|\mathrm{d}s}{t}=\mathbf{E}\left[|z(\omega)|\right]=\frac{1}{\sqrt{\pi}},\\ \lim\limits_{t\to\pm\infty}\frac{\int_0^t|z(\theta_s\omega)|^2\mathrm{d}s}{t}=\mathbf{E}\left[|z(\omega)|^2\right]=\frac{1}{2},\\ \mathbf{E}[\mathrm{e}^{\varepsilon z(\theta_s\omega)}]\le\frac{4\sqrt{\pi}+3\mathrm{e}}{3\sqrt{\pi}},\quad\forall s\in\mathbf{R},|\varepsilon|\le1,\\ \mathbf{E}[\mathrm{e}^{\varepsilon\int_s^{s+r}|z(\theta_l\omega)|\mathrm{d}l}]\le\mathrm{e}^{\varepsilon r},\quad\forall s\in\mathbf{R},r\ge0,\varepsilon^2\le1,\end{cases}\tag{4}$$

其中$\mathbf{E}$表示期望. 对于给定的$\tau\in\mathbf{R}$, $\omega\in\Omega$, 引入新变量

$$\varphi(t,\omega)=\varphi(t,\tau,\omega)=\mathrm{e}^{-(a+b\mathrm{i})z(\theta_t\omega)}u(t,\tau,\omega)=\mathrm{e}^{-(a+b\mathrm{i})z(\theta_t\omega)}u(t,\omega),t\ge\tau,$$

其中$u(t,\omega)=u(t,\tau,\omega,u^\tau)=(u_k(t,\tau,\omega,u^\tau))_{k\in\mathbf{Z}}$是(2)的解, 则$|u_k|=|\varphi_k|\mathrm{e}^{az(\theta_t\omega)}$.

对任意$\omega\in\Omega$, 记$\mathrm{e}^{-(a+bi)z(\omega)}=\mathbf{s}(\omega)$, 则

$$\varphi(t,\omega)=\mathbf{s}(\theta_t\omega)u(t,\omega),u(t,\omega)=\mathbf{s}^{-1}(\theta_t\omega)\varphi(t,\omega),\mathbf{s}^{-1}(\omega)=\mathrm{e}^{(a+b\mathrm{i})z(\omega)}.$$

且$\mathbf{s}(\omega)I$, $\mathbf{s}^{-1}(\omega)I$为l_ρ^2上的同构, 它们的算子范数满足

$$\|\mathbf{s}^{-1}(\omega)I\|_{L(l_\rho^2,l_\rho^2)}\le|\mathrm{e}^{(a+b\mathrm{i})z(\omega)}|=\mathrm{e}^{az(\omega)},\|\mathbf{s}(\omega)I\|_{L(l_\rho^2,l_\rho^2)}\le|\mathrm{e}^{-(a+bi)z(\omega)}|=\mathrm{e}^{-az(\omega)}.$$

于是$\|\mathbf{s}^{-1}(\omega)I\|_{L(l_\rho^2,l_\rho^2)}$和$\|\mathbf{s}(\omega)I\|_{L(l_\rho^2,l_\rho^2)}$关于$\{\theta_t\}_{t\in\mathbf{R}}$都是缓增的. 由于

$$u(\tau,\tau,\omega,u^\tau)=\mathbf{s}^{-1}(\omega)\varphi(\tau,\tau,\omega,\mathbf{s}(\omega)u^\tau)=\mathbf{s}^{-1}(\omega)\mathbf{s}(\omega)u^\tau=u^\tau,$$

且在Stratonovich随机积分意义下, 有

$$\begin{aligned}\mathrm{d}u(t)\quad&=\quad\mathrm{d}[\mathbf{s}^{-1}(\theta_t\omega)\varphi(t)]=\mathbf{s}^{-1}(\theta_t\omega)\mathrm{d}\varphi(t)+(a+bi)\mathbf{s}^{-1}(\theta_t\omega)\varphi(t)\circ\mathrm{d}z(\theta_t\omega)=\\&\quad[\mathrm{i}Au-\lambda u+\mathrm{i}f(|u|^2)u-\mathrm{i}g(t)]\mathrm{d}t+(a+bi)u\circ\mathrm{d}W.\end{aligned}$$

从而方程(2)可化为

$$\begin{cases}\dot\varphi=\mathrm{i}A\varphi-\lambda\varphi+\mathrm{i}f(|\varphi|^2\mathrm{e}^{2az(\theta_t\omega)})\varphi-\mathrm{i}g(t)\mathrm{e}^{-(a+b\mathrm{i})z(\theta_t\omega)}+(a+b\mathrm{i})z(\theta_t\omega)\varphi,\\ \varphi(\tau,\omega)=\varphi^\tau(\omega)=\mathrm{e}^{-(a+b\mathrm{i})z(\omega)}u^\tau,\quad t>\tau,a,b,\tau\in\mathbf{R}.\end{cases}\tag{5}$$

由此可知, (2)的解$u(t)=u(t,\tau,\omega,u^\tau)$和(5)的解$\varphi(t)=\varphi(t,\tau,\omega,\varphi^\tau)$有如下关系:

$$u(t,\tau,\omega,u^\tau)=\mathbf{s}^{-1}(\theta_t\omega)\varphi(t,\tau,\omega,\mathbf{s}(\omega)u^\tau)=\mathbf{s}^{-1}(\theta_t\omega)\varphi(t,\tau,\omega,\varphi^\tau),\varphi^\tau=\mathbf{s}(\omega)u^\tau.\tag{6}$$

3.3 解的存在唯一性与连续余圈的生成

先证明(5)的解能定义一个连续余圈, 再由(6)可得到随机系统(2)或(3)的解也能确定一个连续余圈. 注意到(5)中的随机微分方程(RDE)对于给定$\omega\in\Omega$是确定性的, 则其初值问题的解的存在唯一性有以下结论.

引理3.1 假设(A1)-(A6)成立, 则对任意的$\tau\in\mathbf{R}$, $\omega\in\Omega$, $\varphi^\tau\in l_\rho^2$, 系统(5)存在唯一解$\varphi(\cdot,\tau,\omega,\varphi^\tau)\in C^1([\tau,+\infty),l_\rho^2)$, $\varphi(t,\tau,\omega,\varphi^\tau)$关于$\varphi^\tau$连续, 关于$\omega$可测, 且解$\varphi(t,\tau,\omega,\varphi^\tau)$可以确定一个由$\mathbf{R}$和$(\Omega,\mathcal{F},\mathbb{P},(\theta_t)_{t\in\mathbf{R}})$驱动的取值于$l_\rho^2$的连续余圈$\Phi=\{\Phi(t,\tau,\omega)\}_{t\ge0,\,\tau\in\mathbf{R},\,\omega\in\Omega}$, 有

$$\Phi(t,\tau,\omega,\varphi^\tau)=\Phi(t,\tau,\omega)\varphi^\tau=\varphi(t+\tau,\tau,\theta_{-\tau}\omega,\varphi^\tau),t\ge0,\tau\in\mathbf{R},\omega\in\Omega.$$

证 记

$$\widetilde{F}(t,\omega,\varphi)=\mathrm{i}A\varphi-\lambda\varphi+\mathrm{i}f(|\varphi|^2\mathrm{e}^{2az(\theta_t\omega)})\varphi-\mathrm{i}g(t)\mathrm{e}^{-(a+b\mathrm{i})z(\theta_t\omega)}+(a+bi)z(\theta_t\omega)\varphi.\tag{7}$$

(i) 对任意$\omega\in\Omega$, $t\in\mathbf{R}$, $\varphi=(\varphi_k)_{k\in\mathbf{Z}}\in l^2_\rho$, 则$||\varphi||^2_\rho<+\infty$, $\|\lambda\varphi\|_\rho\le\widehat{\lambda}||\varphi||_\rho<+\infty$,

$$\|\mathrm{i}A\varphi\|_\rho = \|A\varphi\|_\rho=\left(\sum_{k\in\mathbf{Z}}\rho_k(A\varphi)^2_k\right)^{\frac12}\le\frac{\widehat{\rho}^{\frac12}}{\underline{\rho}^{\frac12}}(2m_0+1)^2a_0^2\|\varphi\|_\rho<+\infty,$$

$$\|f(|\varphi|^2\mathrm{e}^{2az(\theta_t\omega)})\|^2_\rho=\sum_{k\in\mathbf{Z}}\rho_k\left|f(|\varphi_k|^2\mathrm{e}^{2az(\theta_t\omega)})\right|^2\le$$
$$\sum_{k\in\mathbf{Z}}\rho_k\left|f'(\epsilon|\varphi_k|^2\mathrm{e}^{2az(\theta_t\omega)})\right|^2|\varphi_k|^4\mathrm{e}^{4az(\theta_t\omega)}(|\epsilon|\le1)\le$$
$$\sum_{k\in\mathbf{Z}}\rho_kf_0^2\left(1+|\varphi_k|^{2p}\mathrm{e}^{2paz(\theta_t\omega)}\right)^2|\varphi_k|^4\mathrm{e}^{4az(\theta_t\omega)}\le$$
$$\frac{f_0^2}{\underline{\rho}}\left(1+\frac{1}{\underline{\rho}^p}||\varphi||^{2p}_\rho\mathrm{e}^{2paz(\theta_t\omega)}\right)^2||\varphi||^4_\rho\mathrm{e}^{4az(\theta_t\omega)},$$
$$\|\mathrm{i}f(|\varphi|^2\mathrm{e}^{2az(\theta_t\omega)})\varphi\|_\rho\le\frac{f_0}{\underline{\rho}^{\frac12}}\left(1+\frac{1}{\underline{\rho}^p}||\varphi||^{2p}_\rho\mathrm{e}^{2paz(\theta_t\omega)}\right)||\varphi||^3_\rho\mathrm{e}^{2az(\theta_t\omega)}<+\infty,$$
$$g(t)\mathrm{e}^{-(a+b\mathrm{i})z(\theta_t\omega)}\in l^2_\rho,\quad(a+b\mathrm{i})z(\theta_t\omega)\varphi\in l^2_\rho,$$

由(7)得$\widetilde{F}(t,\varphi,\omega)$是从$l^2_\rho$到$l^2_\rho$的映射. 并且$\widetilde{F}(t,\varphi,\omega)$关于$(t,\varphi)$连续, 关于$\omega$可测.

(ii) 对固定的$\mathrm{T}>0$, $\tau\in\mathbf{R}$, $\omega\in\Omega$. 令Q是l^2_ρ的有界子集, $h_Q=\sup_{\varphi\in Q}\|\varphi\|_\rho<\infty$. 令$h_1=h_1(\tau,\omega,\mathrm{T})=\sup_{t\in[\tau,\tau+\mathrm{T}]}\mathrm{e}^{2az(\theta_t\omega)}$. 对任意的$\varphi^{(s)}=(\varphi^{(s)}_k)_{k\in\mathbf{Z}}\in Q$, $s=1,2,$有

$$\rho_k|\varphi^{(1)}_k|^2,\rho_k|\varphi^{(2)}_k|^2\le h^2_Q,\forall k\in\mathbf{Z},$$
$$\|\mathrm{i}A(\varphi^{(1)}-\varphi^{(2)})\|_\rho\le\|A(\varphi^{(1)}-\varphi^{(2)})\|_\rho\le\frac{\widehat{\rho}^{\frac12}}{\underline{\rho}^{\frac12}}(2m_0+1)^2a_0^2\|\varphi^{(1)}-\varphi^{(2)}\|_\rho,$$
$$\|f(|\varphi^{(1)}|^2\mathrm{e}^{2az(\theta_t\omega)})\|_\rho\le\frac{f_0}{\underline{\rho}^{\frac12}}\left(1+\frac{1}{\underline{\rho}^p}h^{2p}_Qh^p_1\right)h^2_Qh_1,$$
$$\|[f(|\varphi^{(1)}|^2\mathrm{e}^{2az(\theta_t\omega)})-f(|\varphi^{(2)}|^2\mathrm{e}^{2az(\theta_t\omega)})]\varphi^{(2)}\|^2_\rho\le\frac{4f_0^2}{\underline{\rho}^2}(1+\frac{2^{p+1}}{\underline{\rho}^p}h^{2p}_Qh^p_1)^2h^4_Qh^2_1\|\varphi^{(1)}-\varphi^{(2)}\|^2_\rho,$$
$$\|\mathrm{i}[f(|\varphi^{(1)}|^2\mathrm{e}^{2az(\theta_t\omega)})\varphi^{(1)}-f(|\varphi^{(2)}|^2\mathrm{e}^{2az(\theta_t\omega)})\varphi^{(2)}]\|_\rho\le$$
$$\frac{f_0}{\underline{\rho}^{\frac12}}(1+\frac{1}{\underline{\rho}^p}h^{2p}_Qh^p_1)h^2_Qh_1\|\varphi^{(1)}-\varphi^{(2)}\|_\rho+\frac{2f_0}{\underline{\rho}}(1+\frac{2^{p+1}}{\underline{\rho}^p}h^{2p}_Qh^p_1)h^2_Qh_1\|\varphi^{(1)}-\varphi^{(2)}\|_\rho=$$
$$L_{(Q,\omega,\mathrm{T})}\|\varphi^{(1)}-\varphi^{(2)}\|_\rho,$$

其中$L_{(Q,\omega,\mathrm{T})}=(\frac{f_0}{\underline{\rho}^{\frac12}}(1+\frac{1}{\underline{\rho}^p}h^{2p}_Qh^p_1)h^2_Qh_1+\frac{2f_0}{\underline{\rho}}(1+\frac{2^{p+1}}{\underline{\rho}^p}h^{2p}_Qh^p_1)h^2_Qh_1)<+\infty$. 因此对任意$\tau\in\mathbf{R}$, $\omega\in\Omega$, $t\in[\tau,\tau+\mathrm{T}]$, 有$\|\widetilde{F}(t,\varphi^{(1)},\omega)-\widetilde{F}(t,\varphi^{(2)},\omega)\|_\rho\le[\widehat{\lambda}+\frac{\widehat{\rho}^{\frac12}}{\underline{\rho}^{\frac12}}(2m_0+1)^2a_0^2+L_{(Q,\omega,\mathrm{T})}+\sqrt{a^2+b^2}\sup_{t\in[\tau,\tau+\mathrm{T}]}|z(\theta_t\omega)|]\|\varphi^{(1)}-\varphi^{(2)}\|_\rho$. 所以$\widetilde{F}(t,\varphi,\omega)$关于$\varphi$满足局部Lipschitz条件. 由抽象空间中常微分方程解的存在唯一性定理知, 存在$\mathrm{T}_{\max}>\tau$, 使得对任意的$\tau\in\mathbf{R}$, $\omega\in\Omega$, $\varphi^\tau\in l^2_\rho$, 系统(5)存在唯一局部解$\varphi(\cdot,\tau,\omega,\varphi^\tau)\in C^1([\tau,\mathrm{T}_{\max}),l^2_\rho)$, 关于$\varphi^\tau$连续及关于$\omega$可测.

(iii) 证明$\mathrm{T}_{\max}=+\infty$. 在$t\in[\tau,\mathrm{T}_{\max})$内, 做内积$((5),\varphi(t))_\rho$, 取其实部得

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\varphi\|^2_\rho=2\mathrm{Re}(\mathrm{i}A\varphi,\varphi)_\rho-2(\lambda\varphi,\varphi)_\rho-2\mathrm{Re}(\mathrm{i}g(t)\mathrm{e}^{-(a+b\mathrm{i})z(\theta_t\omega)},\varphi)_\rho+2az(\theta_t\omega)\|\varphi\|^2_\rho,$$

其中

$$(A\varphi,\varphi)_\rho=\sum_{k\in\mathbf{Z}}(E\varphi)_k\overline{(E(\rho\varphi))_k}=\sum_{k\in\mathbf{Z}}\rho_k|(E\varphi)_k|^2+\sum_{k\in\mathbf{Z}}(E\varphi)_k[\overline{(E(\rho\varphi))_k}-\overline{\rho_k(E\varphi)_k}],$$

$$\rho_{k+l}-\rho_k\le b_1\rho_{k+l-1}+b_1\rho_{k+l-2}+\cdots+b_1\rho_k\le(1+b_0+\cdots+b_0^{m_0-1})b_1\rho_k=b_1b_2\rho_k.$$

$$|\sum_{k\in\mathbf{Z}}(E\varphi)_k[\overline{(E(\rho\varphi))_k}-\rho_k\overline{(E\varphi)_k}]|\le\frac{a_0b_1b_2b_3(a_0^2+1)(2m_0+1)^2}{2}\|\varphi\|_\rho^2=\frac{b_4}{2}\|\varphi\|_\rho^2,$$

这里$b_4=a_0b_1b_2b_3(a_0^2+1)(2m_0+1)^2$. 则由(A4)可知, $b_4\le\frac{1}{2}\underline{\lambda}$. 从而有

$$2\mathrm{Im}(A\varphi,\varphi)_\rho\ge-2|\sum_{k\in\mathbf{Z}}(E\varphi)_k[\overline{(E(\rho\varphi))_k}-\overline{\rho_k(E\varphi)_k}]|\ge-b_4\|\varphi\|_\rho^2,$$

$$\mathrm{Re}(\mathrm{i}g(t)\mathrm{e}^{-(a+b\mathrm{i})z(\theta_t\omega)},\varphi)_\rho\le\frac{\|g\|_\rho^2}{\underline{\lambda}}\mathrm{e}^{-2az(\theta_t\omega)}+\frac{\lambda}{4}\|\varphi\|_\rho^2,\quad t\in[\tau,\mathrm{T}_{\max}).$$

可得

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\|\varphi\|_\rho^2 &\le \left(b_4-\frac{3}{2}\underline{\lambda}+2|a||z(\theta_t\omega)|\right)\|\varphi\|_\rho^2+\frac{2}{\underline{\lambda}}\|g\|_\rho^2\mathrm{e}^{-2az(\theta_t\omega)}\le\\ &(-\underline{\lambda}+2|a||z(\theta_t\omega)|)\|\varphi\|_\rho^2+\frac{2}{\underline{\lambda}}\|g\|_\rho^2\mathrm{e}^{-2az(\theta_t\omega)},t\in[\tau,\mathrm{T}_{\max}).\end{aligned}\tag{8}$$

对(8)在$[\tau,t](\tau\le t<\mathrm{T}_{\max})$上用Gronwall不等式, 可得

$$\|\varphi(t,\tau,\omega,\varphi^\tau)\|_\rho^2\le\mathrm{e}^{\int_\tau^t(-\underline{\lambda}+2|a||z(\theta_s\omega)|)\mathrm{d}s}\|\varphi^\tau\|_\rho^2+\frac{2}{\underline{\lambda}}\|g\|_\rho^2\int_\tau^t\mathrm{e}^{\int_r^t(-\underline{\lambda}+2|a||z(\theta_s\omega)|)\mathrm{d}s-2az(\theta_r\omega)}\mathrm{d}r.\tag{9}$$

当t为有限数时, $||\varphi(t,\tau,\omega,\varphi^\tau)||_\rho^2$有限, 这表明$\mathrm{T}_{\max}=+\infty$, 所以$\varphi(t,\tau,\omega,\varphi^\tau)$是方程(5)的全局解. 证毕.

令映射$\mathcal{S}:\Omega\times l_\rho^2\to l_\rho^2$为

$$\mathcal{S}(\omega,u)=\mathbf{s}(\omega)u=\varphi,\omega\in\Omega,u=(u_k)_{k\in\mathbf{Z}},\varphi=(\varphi_k)_{k\in\mathbf{Z}}\in l_\rho^2,\tag{10}$$

则$\mathcal{S}^{-1}(\omega,\varphi)=\mathbf{s}^{-1}(\omega)\varphi=u$, 且对任意$\omega\in\Omega,\mathcal{S}(\omega,\cdot)$为$l_\rho^2$上的同胚, 而对任意给定$u$, $\varphi\in l_\rho^2$, $\mathcal{S}(\cdot,u)$与$\mathcal{S}^{-1}(\cdot,\varphi)$关于$\omega$可测. 由文[19]可得如下结果.

引理3.2 假设(A1)-(A6)成立, 则对任意给定$\tau\in\mathbf{R}$, $\omega\in\Omega$, $u^\tau\in l_\rho^2$,系统(2)存在唯一解

$$u(t,\tau,\omega,u^\tau)=\mathcal{S}^{-1}(\theta_t\omega,\varphi(t,\tau,\omega,\mathcal{S}(\omega,u^\tau)))=\mathbf{s}^{-1}(\theta_t\omega)\varphi(t,\tau,\omega,\mathbf{s}(\omega)u^\tau)\in l_\rho^2,t>\tau,\tau\in\mathbf{R},$$

这里$u(\tau,\tau,\omega,u^\tau)=u^\tau$, 且$u(t,\tau,\omega,u^\tau)$关于$u^\tau$连续, 关于$\omega$可测. 解映射族

$$\overline{\Phi}(t,\tau,\omega):u^\tau\to u(t+\tau,\tau,\theta_{-\tau}\omega,u^\tau),l_\rho^2\to l_\rho^2,t\ge0,\tau\in\mathbf{R},\omega\in\Omega$$

确定了由$\mathbf{R}$和$(\Omega,\mathcal{F},\mathbb{P},\{\theta_t\}_{t\in\mathbf{R}})$驱动的取值于$l_\rho^2$的连续余圈$\overline{\Phi}=\{\overline{\Phi}(t,\tau,\omega)\}_{t\ge0,\tau\in\mathbf{R},\omega\in\Omega}$, 并且有

$$\overline{\Phi}(t,\tau,\omega)u^\tau=\mathcal{S}^{-1}(\theta_t\omega,\Phi(t,\tau,\omega)\mathcal{S}(\omega,u^\tau)),$$

即Φ与$\overline{\Phi}$是l_ρ^2上互为共轭的连续余圈.

由此可知, 如果Φ有随机吸引子$\mathcal{K}=\mathcal{K}(\tau,\omega)_{\tau\in\mathbf{R},\omega\in\Omega}$, 则$\widetilde{\mathcal{K}}=\{\mathcal{S}^{-1}(\omega,\mathcal{K}(\tau,\omega))\}_{\tau\in\mathbf{R},\omega\in\Omega}$是$\overline{\Phi}$的随机吸引子. 为此只需要证明$\Phi$在$l_\rho^2$中的随机吸引子$\mathcal{K}$的存在性.

3.4 吸收集

引理3.3 假设(A1)-(A6)成立, 则$\{\Phi(t,\tau,\omega)\}_{t\ge0,\,\tau\in\mathbf{R},\,\omega\in\Omega}$在$l_\rho^2$存在与$\tau\in\mathbf{R}$无关的有界的缓增闭随机吸收集

$$K_0=\{K_0(\tau,\omega):K_0(\tau,\omega)=K_0(\omega)=\{\varphi\in l_\rho^2:\|\varphi\|_\rho\le R_1(\omega)\},\tau\in\mathbf{R},\omega\in\Omega\},\tag{11}$$

其中

$$R_1^2(\omega)=\frac{4}{\underline{\lambda}}\|g\|_\rho^2\int_{-\infty}^0\mathrm{e}^{\int_r^0(-\frac{\lambda}{2}+2|a||z(\theta_s\omega)|)\mathrm{d}s-2az(\theta_r\omega)}\mathrm{d}r.\tag{12}$$

即$K_0(\omega)$满足: 对任意缓增随机集$K(\tau,\omega)\in\mathcal{D}(l_\rho^2)$, $\tau\in\mathbf{R}$, 存在$\mathrm{T}_K(\tau,\omega)\ge0$, 当$t\ge\mathrm{T}_K(\tau,\omega)$,

$\omega \in \Omega$时, 有$\Phi(t,\tau-t,\theta_{-\tau}\omega,K(\tau-t,\theta_{-t}\omega)) \subseteq K_0(\omega)$.

证 设$\varphi(r)=\varphi(r,\tau-t,\theta_{-\tau}\omega,\varphi^{\tau-t}(\theta_{-\tau}\omega))(\tau\in\mathbf{R},\ t\geq 0,\ r\geq\tau-t)$为(5)的解, 初值$\varphi^{\tau-t}(\theta_{-\tau}\omega)\in K(\tau-t,\theta_{-t}\omega)$. 由(8)可得

$$\frac{\mathrm{d}}{\mathrm{d}t}\|\varphi(r)\|_\rho^2+\frac{\lambda}{2}\|\varphi(r)\|_\rho^2\leq(-\frac{\lambda}{2}+2|a||z(\theta_{r-\tau}\omega)|)\|\varphi(r)\|_\rho^2+\frac{2}{\underline{\lambda}}\|g\|_\rho^2\mathrm{e}^{-2az(\theta_{r-\tau}\omega)}, r\geq\tau-t. \tag{13}$$

对(13)在$[\tau-t,\tau]$上用Gronwall不等式得: $\forall t\in\mathbf{R}^+$, $\tau\in\mathbf{R}$, $\omega\in\Omega$,

$$\begin{aligned}&\|\varphi(\tau,\tau-t,\theta_{-\tau}\omega,\varphi^{\tau-t}(\theta_{-\tau}\omega))\|_\rho^2+\\&\quad\frac{\lambda}{2}\int_{\tau-t}^{\tau}\mathrm{e}^{\int_r^\tau(-\frac{\lambda}{2}+2|a||z(\theta_{s-\tau}\omega)|)\mathrm{d}s}\|\varphi(r,\tau-t,\theta_{-\tau}\omega,\varphi^{\tau-t}(\theta_{-\tau}\omega))\|_\rho^2\mathrm{d}r\leq\\&\mathrm{e}^{\int_{\tau-t}^\tau(-\frac{\lambda}{2}+2|a||z(\theta_{s-\tau}\omega)|)\mathrm{d}s}\|\varphi^{\tau-t}(\theta_{-\tau}\omega)\|_\rho^2+\\&\quad\frac{2}{\underline{\lambda}}\|g\|_\rho^2\int_{\tau-t}^{\tau}\mathrm{e}^{\int_r^\tau(-\frac{\lambda}{2}+2|a||z(\theta_{s-\tau}\omega)|)\mathrm{d}s-2az(\theta_{r-\tau}\omega)}\mathrm{d}r\leq\\&\mathrm{e}^{\int_{-t}^0(-\frac{\lambda}{2}+2|a||z(\theta_s\omega)|)\mathrm{d}s}\|\varphi^{\tau-t}(\theta_{-\tau}\omega)\|_\rho^2+\\&\quad\frac{2}{\underline{\lambda}}\|g\|_\rho^2\int_{-\infty}^{0}\mathrm{e}^{\int_0^r(\frac{\lambda}{2}-2|a||z(\theta_s\omega)|)\mathrm{d}s-2az(\theta_r\omega)}\mathrm{d}r.\end{aligned}\tag{14}$$

令

$$R_1(\omega)=\left(\frac{4}{\underline{\lambda}}\|g\|_\rho^2\int_{-\infty}^{0}\mathrm{e}^{\int_0^r(\frac{\lambda}{2}-2|a||z(\theta_s\omega)|)\mathrm{d}s-2az(\theta_r\omega)}\mathrm{d}r\right)^{1/2}. \tag{15}$$

则由(A6), (4)可知, $R_1(\omega)<\infty$且$R_1(\omega)$是缓增的. 由$\varphi^{\tau-t}(\theta_{-\tau}\omega)\in K(\tau-t,\theta_{-t}\omega)$可得

$$\lim_{t\to+\infty}\mathrm{e}^{\int_{-t}^0(-\frac{\lambda}{2}+2|a||z(\theta_s\omega)|)\mathrm{d}s}\|\varphi^{\tau-t}(\theta_{-\tau}\omega)\|_\rho^2=0.$$

3.5 随机吸引子

为了得到随机吸引子的存在性, 下面给出(5)的解的尾估计与Φ在K_0上的渐近零性.

引理3.4 假设(A1)-(A6)成立, 则对任意$\tau\in\mathbf{R}$, $\omega\in\Omega$, 系统(5)的解$\varphi(r)=(\varphi_k(r))_{k\in\mathbf{Z}}=(\varphi_k(r,\tau,\omega,\varphi^\tau(\omega)))_{k\in\mathbf{Z}}(r\geq\tau)$满足

(i) 对任意$\mathrm{I}\in\mathbf{N}$, $t\geq 0$, 有

$$\begin{aligned}&\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|\varphi_k(\tau,\tau-t,\theta_{-\tau}\omega,\varphi^{\tau-t}(\theta_{-\tau}\omega))|^2\leq\\&(1+\frac{2b_6}{\mathrm{I}\underline{\lambda}})\mathrm{e}^{-\int_{-t}^0(\frac{\lambda}{2}-2|a||z(\theta_s\omega)|)\mathrm{d}s}\|\varphi^{\tau-t}(\theta_{-\tau}\omega)\|_\rho^2+(\frac{b_7}{\mathrm{I}}+\varrho_{\mathrm{I}})P_1(\omega),\end{aligned}\tag{16}$$

其中

$$b_6=\gamma_0m_0a_0b_3^2(a_0^2+1)(2m_0+1)^2, b_7=\frac{4b_6}{\underline{\lambda}^2}\|g\|_\rho^2, \varrho_{\mathrm{I}}=\frac{2}{\underline{\lambda}}\sup_{s\in\mathbf{R}}\sum_{|k|\geq\mathrm{I}}\rho_k|g_k(s)|^2,$$

$$P_1(\omega)=\int_{-\infty}^{0}\mathrm{e}^{\int_0^r(\frac{\lambda}{2}-2|a||z(\theta_s\omega)|)\mathrm{d}s-2az(\theta_r\omega)}\mathrm{d}r.$$

(ii) 对任意$\nu>0$及初值$\varphi^{\tau-t}\in K_0(\tau-t,\theta_{-t}\omega)$, 存在$\mathrm{T}_\nu(\omega)$, 使得当$t\geq\mathrm{T}_\nu(\omega)$时, 有

$$\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|\varphi_k(\tau,\tau-t,\theta_{-\tau}\omega,\varphi^{\tau-t}(\theta_{-\tau}\omega))|^2\leq\nu+(\frac{b_7}{\mathrm{I}}+\varrho_{\mathrm{I}})P_1(\omega).$$

并且对任意$\varepsilon > 0$, 存在$\mathrm{T}(\omega,\varepsilon) > 0$, $\mathrm{I}(\tau,\omega,\varepsilon) \in \mathbf{N}$, 使得对$t \geq \mathrm{T}(\omega,\varepsilon)$, 有

$$\sum_{|k|>\mathrm{I}(\tau,\omega,\varepsilon)} \rho_k|\varphi_k(\tau,\tau-t,\theta_{-\tau}\omega,\varphi^{\tau-t}(\theta_{-\tau}\omega))|^2 \leq \varepsilon, \quad \forall t \geq \mathrm{T}(\tau,\omega,\varepsilon). \tag{17}$$

证 构造一个递增的光滑函数$\gamma \in C^1(\mathbf{R}_+,[0,1])$, 满足

$$\begin{cases} \gamma(s) = 0, & 0 \leq s \leq 1; \\ 0 \leq \gamma(s) \leq 1, & 1 \leq s \leq 2; \\ \gamma(s) = 1, & 2 \leq s < +\infty; \\ |\gamma'(s)| \leq \gamma_0, & \forall s \in \mathbf{R}^+, \gamma_0 > 0. \end{cases} \tag{18}$$

令I是正整数, 设$w(t) = (\gamma(\frac{|k|}{\mathrm{I}})\varphi_k(t))_{k\in\mathbf{Z}} \in l_\rho^2$, 做内积$((5),w)_\rho$并取实部得

$$(\dot{\varphi},w)_\rho = 2\mathrm{Re}(\mathrm{i}A\varphi,w)_\rho - 2(\lambda\varphi,w)_\rho - 2\mathrm{Re}(\mathrm{i}g(t)\mathrm{e}^{-(a+b\mathrm{i})z(\theta_t\omega)},w)_\rho + 2az(\theta_t\omega)(\varphi,w)_\rho, \tag{19}$$

其中

$$\begin{aligned} &(A\varphi,w)_\rho = \sum_{k\in\mathbf{Z}}\gamma(\frac{|k|}{\mathrm{I}})\rho_k|(E\varphi)_k|^2 + \sum_{k\in\mathbf{Z}}(E\varphi)_k\left[\overline{(E(\rho w))_k} - \gamma(\frac{|k|}{\mathrm{I}})\rho_k\overline{(E\varphi)_k}\right], \\ &\left|\sum_{k\in\mathbf{Z}}(E\varphi)_k\left[\overline{(E(\rho w))_k} - \gamma(\frac{|k|}{\mathrm{I}})\rho_k\overline{(E\varphi)_k}\right]\right| \leq \\ &\left|\sum_{k\in\mathbf{Z}}(E\varphi)_k\left[\sum_{l=-m_0}^{m_0}d_l\rho_{k+l}\gamma(\frac{|k+l|}{\mathrm{I}})\overline{\varphi_{k+l}} - \gamma(\frac{|k|}{\mathrm{I}})\sum_{l=-m_0}^{m_0}d_l\rho_{k+l}\overline{\varphi_{k+l}}\right]\right| + \\ &\left|\sum_{k\in\mathbf{Z}}(E\varphi)_k\left[\gamma(\frac{|k|}{\mathrm{I}})\sum_{l=-m_0}^{m_0}d_l\rho_{k+l}\overline{\varphi_{k+l}} - \rho_k\gamma(\frac{|k|}{\mathrm{I}})\sum_{l=-m_0}^{m_0}d_l\overline{\varphi_{k+l}}\right]\right| \leq \\ &\frac{b_6}{2\mathrm{I}}\|\varphi\|_\rho^2 + \frac{b_4}{2}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|\varphi_k(t)|^2, \end{aligned}$$

$$2\mathrm{Re}(\mathrm{i}A\varphi,w)_\rho = -2\mathrm{Im}(A\varphi,w)_\rho \leq \frac{b_6}{\mathrm{I}}\|\varphi\|_\rho^2 + \frac{\underline{\lambda}}{2}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|\varphi_k(t)|^2,$$

$$-2\lambda(\varphi,w)_\rho \leq -2\underline{\lambda}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|\varphi_k(t)|^2,$$

$$-2\mathrm{Re}(\mathrm{i}g(t)\mathrm{e}^{-(a+b\mathrm{i})z(\theta_t\omega)},w)_\rho \leq \frac{2}{\underline{\lambda}}\mathrm{e}^{-2az(\theta_t\omega)}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|g_k(t)|^2 \quad + \quad \frac{\underline{\lambda}}{2}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|\varphi_k(t)|^2.$$

由(19)可得

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}t}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|\varphi_k(t)|^2 + (\frac{\underline{\lambda}}{2} - 2|az(\theta_t\omega)|)\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|\varphi_k(t)|^2 \leq \\ &\frac{2}{\underline{\lambda}}\mathrm{e}^{-2az(\theta_t\omega)}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|g_k|^2 + \frac{b_6}{\mathrm{I}}\|\varphi\|_\rho^2, \quad t \geq \tau, \tau \in \mathbf{R}. \end{aligned} \tag{20}$$

(i) 对(20)在$[\tau-t,\tau](t \geq 0)$上应用Gronwall不等式, 由(11), (14), (A4)和(A6)可得

$$\begin{aligned} &\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|\varphi_k(\tau,\tau-t,\theta_{-\tau}\omega,\varphi^{\tau-t}(\theta_{-\tau}\omega))|^2 \leq \mathrm{e}^{-\int_{-t}^0(\frac{\underline{\lambda}}{2}-2|a||z(\theta_s\omega)|)\mathrm{d}s}\|\varphi^{\tau-t}(\theta_{-\tau}\omega)\|_\rho^2 + \\ &\frac{2}{\underline{\lambda}}\int_{-\infty}^0\mathrm{e}^{\int_0^r(\frac{\underline{\lambda}}{2}-2|a||z(\theta_s\omega)|)\mathrm{d}s-2az(\theta_r\omega)}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|g_k(r+\tau)|^2\mathrm{d}r + \\ &\frac{b_6}{\mathrm{I}}\int_{\tau-t}^\tau\mathrm{e}^{\int_\tau^r(\frac{\underline{\lambda}}{2}-2|a||z(\theta_s\omega)|)\mathrm{d}s}\|\varphi(r,\tau-t,\theta_{-\tau}\omega,\varphi^{\tau-t}(\theta_{-\tau}\omega))\|_\rho^2\mathrm{d}r \leq \end{aligned}$$

$$(1+\frac{2b_6}{\mathrm{I}\underline{\lambda}})\mathrm{e}^{-\int_{-t}^{0}(\frac{\lambda}{2}-2|a||z(\theta_s\omega)|)\mathrm{d}s}\|\varphi^{\tau-t}(\theta_{-\tau}\omega)\|_\rho^2+(\frac{b_7}{\mathrm{I}}+\varrho_{\mathrm{I}})P_1(\omega). \tag{21}$$

(ii) 由(21), 对$\varphi^{\tau-t}\in K_0(\tau-t,\theta_{-t}\omega)$及$t\geq 0$, 有

$$\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|\varphi_k(\tau,\tau-t,\theta_{-\tau}\omega,\varphi^{\tau-t}(\theta_{-\tau}\omega))|^2\leq$$

$$\left(1+\frac{2b_6}{\underline{\lambda}}\right)\mathrm{e}^{-\int_{-t}^{0}(\frac{\lambda}{2}-2|a||z(\theta_s\omega)|)\mathrm{d}s}R_1^2(\theta_{-t}\omega)+(\frac{b_7}{\mathrm{I}}+\varrho_{\mathrm{I}})P_1(\omega). \tag{22}$$

由(A5)与$\lim\limits_{t\to+\infty}\left(1+\frac{2b_6}{\underline{\lambda}}\right)\mathrm{e}^{-\int_{-t}^{0}(\frac{\lambda}{2}-2|a||z(\theta_s\omega)|)\mathrm{d}s}R_1^2(\theta_{-t}\omega)=0$知, (ii)成立.

结合引理3.1-引理3.4及[20]中定理2.4可得Φ的随机吸引子的存在性.

定理3.5 假设(A1)-(A6)成立, 则$\Phi=\{\Phi(t,\tau,\omega)\}_{t\geq 0,\tau\in\mathbf{R},\omega\in\Omega}$存在$\mathcal{D}(l_\rho^2)$-拉回随机吸引子$\mathcal{K}=\{\mathcal{K}(\tau,\omega)\}_{\tau\in\mathbf{R},\,\omega\in\Omega}$满足: 对任意$\tau\in\mathbf{R}$, $\omega\in\Omega$, 有

$$\mathcal{K}(\tau,\omega)=\bigcap_{s\geq \mathrm{T}_{K_0}(\tau,\omega)}\overline{\bigcup_{t\geq s}\Phi(t,\tau-t,\theta_{-t}\omega)K_0(\theta_{-t}\omega)}.$$

§4 Schrödinger格点系统在加权空间中的随机指数吸引子

假设条件(A1)-(A6)成立. 定理3.5表明连续余圈Φ存在随机吸引子$\mathcal{K}=\{\mathcal{K}(\tau,\omega)\}_{\tau\in\mathbf{R},\omega\in\Omega}$, 但不能断定其中的子集$\mathcal{K}(\tau,\omega)$的维数是有限的. 根据随机指数吸引子的定义, 如其存在, 则可得到随机吸引子中的子集是有限维的. 为此本节将用[21]中的定理2.1考虑Schrödinger格点系统(5)的解确定的连续余圈Φ在加权空间l_ρ^2中的随机指数吸引子的存在性.

取$\nu=\nu_0>0$充分小, 使得

$$\frac{4\sqrt{\pi}+3\mathrm{e}}{3\sqrt{\pi}}(\frac{2\nu_0 f_0}{\underline{\rho}}+\frac{2^{2p+3}\nu_0^{p+1}f_0}{\underline{\rho}^{p+1}})\leq\frac{\underline{\lambda}}{32}. \tag{23}$$

由引理3.3知, 对任意$\omega\in\Omega$, 存在$t_{K_0}(\omega)\geq 0$,使得对任意$\tau\in\mathbf{R}$, 当$t\geq t_{K_0}(\omega)$有$\Phi(t,\tau-t,\theta_{-\tau}\omega,K(\theta_{-t}\omega))\subseteq K_0(\omega)$. 则可得到如下引理.

引理4.1 假设条件(A1)-(A6)成立. 设系统(5)的解为

$$\varphi(r)=(\varphi_k(r))_{k\in\mathbf{Z}}=(\varphi_k(r,\tau,\omega,\varphi^\tau(\omega)))_{k\in\mathbf{Z}}, r\geq\tau.$$

对任意$\tau\in\mathbf{R}$, $\omega\in\Omega$, $s>0$, 令

$$\eta(\tau-s,\theta_{-s}\omega)=\overline{\bigcup_{t\geq\max\{t_{K_0}(\omega),\mathrm{T}_{\nu_0}(\omega),t_{K_0}(\theta_{-s}\omega),\mathrm{T}_{\nu_0}(\theta_{-s}\omega)\}}\varphi(\tau,\tau-t-s,\theta_{-t-s}\omega)K_0(\theta_{-t-s}\omega)}\subseteq K_0(\theta_{-s}\omega).$$

则对任意$\tau\in\mathbf{R}$, $\omega\in\Omega$, 有

(i) 有界性: $\sup_{\tau\in\mathbf{R}}\sup_{m,n\in\eta(\tau,\omega)}\|m-n\|_\rho\leq 2R_1(\omega)<+\infty$, 且$R_1(\theta_t\omega)$关于$t$连续;

(ii) 正不变性: $\Phi(t,\tau-t,\theta_{-t}\omega)\eta(\tau-t,\theta_{-t}\omega)\subseteq\eta(\tau,\omega)$, $\forall t\geq 0$;

(iii) 拉回吸收性: 对任意$K=\{K(\tau,\omega)\}_{\tau\in\mathbf{R},\omega\in\Omega}\in\mathcal{D}$, 存在$\mathrm{T}_K(\tau,\omega)\geq 0$, 使得

$$\Phi(t,\tau-t,\theta_{-t}\omega)K(\tau-t,\theta_{-t}\omega)\subseteq\eta(\tau,\omega),\forall t\geq\mathrm{T}_K(\tau,\omega);$$

(iv) 对任意$\varphi\in\eta(\tau,\omega)$, $\mathrm{I}\in\mathbf{N}$, 有$\Sigma_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|\varphi_k|^2\leq\nu_0+(\frac{b_7}{\mathrm{I}}+\varrho_{\mathrm{I}})P_1(\omega)$.

由此可知, Φ满足[21]中定理2.1的条件(H1). 下面验证Φ满足[21]中定理2.1的条件(H2).

引理4.2 假设条件(A1)-(A6)成立, 则$\forall\tau\in\mathbf{R}$, $\omega\in\Omega$, $t\geq 0$, $\varphi^{s,\tau-t}(\theta_{-\tau}\omega)\in\eta(\tau-t,\theta_{-t}\omega)$,

其中$s=1,2$, 存在一个随机变量$B_0(\omega)>0$,使得(5)的解满足

$$\|\varphi(\tau,\tau-t,\theta_{-\tau}\omega,\varphi^{1,\tau-t}(\theta_{-\tau}\omega))-\varphi(\tau,\tau-t,\theta_{-\tau}\omega,\varphi^{2,\tau-t}(\theta_{-\tau}\omega))\|_\rho\leq \mathrm{e}^{\int_{-t}^{0}B_0(\theta_s\omega)\mathrm{d}s}\|\varphi^{1,\tau-t}(\theta_{-\tau}\omega)-\varphi^{2,\tau-t}(\theta_{-\tau}\omega)\|_\rho, \tag{24}$$

其中

$$B_0(\omega)=(-\frac{3}{4}\underline{\lambda}+\widehat{L}(\omega)+|a||z(\omega)|),\ \ \widehat{L}(\omega)=\frac{f_0}{\underline{\rho}}\left(3+\frac{(1+2^{p+2})}{\underline{\rho}^p}R_1^{2p}(\omega)\mathrm{e}^{2p|a||z(\omega)|}\right)R_1^2(\omega)\mathrm{e}^{2az(\omega)}.$$

证 对任意$\tau\in\mathbf{R}$, $\omega\in\Omega$, $t\geq 0$, $\varphi^{s,\tau-t}(\theta_{-\tau}\omega)\in\eta(\tau-t,\theta_{-t}\omega)\subseteq K_0(\theta_{-t}\omega)$, 其中$s=1,2$, 令

$$\varphi^{(s)}(r)=\varphi(r,\tau-t,\theta_{-\tau}\omega,\varphi^{s,\tau-t}(\theta_{-\tau}\omega)),y(r)=\varphi^{(1)}(r)-\varphi^{(2)}(r),r\geq\tau-t.$$

则$y(r)$满足

$$\begin{aligned}\dot{y}=\mathrm{i}Ay-\lambda y+\mathrm{i}(f(|\varphi^{(1)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(1)}-f(|\varphi^{(2)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(2)})+\\(a+b\mathrm{i})z(\theta_{r-\tau}\omega)y,y(\tau,\omega)=\varphi^{1,\tau}(\omega)-\varphi^{2,\tau}(\omega),\quad t\geq0,r>\tau-t.\end{aligned}\tag{25}$$

用(25)与$y(r)$做内积, 取实部得

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\|y\|_\rho^2 &= 2\mathrm{Re}(\mathrm{i}Ay,y)_\rho-2\lambda\|y\|_\rho^2+2az(\theta_{r-\tau}\omega)\|y\|_\rho^2+\\&\quad 2\mathrm{Re}(\mathrm{i}(f(|\varphi^{(1)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(1)}-f(|\varphi^{(2)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(2)}),y)_\rho,\end{aligned}$$

其中

$$2\mathrm{Re}(\mathrm{i}Ay,y)_\rho\leq\frac{\lambda}{2}\|y\|_\rho^2,$$

$$\begin{aligned}&\mathrm{Re}(\mathrm{i}(f(|\varphi^{(1)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(1)}-f(|\varphi^{(2)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(2)}),y)_\rho\leq\\&\frac{f_0}{\underline{\rho}}\left(1+\frac{1}{\underline{\rho}^p}R_1^{2p}(\theta_{r-\tau}\omega)\mathrm{e}^{2paz(\theta_{r-\tau}\omega)}\right)R_1^2(\theta_{r-\tau}\omega)\mathrm{e}^{2az(\theta_{r-\tau}\omega)}\|y\|_\rho^2+\\&\frac{2f_0}{\underline{\rho}}\left(1+\frac{2^{p+1}}{\underline{\rho}^p}R_1^{2p}(\theta_{r-\tau}\omega)\mathrm{e}^{2paz(\theta_{r-\tau}\omega)}\right)R_1^2(\theta_{r-\tau}\omega)\mathrm{e}^{2az(\theta_{r-\tau}\omega)}\|y\|_\rho^2=\widehat{L}(\theta_{r-\tau}\omega)\|\varphi^{(1)}-\varphi^{(2)}\|_\rho^2.\end{aligned}$$

所以

$$\frac{\mathrm{d}}{\mathrm{d}t}\|y(r)\|_\rho^2\leq\left(-\frac{3}{2}\underline{\lambda}+2\widehat{L}(\theta_{r-\tau}\omega)+2|a||z(\theta_{r-\tau}\omega)|\right)\|y(r)\|_\rho^2=2B_0(\theta_{r-\tau}\omega)\|y(r)\|_\rho^2,r>\tau-t.$$

对上式在$[\tau-t,r]$上用Gronwall不等式, 得

$$\|y(r)\|_\rho^2\leq\mathrm{e}^{2\int_{\tau-t}^{r}B_0(\theta_{s-\tau}\omega)\mathrm{d}s}\|y(\tau-t)\|_\rho^2.\tag{26}$$

当取$r=\tau$时即得(24).

引理4.3 假设条件(A1)-(A6)成立, 则对任意$\tau\in\mathbf{R}$, $\omega\in\Omega$, $t\geq0$, $\mathrm{I}\in\mathbf{N}$, 存在随机变量$B_1(\omega)$, $B_2(\omega)>0$, 使得对任意$\varphi^{s,\tau-t}(\theta_{-\tau}\omega)\in\eta(\tau-t,\theta_{-t}\omega)$, 其中$s=1,2$,有

$$\begin{aligned}&\sum_{|k|>2\mathrm{I}}\rho_k\left|\varphi_k(\tau,\tau-t,\theta_{-\tau}\omega,\varphi^{1,\tau-t}(\theta_{-\tau}\omega))-\varphi_k(\tau,\tau-t,\theta_{-\tau}\omega,\varphi^{2,\tau-t}(\theta_{-\tau}\omega))\right|^2\leq\\&(\mathrm{e}^{\int_{-t}^{0}(-\frac{3}{4}\underline{\lambda}+B_1(\theta_s\omega))\mathrm{d}s}+\frac{\delta_\mathrm{I}}{2}\mathrm{e}^{\int_{-t}^{0}B_2(\theta_s\omega)\mathrm{d}s})^2\|\varphi^{1,\tau-t}(\theta_{-\tau}\omega)-\varphi^{2,\tau-t}(\theta_{-\tau}\omega)\|_\rho^2,\end{aligned}\tag{27}$$

其中

$$\frac{\delta_\mathrm{I}^2}{4}=\frac{1}{\sqrt{3\underline{\lambda}}}(\frac{1}{\mathrm{I}}+\varrho_\mathrm{I}+\frac{1}{\mathrm{I}^{p+1}}+\varrho_\mathrm{I}^{p+1}),\quad B_1(\omega)=|az(\omega)|+\frac{2\nu_0}{\underline{\rho}}f_0\mathrm{e}^{2az(\omega)}+\frac{2^{2p+3}}{\underline{\rho}^{p+1}}\nu_0^{p+1}f_0\mathrm{e}^{2(p+1)az(\omega)},$$

$$B_2(\omega)=B_0(\omega)+B_1(\omega)+b_{10}^2(1+\mathrm{e}^{4(p+1)az(\omega)}P_1^{2(p+1)}(\omega)).$$

证 令$\widetilde{w}=(\widetilde{w}_k)_{k\in\mathbf{Z}}=(\gamma(\frac{|k|}{\mathrm{I}})y_k)_{k\in\mathbf{Z}}$, 用其与(25)做内积, 取实部得

$$(\dot{y},\widetilde{w})_\rho\ =\ \mathrm{Re}(\mathrm{i}Ay,\widetilde{w})_\rho-(\lambda y,\widetilde{w})_\rho+az(\theta_{r-\tau}\omega)(y,\widetilde{w})_\rho+$$

$$\mathrm{Re}(\mathrm{i}(f(|\varphi^{(1)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(1)}-f(|\varphi^{(2)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(2)}),\widetilde{w})_\rho,$$

其中

$$\mathrm{Re}(\mathrm{i}Ay,\widetilde{w})_\rho\leq\frac{b_6}{2\mathrm{I}}\|y\|_\rho^2+\frac{\lambda}{4}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|y_k(r)|^2. \tag{28}$$

由(A3)得

$$\begin{aligned}
&\mathrm{Re}(\mathrm{i}(f(|\varphi^{(1)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(1)}-f(|\varphi^{(2)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(2)}),\widetilde{w})_\rho\leq\\
&(f(|\varphi^{(1)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(1)}-f(|\varphi^{(1)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(2)},\widetilde{w})_\rho+\\
&(f(|\varphi^{(1)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(2)}-f(|\varphi^{(2)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(2)},\widetilde{w})_\rho\leq\\
&\Sigma_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})f_0\left(1+|\varphi_k^{(1)}|^{2p}\mathrm{e}^{2paz(\theta_{r-\tau}\omega)}\right)|\varphi_k^{(1)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)}|y_k(r)|^2+\\
&\Sigma_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})f_0\left(1+(|\varphi_k^{(1)}|^2+|\varphi_k^{(2)}|^2)^p\mathrm{e}^{2paz(\theta_{r-\tau}\omega)}\right)\times\\
&\left(|\varphi_k^{(1)}||\varphi_k^{(2)}|+|\varphi_k^{(2)}|^2\right)\mathrm{e}^{2az(\theta_{r-\tau}\omega)}|y_k(r)|^2\leq\\
&2f_0\mathrm{e}^{2az(\theta_{r-\tau}\omega)}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})\left(|\varphi_k^{(1)}|^2+|\varphi_k^{(2)}|^2\right)|y_k(r)|^2+\\
&2^{p+2}f_0\mathrm{e}^{2(p+1)az(\theta_{r-\tau}\omega)}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})\left(|\varphi_k^{(1)}|^{2(p+1)}+|\varphi_k^{(2)}|^{2(p+1)}\right)|y_k(r)|^2.
\end{aligned} \tag{29}$$

由引理4.1可知

$$\sum_{|k|>2\mathrm{I}}(|\varphi_k^{(1)}(r)|^2+|\varphi_k^{(2)}(r)|^2)\leq\frac{2}{\underline{\rho}}\nu_0+\frac{2}{\underline{\rho}}(\frac{b_7}{\mathrm{I}}+\varrho_\mathrm{I})P_1(\theta_{r-\tau}\omega), \tag{30}$$

$$\sum_{|k|>2\mathrm{I}}(|\varphi_k^{(1)}(r)|^{2(p+1)}+|\varphi_k^{(2)}(r)|^{2(p+1)})\leq\frac{2^{p+2}}{\underline{\rho}^{p+1}}\nu_0^{p+1}+\frac{2^{p+2}}{\underline{\rho}^{p+1}}(\frac{b_7}{\mathrm{I}}+\varrho_\mathrm{I})^{p+1}P_1^{p+1}(\theta_{r-\tau}\omega). \tag{31}$$

因此, 当$|k|>2\mathrm{I}$, $\mathrm{I}\in\mathbf{N}$时有

$$\begin{aligned}
&\mathrm{Re}(\mathrm{i}(f(|\varphi^{(1)}|^2\mathrm{e}^{2az(\theta_{r-\tau}\omega)})\varphi^{(1)}-f(|\varphi^{(2)}|^2\mathrm{e}^{2az(\theta_t\omega)})\varphi^{(2)}),\widetilde{w})_\rho\leq\\
&2f_0\mathrm{e}^{2az(\theta_{r-\tau}\omega)}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})(\frac{2\nu}{\underline{\rho}}+\frac{2}{\underline{\rho}}(\frac{b_7}{\mathrm{I}}+\varrho_\mathrm{I})P_1(\theta_{r-\tau}\omega))|y_k(r)|^2+\\
&2^{p+2}f_0\mathrm{e}^{2(p+1)az(\theta_{r-\tau}\omega)}\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})(\frac{2^{p+2}}{\underline{\rho}^{p+1}}\nu^{p+1}+\frac{2^{p+2}}{\underline{\rho}^{p+1}}(\frac{b_7}{\mathrm{I}}+\varrho_\mathrm{I})^{p+1}P_1^{p+1}(\theta_{r-\tau}\omega))|y_k(r)|^2\leq\\
&\left(\frac{4\nu}{\underline{\rho}}f_0\mathrm{e}^{2az(\theta_{r-\tau}\omega)}+\frac{2^{2p+4}}{\underline{\rho}^{p+1}}\nu^{p+1}f_0\mathrm{e}^{2(p+1)az(\theta_{r-\tau}\omega)}\right)\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|y_k(r)|^2+\\
&\frac{4}{\underline{\rho}}f_0\mathrm{e}^{2az(\theta_t\omega)}(\frac{b_7}{\mathrm{I}}+\varrho_\mathrm{I})P_1(\theta_{r-\tau}\omega)\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|y_k(r)|^2+\\
&\frac{2^{3p+5}}{\underline{\rho}^{p+1}}f_0\mathrm{e}^{2(p+1)az(\theta_{r-\tau}\omega)}(\frac{b_7^{p+1}}{\mathrm{I}^{p+1}}+\varrho_\mathrm{I}^{p+1})P_1^{p+1}(\theta_{r-\tau}\omega)\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|y_k(r)|^2.
\end{aligned} \tag{32}$$

由此可得

$$\begin{aligned}
&\frac{\mathrm{d}}{\mathrm{d}t}\sum_{k\in\mathbf{Z}}\rho_k\gamma\left(\frac{|k|}{\mathrm{I}}\right)|y_k(r)|^2+\left(\frac{3}{2}\underline{\lambda}-2B_1(\theta_{r-\tau}\omega)\right)\sum_{k\in\mathbf{Z}}\rho_k\gamma\left(\frac{|k|}{\mathrm{I}}\right)|y_k(r)|^2\leq\\
&\left(\frac{b_6}{\mathrm{I}}+\frac{4}{\underline{\rho}}f_0\mathrm{e}^{2az(\theta_{r-\tau}\omega)}(\frac{b_7}{\mathrm{I}}+\varrho_\mathrm{I})P_1(\theta_{r-\tau}\omega)\right)\|y(r)\|_\rho^2+
\end{aligned}$$

$$\frac{2^{3p+5}}{\underline{\rho}^{p+1}}f_0\mathrm{e}^{2(p+1)az(\theta_{r-\tau}\omega)}(\frac{b_7^{p+1}}{\mathrm{I}^{p+1}}+\varrho_{\mathrm{I}}^{p+1})P_1^{p+1}(\theta_{r-\tau}\omega)\|y(r)\|_\rho^2\leq$$

$$b_8(\frac{1}{\mathrm{I}}+\varrho_{\mathrm{I}})(1+\mathrm{e}^{2az(\theta_{r-\tau}\omega)}P_1(\theta_{r-\tau}\omega))\mathrm{e}^{2\int_{\tau-t}^{r}B_0(\theta_{s-\tau}\omega)\mathrm{d}s}\|y(\tau-t)\|_\rho^2+$$

$$b_9(\frac{1}{\mathrm{I}^{p+1}}+\varrho_{\mathrm{I}}^{p+1})\mathrm{e}^{2(p+1)az(\theta_{r-\tau}\omega)}P_1^{p+1}(\theta_{r-\tau}\omega)\mathrm{e}^{2\int_{\tau-t}^{r}B_0(\theta_{s-\tau}\omega)\mathrm{d}s}\|y(\tau-t)\|_\rho^2\leq$$

$$b_{10}(\frac{1}{\mathrm{I}}+\varrho_{\mathrm{I}}+\frac{1}{\mathrm{I}^{p+1}}+\varrho_{\mathrm{I}}^{p+1})(1+\mathrm{e}^{2(p+1)az(\theta_{r-\tau}\omega)}P_1^{p+1}(\theta_{r-\tau}\omega))\mathrm{e}^{2\int_{\tau-t}^{r}B_0(\theta_{s-\tau}\omega)\mathrm{d}s}\|y(\tau-t)\|_\rho^2, \tag{33}$$

其中$b_8=\max\{(b_6+\frac{4f_0}{\underline{\rho}})b_7,\frac{4f_0}{\underline{\rho}}\}$, $b_9=\max\{\frac{2^{3p+5}}{\underline{\rho}^{p+1}}f_0,\frac{2^{3p+5}}{\underline{\rho}^{p+1}}f_0b_7^{p+1}\}$, $b_{10}=\max\{b_8,b_9,\frac{2p+1}{p},\frac{p+2}{p+1}\}$.
对(33)在$[\tau-t,r]$上用Gronwall不等式, 则当$|k|\geq 2\mathrm{I}$时, 有

$$\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})\left|y_k(\tau,\tau-t,\theta_{-\tau}\omega,y^{\tau-t}(\theta_{-\tau}\omega))\right|^2\leq\mathrm{e}^{\int_{-t}^{0}(-\frac{3}{2}\underline{\lambda}+2B_1(\theta_s\omega))\mathrm{d}s}\|y(\tau-t)\|_\rho^2+$$

$$(\frac{1}{\mathrm{I}}+\varrho_{\mathrm{I}}+\frac{1}{\mathrm{I}^{p+1}}+\varrho_{\mathrm{I}}^{p+1})\|y(\tau-t)\|_\rho^2\mathrm{e}^{\int_{-t}^{0}(2B_0(\theta_s\omega)+2B_1(\theta_s\omega))\mathrm{d}s}\times$$

$$\int_{-t}^{0}b_{10}\left(1+\mathrm{e}^{2(p+1)az(\theta_r\omega)}P_1^{p+1}(\theta_r\omega)\right)\mathrm{e}^{\frac{3}{2}\underline{\lambda}r}\mathrm{d}r, \tag{34}$$

其中

$$\int_{-t}^{0}b_{10}\left(1+\mathrm{e}^{2(p+1)az(\theta_r\omega)}P_1^{p+1}(\theta_r\omega)\right)\mathrm{e}^{\frac{3}{2}\underline{\lambda}r}\mathrm{d}r\leq\frac{1}{\sqrt{3\underline{\lambda}}}\mathrm{e}^{\int_{-t}^{0}2b_{10}^2(1+\mathrm{e}^{4(p+1)az(\theta_r\omega)}P_1^{2(p+1)}(\theta_r\omega))\mathrm{d}r}. \tag{35}$$

所以, 当$|k|\geq 2\mathrm{I}$时有

$$\sum_{|k|\geq 2\mathrm{I}}\rho_k|y_k(\tau,\tau-t,\theta_{-\tau}\omega,y^{\tau-t}(\theta_{-\tau}\omega))|^2\leq\sum_{k\in\mathbf{Z}}\rho_k\gamma(\frac{|k|}{\mathrm{I}})|y_k(\tau,\tau-t,\theta_{-\tau}\omega,y^{\tau-t}(\theta_{-\tau}\omega))|^2\leq$$

$$(\mathrm{e}^{\int_{-t}^{0}(-\frac{3}{2}\underline{\lambda}+2B_1(\theta_s\omega))\mathrm{d}s}+\frac{\delta_{\mathrm{I}}^2}{4}\mathrm{e}^{\int_{-t}^{0}2B_2(\theta_s\omega)\mathrm{d}s})\|y(\tau-t)\|_\rho^2, \tag{36}$$

即(27)成立.

引理4.4 假设条件(A1)-(A5)成立, 取$|a|$充分小使得

$$|a|\leq\min\left\{\frac{\underline{\lambda}\sqrt{\pi}}{32},\frac{1}{32p+32}\right\}, \tag{37}$$

那么, $0\leq\mathbf{E}[B_1(\omega)]\leq\frac{\underline{\lambda}}{16}$, $\quad 0\leq\mathbf{E}[B_2^2(\omega)]<+\infty$.

证 显然(37)蕴含(A6)成立. 由(4), (23)与(37)得

$$\begin{aligned}\mathbf{E}[B_1(\omega)] &= |a|\mathbf{E}[z(\omega)]+\frac{2\nu_0f_0}{\underline{\rho}}\mathbf{E}[\mathrm{e}^{2|az(\omega)|}]+\frac{2^{2p+3}\nu_0^{p+1}f_0}{\underline{\rho}^{p+1}}\mathbf{E}[\mathrm{e}^{2(p+1)|az(\omega)|}]\leq\\ &\frac{|a|}{\sqrt{\pi}}+\frac{4\sqrt{\pi}+3\mathrm{e}}{3\sqrt{\pi}}\left(\frac{2\nu_0f_0}{\underline{\rho}}+\frac{2^{2p+3}\nu_0^{p+1}f_0}{\underline{\rho}^{p+1}}\right)\leq\frac{\underline{\lambda}}{16}.\end{aligned} \tag{38}$$

$$\begin{aligned}B_2^2(\omega) &\leq 3\left[B_0^2(\omega)+B_1^2(\omega)+4b_{10}^4(\varepsilon)+2b_{10}^4(\varepsilon)\left(\mathrm{e}^{16(p+1)az(\omega)}+P_1^{8(p+1)}(\omega)\right)\right]\leq\\ &b_{11}\left(1+a^2|z(\omega)|^2+P_2^2(\omega)+\mathrm{e}^{16(p+1)|az(\omega)|}+P_1^{8(p+1)}(\omega)\right),\end{aligned} \tag{39}$$

其中

$$P_2(\omega)=\frac{2\nu_0f_0}{\underline{\rho}}\mathrm{e}^{2az(\omega)}+\frac{2^{2p+3}\nu_0^{p+1}f_0}{\underline{\rho}^{p+1}}\mathrm{e}^{2(p+1)az(\omega)}.$$

$$\mathbf{E}[P_1^{8p+8}(\omega)]\leq\mathbf{E}[\left(\int_{-\infty}^{0}\mathrm{e}^{-\int_r^0(\frac{\underline{\lambda}}{2}-2az(\theta_s\omega)\mathrm{d}s-2az(\theta_r\omega))}\mathrm{d}r\right)^{8p+8}]\leq$$

$$\mathbf{E}[\left(\int_{-\infty}^{0}\mathrm{e}^{\frac{4p+4}{8p+7}\underline{\lambda}r}\mathrm{d}r\right)^{8p+7}\int_{-\infty}^{0}\mathrm{e}^{4\underline{\lambda}+16|a|\int_r^0|z(\theta_s\omega)|\mathrm{d}s+16|a||z(\theta_r\omega)|}\mathrm{d}r]\leq$$

$$\left(\frac{8p+7}{(4p+4)\underline{\lambda}}\right)^{8p+7}\cdot\int_{-\infty}^{0}\mathrm{e}^{4\underline{\lambda}r}\left(\mathrm{e}^{-32|a|r}+\frac{4\sqrt{\pi}+3\mathrm{e}}{3\sqrt{\pi}}\right)\mathrm{d}r=b_{12}<\infty, \tag{40}$$

则

$$\mathbf{E}[B_2^2(\omega)]\leq b_{11}\left(1+\frac{a^2}{2\sqrt{\pi}}+b_{13}+\frac{4\sqrt{\pi}+3\mathrm{e}}{3\sqrt{\pi}}+b_{12}\right)<+\infty. \tag{41}$$

定理4.5 假设条件(A1)-(A5)及(37)成立, 则连续余圈$\Phi=\{\Phi(t,\tau,\omega)\}_{t\geq0,\,\tau\in\mathbf{R},\,\omega\in\Omega}$有一个随机指数吸引子$\mathcal{H}=\{\mathcal{H}(\tau,\omega)\}_{\tau\in\mathbf{R},\omega\in\Omega}$, 满足: 对任意$\tau\in\mathbf{R}$, $\omega\in\Omega$, $t\geq0$,

(1) $\mathcal{H}(\tau,\omega)(\subseteq\eta(\tau,\omega))$是$l_\rho^2$中的紧集, 关于$\omega$可测;

(2) $\Phi(t,\tau,\omega)\mathcal{H}(\tau,\omega)\subseteq\mathcal{H}(t+\tau,\theta_t\omega)$;

(3) 存在$\bar{\mathrm{I}}\in\mathbf{N}$, 使得分形维数$\dim_f\mathcal{H}(\tau,\omega)\leq\frac{2(4\bar{\mathrm{I}}+2)\ln(\frac{2\sqrt{4\bar{\mathrm{I}}+2}}{\delta}+1)}{\ln\frac{4}{3}}<\infty$;

(4) 对$\mathcal{D}(l_\rho^2)$中的任意集合K, 存在随机变量$t_K(\tau,\omega)\geq0$和缓增随机变量$\xi_{K,\omega}>0$使得

$$d_h(\Phi(t,\tau,\omega)K(\tau,\omega),\mathcal{H}(t+\tau,\theta_t\omega))\leq\xi_{K,\omega}\mathrm{e}^{-\frac{\lambda\ln\frac{4}{3}}{64\ln2}t},t\geq t_K(\tau,\omega).$$

证 在(24), (27)中取$t=t_1=\frac{16\ln2}{\underline{\lambda}}$. 由引理4.4知

$$0<\widetilde{\nu}=t_1^2\left(2\mathbf{E}[B_2^2(\omega)]+\frac{\lambda}{2}\mathbf{E}[B_2(\omega)]\right)<\infty.$$

由$\delta_{\bar{\mathrm{I}}}^2$的表达式知, 存在充分大的正整数$\bar{\mathrm{I}}\in\mathbf{N}$, 使得

$$0\leq\delta_{\bar{\mathrm{I}}}^2=\frac{4}{\sqrt{3\underline{\lambda}}}\left(\frac{1}{\bar{\mathrm{I}}}+\frac{1}{\bar{\mathrm{I}}^{p+1}}+\varrho_{1,\bar{\mathrm{I}}}+\varrho_{1,\bar{\mathrm{I}}}^{p+1}\right)\leq\min\{\frac{1}{16},\mathrm{e}^{-\frac{2}{\ln\frac{3}{2}}\widetilde{\nu}}\}. \tag{42}$$

则由引理4.1-4.4和[21]中定理2.1可知, 连续余圈Φ存在随机指数吸引子$\mathcal{H}=\{\mathcal{H}(\tau,\omega)\}_{\tau\in\mathbf{R},\omega\in\Omega}$满足定理4.5的性质(1)-(4).

附注4.1 在定理4.5的条件下, 带可乘白噪声的非自治Schrödinger格点系统(1)存在随机指数吸引子, 这表明此时系统(1)的解的极限行为可以用有限个参数来描述, 即原来无穷维的系统最终可退化为有限维系统. 注意到条件(37)需要随机项系数中的$|a|$充分小, 而对b的取值无限制, 这是因为(1)中起作用的项是虚数“i”项.

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Random attractor and random exponential attractor for Schrödinger lattice system in weighted spaces

ZHANG Si-jia, ZHOU Sheng-fan

(College of Mathematical Sciences, Zhejiang Normal University, Jinhua 321004, China)

Abstract: It mainly study the existence of random attractors and random exponential attractors for a non-autonomous schrödinger lattice system with multiplicative white noise and weak damping in weighted spaces of infinite sequences (the weights of components of elements are not exactly the same in their norm). It is shown that under certain conditions, the limit behavior of the solutions of the original infinite dimensional system can be described by a finite number of parameters, that is, the original infinite dimensional system can be reduced into a finite dimensional system.

Keywords: Schrödinger lattice system; weighted space of infinite sequences; random attractor; random exponential attractor

MR Subject Classification: 37L55; 37L60; 34F05