高校应用数学学报
2025, 40(3): 266-280

Hawkes跳扩散模型下具有随机相关的期权定价

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摘 要: 在随机波动率模型基础上, 假设资产价格的跳跃由具有自刺激性的Hawkes过程所驱动, 无风险利率是随机的, 且标的资产价格与其波动率是随机相关的. 针对所构建的标的资产价格模型, 求得了标准欧式期权价值的解析表达式. 数值分析中, 发现相比常利率模型和常跳跃强度模型, 文中所提模型的期权价值更高, 但当到期时间较短时, 随机利率和随机跳强度对期权价值影响甚微; 此外, 文中的模型能生成形态丰富的隐含波动率曲面, 因此具备同时拟合期权市场中普遍观察到的隐含波动率微笑和隐含波动率倾斜的能力.

关键词: 期权定价; Hawkes过程; 随机波动率; 随机利率; 随机相关
中图分类号: F830
文献标识码: A **文章编号**: 1000-4424(2025)03-0266-15

§1 引 言

1973年, Black和Scholes[1]提出了基于几何布朗运动的期权定价模型. 然而, 该模型的一些假设与现实金融市场现象相违背. 比如, 其常数波动率假设与真实市场所观察到的隐含波动率微笑相矛盾. 为弥补Black-Scholes(BS)模型的不足, 更多贴近实际市场的期权定价模型被提出, 其中有两个主要的模型改进方向: 考虑随机波动率和资产价格的跳跃.

Heston[2]为了克服BS模型的缺陷提出了随机波动率模型. 该模型的优势在于它的易处理性以及能够拟合BS模型产生的波动率微笑(或偏斜)与杠杆效应. Heston模型及其变异模型尽管在很多方面优于BS模型, 但仍不够完美. 例如, 在实际市场中, 利率是随时变化的. Abudy等[3]发现, 引入随机利率能改善模型的期权定价性能. Kim等[4]将Hull-White利率期限结构纳入随机波动率模型, 他们发现考虑随机利率的随机波动率模型表现更优, 尤其是对于短期限的欧式期权. Recchioni等[5]探讨了具有随机利率的Heston组合模型, 实证发现其提出的模型在数据解释方面优于利率Heston模型. 国内学者也对随机利率情形下的期权定价进行了探讨[6-9]. 另一方

收稿日期: 2024-06-25 修回日期: 2024-11-07
基金项目: 湖南省教育厅科学研究优秀青年项目(23B0199); 国家自然科学基金(71971077)

面, Heston模型通过标的资产价格与其波动率变化的负相关性刻画杠杆效应, 进而可以产生隐含波动率微笑或偏斜. 但在Heston模型及其变异模型中, 标的资产价格与其波动率变化之间的常相关性不能在隐含波动率曲面上产生足够的微笑或偏斜, 尤其对于期限较短的期权[5]. 于是, 人们在Heston模型中考虑构建两者之间随机相关的模型. 例如, Da Fonseca等[10]考虑了标的资产与其波动率具有线性相关结构, 他们发现该模型得到的隐含波动率与实际市场所观察到的微笑和偏度是一致的. Teng等[11]在纯Heston模型中使用随机过程模拟随机相关性; 实证表明, 相比纯Heston模型和双重Heston模型, 引入随机相关性可以更好地拟合市场波动率, 特别是能够产生短期限期权所需的波动率微笑. Pacati等[12]在Heston++模型中假设标的资产与其波动率之间的相关系数是关于时间的位移函数, 他们发现该模型在VIX期货和期权的期限结构和隐含波动率曲面的校准中发挥着至关重要的作用, 并且能显著改善拟合期权价格的能力.

考虑标的资产价格跳跃是BS模型的另一个主要改进方向. Merton[13]最先构建一个资产价格非连续变化的期权定价模型, 其中跳跃由Poisson(泊松)过程驱动. 之后, 大部分基于跳跃扩散过程的期权定价模型都是采用泊松过程等其他Lévy(列维)过程刻画跳跃[14-16]. 列维过程的一个重要特征是独立增量性, 将其用于跳跃建模则意味着跳跃的到达时间间隔是相互独立的. 但是, 学者们近年来发现跳跃并非独立到达. Ait-Sahalia等[17]认为, 肇始于美国的"次贷危机"表明资本市场中的跳跃具有时间和空间上的聚集特征; Fulop等[18]通过实证发现市场广泛存在跳跃聚集. 国内学者如陈浪南等[19]、陈国进等[20]、马勇等[21], 他们通过实证发现我国股市的跳跃发生强度依赖历史信息且跳跃具有显著的聚集性. 于是, 在进行期权定价与风险管理时, 学者们开始采用能刻画跳跃聚集特征的点过程如Hawkes过程对资产价格跳跃进行建模. Ma等[22]基于Hawkes 跳跃扩散过程对脆弱欧式期权进行估值, 结果表明跳跃聚集对期权价值有显著影响; Ma等[23]和Jing等[24]分别利用Hawkes跳跃扩散模型对互换期权和VIX期权定价, 实证结果表明, 考虑跳跃聚集的期权定价模型能显著改善定价能力. 这些模型考虑了跳跃聚集性, 但没有将随机利率与随机相关, 甚至未将随机波动率纳入考虑.

本文将在跳跃扩散模型框架内, 利用能刻画跳跃聚集特征的Hawkes过程对跳跃进行建模. 此外, 标的资产价格波动率、无风险利率、标的资产价格与其波动率的相关性都是随机变化的. 本文构建的期权定价模型囊括随机波动率和聚集跳跃两个核心要素, 推广了以往的诸多重要模型. 尽管本模型因综合较多因素而形式上略显复杂, 但仍能得到解析形式的欧式期权定价公式. 因此, 相比基于数值方法的期权定价模型, 本文构建的更合理定价模型是建立在不牺牲定价速度基础上.

§2 理论模型构建

为简化定价过程、满足无套利原则等优点, 本文考虑在风险中性测度下对标的资产进行定价. 令$(\Omega,\mathcal{F},(\mathcal{F}(t))_{t\geq 0},\mathbb{Q})$为完备概率空间, 其中$\mathcal{F}(t)$是由Brownian(布朗)运动与跳过程在时间$t$所生成的域流, $\mathbb{Q}$是风险中性测度, 令E表示风险中性测度$\mathbb{Q}$下的期望. 假设标的资产价格过程$S(t)$满足随机微分方程

$$\mathrm{d}S(t)=(r(t)-d-\lambda(t)m_1)S(t)\mathrm{d}t+\sqrt{\nu(t)}S(t)\mathrm{d}W^s(t)+(\mathrm{e}^{\varepsilon_t}-1)S(t-)\mathrm{d}N(t), \tag{1}$$

其中d是常数红利率, 瞬时方差$\nu(t)$和随机利率$r(t)$满足随机微分方程

$$\mathrm{d}\nu(t) = \kappa_\nu(\theta_\nu-\nu(t))\mathrm{d}t+\eta_\nu\sqrt{\nu(t)}\mathrm{d}W^\nu(t), \tag{2}$$

$$\mathrm{d}r(t) = \beta(\theta_r-r(t))\mathrm{d}t+\eta_r\mathrm{d}W^r(t). \tag{3}$$

跳跃幅度ε_t为独立同分布的随机变量且$\varepsilon_t \sim N(\mu,\sigma^2)$, $m_1=\mathrm{E}(\mathrm{e}^{\varepsilon_t}-1)$为平均跳跃幅度. $N(t)$是强度为$\lambda(t)$的Hawkes过程, 其中

$$\mathrm{d}\lambda(t)=\kappa_\lambda\left(\theta_\lambda-\lambda(t)\right)\mathrm{d}t+\eta_\lambda \mathrm{d}N(t), \tag{4}$$

其中κ_λ, θ_λ和η_λ均为正值参数, $W^s(t)$, $W^\nu(t)$, $W^r(t)$都是标准布朗运动, 且$\mathrm{d}W^s(t)\mathrm{d}W^\nu(t)=\rho(t)\mathrm{d}t$, 其中$\rho(t)$是满足方程

$$\mathrm{d}\rho(t)=\kappa_\rho\left(\theta_\rho-\rho(t)\right)\mathrm{d}t+\eta_\rho\sqrt{1-\rho^2(t)}\mathrm{d}W^\rho(t) \tag{5}$$

的有界Jacobi过程. 以上涉及的参数κ_ν, β, κ_λ 和κ_ρ分别表示方差、利率、Hawkes过程和随机相关过程的均值回复速度, θ_ν, θ_r, θ_λ和θ_ρ表示它们的均值回复水平. 此外, $W^\rho(t)$是一个标准布朗运动, 且$\mathrm{d}W^\rho(t)\mathrm{d}W^\nu(t)=0$, $\mathrm{d}W^s(t)\mathrm{d}W^\rho(t)=\rho_1\mathrm{d}t$, 其中$\rho_1$为常数; $W^r(t)$独立于任何其它布朗运动.

令$X(t)=\log S(t)$. 根据Itô公式有

$$\mathrm{d}X(t)=(r(t)-d-\lambda(t)m_1-\frac{1}{2}\nu(t))\mathrm{d}t+\sqrt{\nu(t)}\mathrm{d}W^s(t)+\mathrm{d}\tilde{J}(t). \tag{6}$$

其中$\mathrm{d}\tilde{J}(t)=\varepsilon_t\mathrm{d}N(t)$.

注 根据文献[11], 为使(5)中$\rho(t)\neq\pm1$, 假设$\kappa_\rho>\eta_\rho^2/(1\pm\theta_\rho)$.

令$P(t,T)$表示到期时间为T的零息债券在时间t的价格, 则

$$P(t,T)=\mathrm{E}_t\left[\mathrm{e}^{-\int_t^T r(u)\mathrm{d}u}\right].$$

若无风险利率$r(t)$由Hull-White利率模型决定, 即其满足(3), 则

$$P(t,T)=\exp\{M(t,T)-L(t,T)r(t)\}, \tag{7}$$

其中

$$M(t,T)=(\theta_r-\frac{\eta_r^2}{2\beta^2})(L(t,T)-(T-t))-\frac{\eta_r^2}{4\beta}(L(t,T))^2,\quad L(t,T)=\frac{1}{\beta}(1-\mathrm{e}^{-\beta(T-t)}).$$

通过Radon-Nikodym导数定义前向测度$\mathbb{Q}^\top$, 即

$$\frac{\mathrm{d}\mathbb{Q}^\top}{\mathrm{d}\mathbb{Q}}=\frac{\mathrm{e}^{-\int_0^T r(u)\mathrm{d}u}}{P(0,T)}. \tag{8}$$

在测度$\mathbb{Q}^\top$下, 随机利率$r(t)$满足

$$\mathrm{d}r(t)=(\beta(\theta_r-r(t))-\eta_r^2L(t,T))\mathrm{d}t+\eta_r\mathrm{d}W_r^\top(t), \tag{9}$$

其中$\mathrm{d}W_r^\top(t)=\mathrm{d}W^r(t)+\eta_rL(t,T)\mathrm{d}t$在测度$\mathbb{Q}^\top$下是标准布朗运动.

§3 特征函数

本节研究标的资产对数价格$X(T)$在时间t的条件特征函数

$$f(\phi;\tau,X,\nu,\rho,\lambda,r)=\mathrm{E}^{\mathbb{Q}^\top}[\mathrm{e}^{\mathrm{i}\phi X(T)}\mid\mathcal{F}(t)], \tag{10}$$

其中$\mathrm{i}=\sqrt{-1}$, $\tau=T-t$.

令$X(t)$满足随机微分方程$\mathrm{d}X(t)=\mathrm{d}X^C(t)+\mathrm{d}X^J(t)$, 其中

$$\mathrm{d}X^C(t)=(r(t)-d-\frac{1}{2}\nu(t))\mathrm{d}t+\sqrt{\nu(t)}\mathrm{d}W^s(t),\quad \mathrm{d}X^J(t)=-m_1\lambda(t)\mathrm{d}t+\mathrm{d}\tilde{J}(t),$$

因为$\tilde{J}(t)$与$W^\nu(t)$, $W^\rho(t)$, $W^s(t)$, $W_r^\top(t)$相互独立, 所以

$$f(\phi;\tau,X,\nu,\rho,\lambda,r)=f_C(\phi;\tau,X,\nu,\rho,\lambda,r)f_J(\phi;\tau,X,\nu,\rho,\lambda,r), \tag{11}$$

其中

$$f_C(\phi;\tau,X,\nu,\rho,\lambda,r)=\mathrm{E}^{\mathbb{Q}^\top}[\mathrm{e}^{\mathrm{i}\phi X^C(T)}\mid\mathcal{F}(t)],\quad f_J(\phi;\tau,X,\nu,\rho,\lambda,r)=\mathrm{E}^{\mathbb{Q}^\top}[\mathrm{e}^{\mathrm{i}\phi X^J(T)}\mid\mathcal{F}(t)].$$

首先求$f_J(\phi;\tau,X,\nu,\rho,\lambda,r)$. 令

$$Y(t)=(\lambda(t),-\int_0^t m_1\lambda(s)\mathrm{d}s+\tilde{J}(t))'.$$

则$Y(t)$是满足方程

$$\mathrm{d}Y(t)=(\tilde{\alpha}+\tilde{\beta}Y(t))\mathrm{d}t+\gamma\mathrm{d}K(t)$$

和

$$\lambda(t)=\tilde{\delta}Y(t)$$

的二维仿射跳扩散过程, 其中

$$\tilde{\alpha}=\begin{pmatrix}\kappa_\lambda\theta_\lambda\\0\end{pmatrix},\tilde{\beta}=\begin{pmatrix}-\kappa_\lambda & 0\\-m_1 & 0\end{pmatrix},\gamma=\begin{pmatrix}\eta_\lambda & 0\\0 & 1\end{pmatrix},K(t)=\begin{pmatrix}N(t)\\\tilde{J}(t)\end{pmatrix},\tilde{\delta}=\begin{pmatrix}1 & 0\end{pmatrix}.$$

根据文献[25], $Y(T)$在时间t的条件特征函数为

$$f_Y(\varepsilon;\tau,X,\nu,\rho,\lambda,r)=\mathrm{E}^{\mathbb{Q}^\top}[\mathrm{e}^{\varepsilon'Y(T)}\mid\mathcal{F}(t)]=\mathrm{e}^{G_1(\varepsilon,\tau)+G_2(\varepsilon,\tau)'Y(t)},\quad\varepsilon\in\mathbf{C}^2.$$

$G_2(\varepsilon,\tau)=(G_{21}(\varepsilon,\tau),G_{22}(\varepsilon,\tau))'$满足微分方程

$$\begin{cases}\frac{\partial G_1(\varepsilon,\tau)}{\partial t}=-\tilde{\alpha}'G_2(\varepsilon,\tau),\\\frac{\partial G_2(\varepsilon,\tau)}{\partial t}=-\tilde{\beta}'G_2(\varepsilon,\tau)-(\Psi(\gamma G_2(\varepsilon,\tau))-1)\tilde{\delta}',\end{cases}\tag{12}$$

其中

$$\Psi(\omega)=\int_{\mathbf{R}}\mathrm{e}^{\omega'\varphi}\mathrm{d}F(y),\quad\omega\in\mathbf{C}^2,\quad\varphi=(1,y)'.$$

此外, 它们的边界条件为$G_1(\varepsilon,0)=0$, $G_2(\varepsilon,0)=\varepsilon$. 可得

$$f_J(\phi;\tau,X,\nu,\rho,\lambda,r)=f_Y((0,\mathrm{i}\phi)';\tau,X,\nu,\rho,\lambda,r).\tag{13}$$

方程组(12)通常没有闭形式解的, 本文将采用Runge-Kutta算法对其进行求解.

接下来, 求解$f_C(\phi;\tau,X,\nu,\rho,\lambda,r)$. 在测度$\mathbb{Q}^\top$下, 标的资产连续部分等价于

$$\mathrm{d}X^C(t)=(r(t)-d-\frac{1}{2}\nu(t))\mathrm{d}t+\rho(t)\sqrt{\nu(t)}\mathrm{d}\tilde{W}^\nu(t)+\rho_1\sqrt{\nu(t)}\mathrm{d}\tilde{W}^\rho(t)+\sqrt{1-\rho^2(t)-\rho_1^2}\sqrt{\nu(t)}\mathrm{d}\tilde{W}^s(t),\tag{14}$$

其中

$$\begin{aligned}\mathrm{d}\nu(t)&=\kappa_\nu(\theta_\nu-\nu(t))\mathrm{d}t+\eta_\nu\sqrt{\nu(t)}\mathrm{d}\tilde{W}^\nu(t),\\\mathrm{d}\rho(t)&=\kappa_\rho(\theta_\rho-\rho(t))\mathrm{d}t+\eta_\rho\sqrt{1-\rho^2(t)}\mathrm{d}\tilde{W}^\rho(t),\\\mathrm{d}r(t)&=(\beta(\theta_r-r(t))-\eta_r^2L(t,T))\mathrm{d}t+\eta_r\mathrm{d}W_r^\top(t),\end{aligned}$$

$\tilde{W}^\nu(t)$, $\tilde{W}^\rho(t)$, $\tilde{W}^s(t)$是三个相互独立的标准布朗运动.

有如下结论.

命题1 如果$X^C(t)$由(14)确定, 则其特征函数为

$$f_C(\phi;\tau,X,\nu,\rho,\lambda,r)=\mathrm{e}^{A(\phi,\tau)+B(\phi,\tau)\nu+C(\phi,\tau)\rho+D(\phi,\tau)r+\mathrm{i}\phi X^C},\tag{15}$$

其中

$$\begin{aligned}B(\phi,\tau)&=\frac{\kappa_\nu-b_1(\phi)}{\eta_\nu^2}\frac{1-\mathrm{e}^{-b_1(\phi)\tau}}{1-g_1(\phi)\mathrm{e}^{-b_1(\phi)\tau}},\\C(\phi,\tau)&=\frac{C_1\eta_\nu\left(\theta_\nu-\nu(0)\right)}{\kappa_\nu+\kappa_\rho-l_1}\mathrm{e}^{(\kappa_\nu-l_1)\tau-\kappa_\nu T}+\frac{C_1\eta_\nu\left(\nu(0)-\theta_\nu\right)}{\kappa_\nu+\kappa_\rho}\mathrm{e}^{\kappa_\nu(\tau-T)}+\end{aligned}$$

$$\frac{C_1\eta_\nu\theta_\nu}{\kappa_\rho}-\frac{C_1\eta_\nu\theta_\nu}{\kappa_\rho-l_1}\mathrm{e}^{-l_1\tau}+C_1C_2\mathrm{e}^{-\kappa_\rho\tau},$$

$$D(\phi,\tau)=\frac{\mathrm{i}\phi(1-\mathrm{e}^{-\beta\tau})}{\beta},$$

$$A(\phi,\tau)=\kappa_\nu\theta_\nu H_0(\phi,\tau)+(\kappa_\rho\theta_\rho+\eta_\rho\rho_1 m a_3\mathrm{i}\phi)H_1(\phi,\tau)+\beta\theta_r H_2(\phi,\tau)-\eta_r^2H_3(\phi,\tau)-d\mathrm{i}\phi\tau+\frac{1}{2}\eta_r^2H_4(\phi,\tau)+\frac{1}{2}\eta_\rho^2(1-a_2)H_5(\phi,\tau)+\eta_\rho\rho_1 m\mathrm{i}\phi H_6(\phi,\tau)+\eta_\rho\rho_1 m b_3\mathrm{i}\phi H_7(\phi,\tau)+\eta_\rho\rho_1 n\mathrm{i}\phi H_8(\phi,\tau)+\eta_\rho\rho_1 n b_3\mathrm{i}\phi H_9(\phi,\tau)+\eta_\rho\rho_1 n a_3\mathrm{i}\phi H_{10}(\phi,\tau)-\frac{1}{2}\eta_\rho^2H_{11}(\phi,\tau)-\frac{1}{2}\eta_\rho^2b_2H_{12}(\phi,\tau)$$

和

$$\begin{aligned}
b_1(\phi)&=\sqrt{\kappa_\nu^2+\eta_\nu^2(\mathrm{i}\phi+\phi^2)},\quad g_1(\phi)=\frac{\kappa_\nu-b_1(\phi)}{\kappa_\nu+b_1(\phi)},\quad C_1=\mathrm{i}\phi\frac{\kappa_\nu-b_1(\phi)}{\eta_\nu^2},\\
C_2&=-\frac{\eta_\nu(\theta_\nu-\nu(0))}{\kappa_\nu+\kappa_\rho-l_1}\mathrm{e}^{-\kappa_\nu T}-\frac{\eta_\nu(\nu(0)-\theta_\nu)}{\kappa_\nu+\kappa_\rho}\mathrm{e}^{-\kappa_\nu T}-\frac{\eta_\nu\theta_\nu}{\kappa_\rho}+\frac{\eta_\nu\theta_\nu}{\kappa_\rho-l_1},\\
l_1&=-\ln\left(\frac{\mathrm{e}^{-b_1(\phi)}-g_1(\phi)\mathrm{e}^{-b_1(\phi)}}{1-g_1(\phi)\mathrm{e}^{-b_1(\phi)}}\right),
\end{aligned}$$

$H_i(\phi,\tau)$, $i=0,1,\cdots,12$和证明见附录A. 最终$X(t)$的特征函数表达式为

$$f(\phi;\tau,X,\nu,\rho,\lambda,r)=f_C(\phi;\tau,X,\nu,\rho,\lambda,r)f_J(\phi;\tau,X,\nu,\rho,\lambda,r).$$

§4 欧式期权定价

本节将计算基于本文模型下的欧式看涨期权价值(欧式看跌期权可以相应得到). 根据风险中性定价方法, 行权价为K、到期时间为T的欧式看涨期权在时间t的价格为

$$C(X(t),\nu(t),\rho(t),\lambda(t),r(t))=\mathrm{E}^{\mathbb{Q}}[\mathrm{e}^{-\int_t^T r(s)\mathrm{d}s}(\mathrm{e}^{X_T}-K)^+\mid\mathcal{F}(t)].\tag{16}$$

基于式(8)的测度变换, 该期权在前向测度$\mathbb{Q}^\top$下的定价表达式为

$$\begin{aligned}&C(X(t),\nu(t),\rho(t),\lambda(t),r(t))=\\&P(t,T)\left(\mathrm{E}^{\mathbb{Q}^\top}[\mathrm{e}^{X(T)}\mathbf{1}_{\{X(T)\ge k\}}\mid\mathcal{F}(t)]-\mathrm{e}^k\mathbb{Q}^\top\{X(T)\ge k\mid\mathcal{F}(t)\}\right),\end{aligned}\tag{17}$$

其中$k=\log K$.

命题2 若标的资产对数价格$X(t)$的条件特征函数为$f(\phi;\tau,X,\nu,\rho,\lambda,r)$, 则

$$\begin{aligned}&C(X(t),\nu(t),\rho(t),\lambda(t),r(t))=\\&P(t,T)\Bigg\{f(-\mathrm{i};\tau,X,\nu,\rho,\lambda,r)\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty\Re\left(\frac{\mathrm{e}^{-\mathrm{i}\phi k}f(\phi-\mathrm{i};\tau,X,\nu,\rho,\lambda,r)}{\mathrm{i}\phi f(-\mathrm{i};\tau,X,\nu,\rho,\lambda,r)}\right)\mathrm{d}\phi\right)-\\&\mathrm{e}^k\left(\frac{1}{2}+\frac{1}{\pi}\int_0^\infty\Re\left(\frac{\mathrm{e}^{-\mathrm{i}\phi k}f(\phi;\tau,X,\nu,\rho,\lambda,r)}{\mathrm{i}\phi}\right)\mathrm{d}\phi\right)\Bigg\},\end{aligned}\tag{18}$$

其中$\Re(\cdot)$表示复数的实部.

证 令$\varphi(\phi,t)=f(\phi;\tau,X,\nu,\rho,\lambda,r)$, 即$\varphi(\phi,t)$是$X(T)$在$t$时刻的条件特征函数, 则

$$\mathbb{Q}^\top\{X(T)\ge k\mid\mathcal{F}(t)\}=\frac{1}{2}+\frac{1}{\pi}\int_0^\infty\Re\left(\frac{\mathrm{e}^{-\mathrm{i}\phi k}\varphi(\phi,t)}{\mathrm{i}\phi}\right)\mathrm{d}\phi.\tag{19}$$

令$p(x)$为$X(T)$的密度函数. 设Y是密度函数为$\frac{\mathrm{e}^x p(x)}{\varphi(-\mathrm{i},t)}$的随机变量, 因为

$$\mathrm{E}^{\mathbb{Q}^\top}[\mathrm{e}^{\mathrm{i}\phi Y}\mid\mathcal{F}(t)]=\int_{-\infty}^\infty\mathrm{e}^{\mathrm{i}\phi y}\frac{\mathrm{e}^y p(y)}{\varphi(-\mathrm{i},t)}\mathrm{d}y=\frac{1}{\varphi(-\mathrm{i},t)}\int_{-\infty}^\infty\mathrm{e}^{\mathrm{i}(\phi-\mathrm{i})y}p(y)\mathrm{d}y=\frac{\varphi(\phi-\mathrm{i},t)}{\varphi(-\mathrm{i},t)},$$

所以Y的特征函数是$\frac{\varphi(\phi-\mathrm{i},t)}{\varphi(-\mathrm{i},t)}$. 根据傅里叶逆变换,

$$\begin{aligned}\mathbb{Q}^{\top}\{Y\geq k\mid\mathcal{F}(t)\} &= \int_k^{\infty}\frac{\mathrm{e}^{y}p(y)}{\varphi(-\mathrm{i},t)}\mathrm{d}y= \\ &\quad \frac{1}{2}+\frac{1}{\pi}\int_0^{\infty}\Re\left(\frac{\mathrm{e}^{-\mathrm{i}\phi k}\varphi(\phi-\mathrm{i},t)}{\mathrm{i}\phi\varphi(-\mathrm{i},t)}\right)\mathrm{d}\phi. \end{aligned}\tag{20}$$

因此

$$\begin{aligned}\mathrm{E}^{\mathbb{Q}^{\top}}\left[\mathrm{e}^{X(T)}\mathbf{1}_{\{X(T)\geq k\}}\mid\mathcal{F}(t)\right] &= \int_k^{\infty}\mathrm{e}^{x}p(x)\mathrm{d}x= \\ &\quad \varphi(-\mathrm{i},t)\left(\frac{1}{2}+\frac{1}{\pi}\int_0^{\infty}\Re\left(\frac{\mathrm{e}^{-\mathrm{i}\phi k}\varphi(\phi-\mathrm{i},t)}{\mathrm{i}\phi\varphi(-\mathrm{i},t)}\right)\mathrm{d}\phi\right). \end{aligned}\tag{21}$$

将(20)-(21)代入(17)即可得(18).

§5 数值分析

本节针对本文的期权定价模型进行数值分析. 基于Heston[2], Teng等[11], Ait-Sahalia等[17], Forde等[27]和Kokholm[28]等相关文献, 设定模型基准参数值, 见表1. 在下面的模型对比分析中, 固定利率模型中的利率设定为随机利率模型中的长期均值水平(0.05), Poisson跳模型的强度设定为Hawkes跳模型中的长期均值水平(0.5).

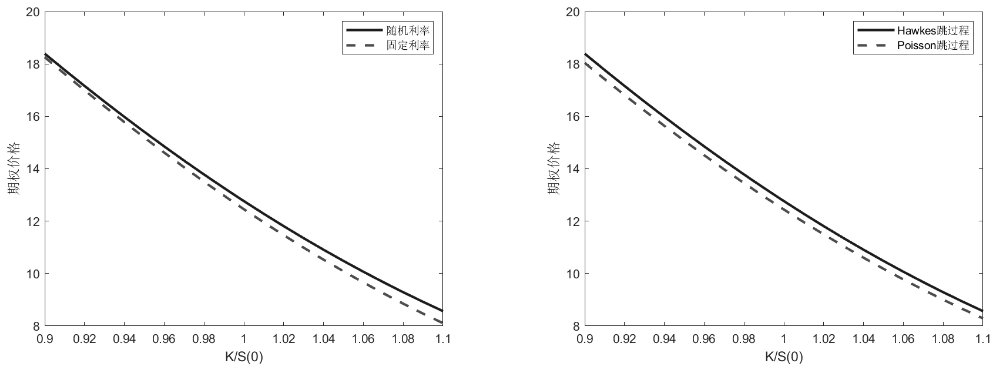
表1 基准参数值

$S(0)$	K	$\nu(0)$	$\rho(0)$	$\lambda(0)$	$r(0)$	d	ρ_1
100	100	0.04	-0.3	0.55	0.05	0.05	-0.5
κ_ν	κ_ρ	κ_λ	β	θ_ν	θ_ρ	θ_λ	θ_r
3	2	22	0.2	0.02	-0.5	0.5	0.05
η_ν	η_ρ	η_λ	η_r	μ	σ	T	
0.4	0.45	4	0.15	-0.22	0.27	1	

图1中(a)和(b)展示了本文模型与固定利率模型和Poisson跳模型下的(看涨)期权价格与期权价值状态之间的关系. 由图可知, 期权价格都随期权价值状态($K/S(0)$)递增而递减, 而且随机利率模型比固定利率模型所得期权价格更高、Hawkes跳模型比Poisson跳模型所得期权价格更高, 这是因为随机利率模型或者随机跳强度模型能产生更大的资产价格不确定性. 图2(c)和(d)展示了期权价值与到期期限之间的关系. 当到期期限较短时, 本文与参考模型的期权价格差异较小; 当到期期限增大时, 不同模型下的期权价格都增大, 但它们之间的差异变大. 这是因为当到期期限较短时, 随机利率和Hawkes跳过程的强度都不会明显变化, 然而当到期期限变长时, 随机利率和Hawkes跳过程会产生较大的利率和跳强度的随机性, 进而提高了资产价格变化的随机性, 导致了更大的期权价格差异.

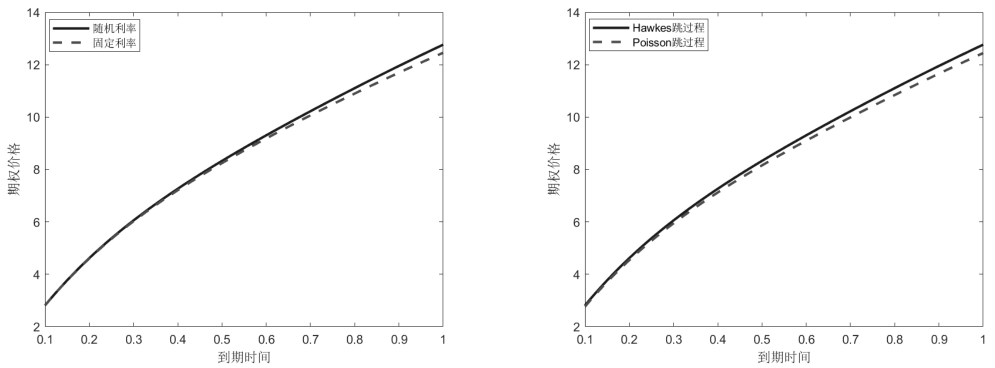
众所周知, 隐含波动率微笑和隐含波动率倾斜是期权市场中的典型经验特征. 从图3可知, 随着期权到期期限的变长, 隐含波动率关于价值状态的曲线形态从“微笑”逐步变化为“倾斜”. 因此本文模型可产生丰富的隐含波动率曲线形态, 具备拟合隐含波动率微笑与隐含波动率倾斜的能力.

图4(e)和(f)分别展示了欧式看涨期权价格对跳跃幅度均值μ和标准差σ的敏感性. 该期权的价格是关于跳跃幅度绝对均值的增函数, 但它对正的跳跃幅度均值变化更为敏感; 这是因为期权是非线性产品, 对于看涨期权而言, 它对标的资产价格的上涨更为敏感. 此外该期权价格也是关


随机利率
固定利率
期权价格
K/S(0)
Hawkes跳过程
Poisson跳过程
期权价格
K/S(0)

(a) (b)

图1


随机利率
固定利率
期权价格
到期时间
Hawkes跳过程
Poisson跳过程
期权价格
到期时间

(c) (d)

图2

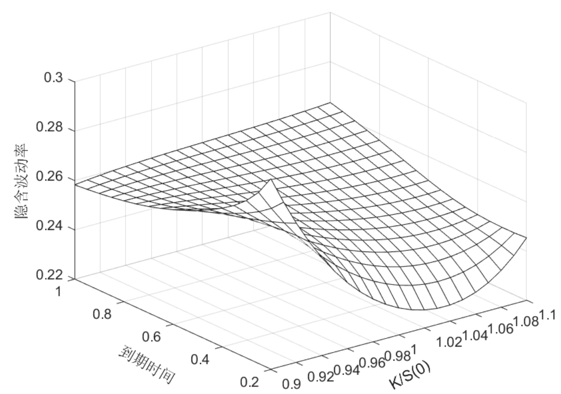
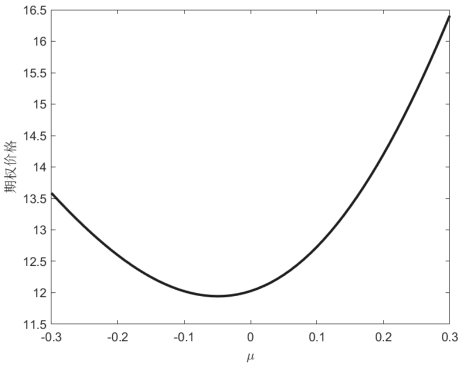
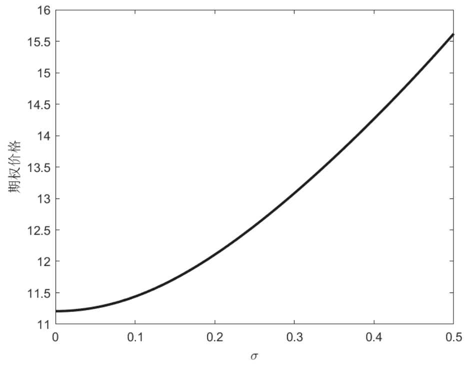

隐含波动率
到期时间
K/S(0)

图3

于跳跃幅度标准差的增函数. 图5表明, 该期权价格随着Hakwes过程的初始强度θ_λ和强度跳跃幅



(e)



(f)

图4

度η_λ的增加而增加, 且随着强度衰减率κ_λ的增大而下降. 这些是非常符合直觉的, 因为初始强度和强度跳跃幅度的增大会在固定的时期内使得发生跳跃的平均次数增加, 进而提高标的资产价格变化的不确定性和增大期权价值. 相反, 强度衰减率的增大会降低发生跳跃的次数, 从而降低期权价值.

§6 结论

鉴于近期实证研究广泛发现的资产价格跳跃聚集现象, 本文在跳跃扩散模型框架内采用Hawkes过程对资产价格跳跃建模. 此外, 将Heston随机波动率和Hull-White随机利率纳入模型, 并假设标的资产价格与其波动率之间的相关性由有界Jacobi过程所驱动. 基于该期权定价模型, 求得了欧式(看涨)期权价值的解析表达式. 数值分析结果表明, 首先, 相比常利率模型和常跳跃强度模型, 本文模型下的期权价值更高, 但当到期时间较短时, 随机利率和随机跳强度对期权价值影响甚微; 其次, 本文模型能生成形态丰富的隐含波动率曲面, 具备同时拟合隐含波动率微笑和隐含波动率倾斜的能力; 最后, 发现资产价格跳跃幅度的均值对期权价值的影响是非对称的, 以及期权价值是关于Hawkes过程的强度的跳跃幅度(强度衰减率)的增(减)函数.

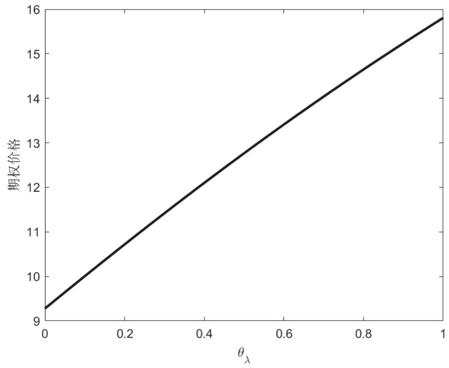
附录A

令$\boldsymbol{X}(t)=(\nu(t),\rho(t),X^C(t),r(t))'$, 则其对称瞬时协方差矩阵为

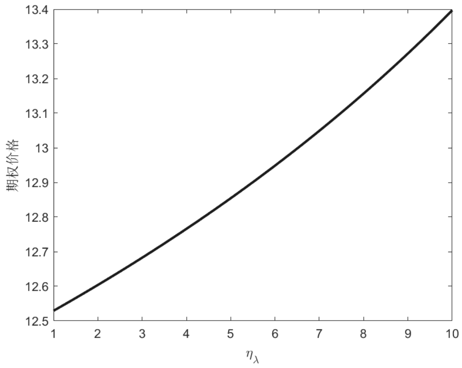
$$\sigma(\boldsymbol{X}(t))\sigma(\boldsymbol{X}(t))'=\begin{pmatrix} \eta_\nu^2\nu(t) & 0 & \eta_\nu\rho(t)\nu(t) & 0 \\ * & \eta_\rho^2(1-\rho^2(t)) & \rho_1\eta_\rho\sqrt{1-\rho^2(t)}\sqrt{\nu(t)} & 0 \\ * & * & \nu(t) & 0 \\ * & * & * & \eta_r^2 \end{pmatrix}. \tag{A.1}$$

根据Kolmogorov后向方程

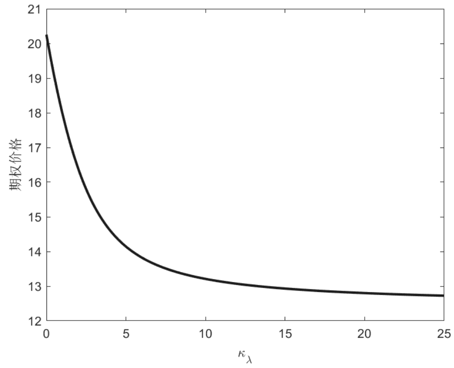
$$-\frac{\partial f_C}{\partial\tau}+(r-d-\frac{1}{2}\nu)\frac{\partial f_C}{\partial X^C}+\kappa_\nu(\theta_\nu-\nu)\frac{\partial f_C}{\partial\nu}+\kappa_\rho\left(\theta_\rho-\rho(t)\right)\frac{\partial f_C}{\partial\rho}+$$
$$(\beta(\theta_r-r)-\eta_r^2L)\frac{\partial f_C}{\partial r}+\frac{1}{2}\nu\frac{\partial^2 f_C}{\partial(X^C)^2}+\frac{1}{2}\eta_\nu^2\nu\frac{\partial^2 f_C}{\partial\nu^2}+\frac{1}{2}\eta_\rho^2(1-\rho^2(t))\frac{\partial^2 f_C}{\partial\rho^2}+$$



(g)



(h)



(i)

图5

$$\frac{1}{2}\eta_r^2\frac{\partial^2 f_C}{\partial r^2}+\eta_\nu\rho(t)\nu(t)\frac{\partial^2 f_C}{\partial X^C\partial\nu}+\rho_1\eta_\rho\sqrt{1-\rho^2(t)}\sqrt{\nu(t)}\frac{\partial^2 f_C}{\partial X^C\partial\rho}=0. \tag{A.2}$$

(A.2)是非线性偏微分方程, 仿射过程的一般结果在这里不再适用, 需对其非仿射项进行合理近似以便产生仿射形式. 由(A.1)可知, 非仿射项为$\eta_\nu\rho(t)\nu(t)$, $\eta_\rho^2(1-\rho^2(t))$和$\rho_1\eta_\rho\sqrt{1-\rho^2(t)}\sqrt{\nu(t)}$. 由文献[11]可得以下近似

$$\begin{cases}\eta_\nu\rho(t)\nu(t)\approx\eta_\nu\rho(t)\mathrm{E}^{\mathbb{Q}^\top}[\nu(t)],\\ \eta_\rho^2(1-\rho^2(t))\approx\eta_\rho^2(1-\mathrm{E}^{\mathbb{Q}^\top}[\rho^2(t)]),\\ \rho_1\eta_\rho\sqrt{1-\rho^2(t)}\sqrt{\nu(t)}\approx\rho_1\eta_\rho\mathrm{E}^{\mathbb{Q}^\top}[\sqrt{1-\rho^2(t)}]\mathrm{E}^{\mathbb{Q}^\top}[\sqrt{\nu(t)}],\end{cases} \tag{A.3}$$

其中$\mathrm{E}^{\mathbb{Q}^\top}[\nu(t)]=(\nu(0)-\theta_\nu)\,\mathrm{e}^{-\kappa_\nu(T-\tau)}+\theta_\nu$.

根据文献[11]可得

$$\mathrm{E}^{\mathbb{Q}^\top}\left[\rho^2(t)\right]\approx\mathrm{e}^{-m_2t}+b_2\mathrm{e}^{-n_2t}+a_2, \tag{A.4}$$

$$\mathrm{E}^{\mathbb{Q}^\top}[\sqrt{1-\rho^2(t)}]\approx\mathrm{e}^{-m_3t}+b_3\mathrm{e}^{-n_3t}+a_3, \tag{A.5}$$

$$\mathrm{E}^{\mathbb{Q}^\top}[\sqrt{\nu(t)}]\approx m+n\mathrm{e}^{-lt}, \tag{A.6}$$

其中

$$\begin{aligned}
a_2&=\frac{\left(\eta_\rho^2+\kappa_\rho\right)\left(\eta_\rho^2+2\kappa_\rho\theta_\rho^2\right)}{\eta_\rho^4+3\kappa_\rho\eta_\rho^2+2\kappa_\rho^2},\quad b_2=\rho^2(0)-a_2-1,\\
m_2&=-2\log\left(\gamma_1-b_2\mathrm{e}^{-\frac{n_2}{2}}\right),\quad n_2=-2\log\left(\frac{b_2\gamma_1-\sqrt{b_2^2\gamma_1^2-\gamma_2\gamma_3}}{\gamma_2}\right),\\
\gamma_1&=f_2(0.5)-a_2,\quad \gamma_2=b_2+b_2^2,\quad \gamma_3=\gamma_1^2+a_2-f_2(1).\\
a_3&=\sqrt{1-\frac{\left(\eta_\rho^2+\kappa_\rho\right)\left(\eta_\rho^2+2\kappa_\rho\theta_\rho^2\right)-\theta_\rho^4\left(\eta_\rho^4+3\kappa_\rho\eta_\rho^2+2\kappa_\rho^2\right)}{\left(1-\theta_\rho^2\right)\left(\eta_\rho^4+3\kappa_\rho\eta_\rho^2+2\kappa_\rho^2\right)}},\\
b_3&=\sqrt{1-\rho^2(0)}-a_3-1,\\
m_3&=-2\log\left(\eta_1-b_3\mathrm{e}^{-\frac{n_3}{2}}\right),\quad n_3=-2\log\left(\frac{b_3\eta_1-\sqrt{b_3^2\eta_1^2-\eta_2\eta_3}}{\eta_2}\right),\\
\eta_1&=f_3(0.5)-a_3,\quad \eta_2=b_3+b_3^2,\quad \eta_3=\eta_1^2+a_3-f_3(1).
\end{aligned}$$

$$\begin{aligned}
m&=\sqrt{\theta_\nu-\frac{\eta_\nu^2}{8\kappa_\nu}},\quad n=\sqrt{\nu(0)}-m,\quad l=-\log\left(n^{-1}(\hat{d}-m)\right),\\
\hat{d}&=\sqrt{\left(\nu(0)\mathrm{e}^{-\kappa_\nu}-\frac{\eta_\nu^2\left(1-\mathrm{e}^{-\kappa_\nu}\right)}{4\kappa_\nu}\right)+\theta_\nu\left(1-\mathrm{e}^{-\kappa_\nu}\right)+\frac{\eta_\nu^2\theta_\nu\left(1-\mathrm{e}^{-\kappa_\nu}\right)^2}{8\kappa_\nu\theta_\nu+8\kappa_\nu\mathrm{e}^{-\kappa_\nu}\left(\nu(0)-\theta_\nu\right)}}.
\end{aligned}$$

根文献[25], $X^C(t)$的特征函数具有形式

$$f_C(\phi;\tau,X,\nu,\rho,\lambda,r)=\mathrm{e}^{A(\phi,\tau)+B(\phi,\tau)\nu+C(\phi,\tau)\rho+D(\phi,\tau)r+\mathrm{i}\phi X^C}, \tag{A.7}$$

其中$A(\phi,0)=0$, $B(\phi,0)=0$, $C(\phi,0)=0$, $D(\phi,0)=0$. 把(A.7)代入(A.2), 并通过(A.3)-(A.6)近似替换得

$$\frac{\partial B}{\partial\tau}=\frac{1}{2}\eta_\nu^2B^2-\kappa_\nu B-\frac{1}{2}(\mathrm{i}\phi+\phi^2), \tag{A.8}$$

$$\frac{\partial C}{\partial\tau}=-\kappa_\rho C+\eta_\nu\mathrm{i}\phi\mathrm{E}^{\mathbb{Q}^\top}(\nu(t))B, \tag{A.9}$$

$$\frac{\partial D}{\partial\tau}=-\beta D+\mathrm{i}\phi, \tag{A.10}$$

$$\frac{\partial A}{\partial \tau}=\kappa_\nu\theta_\nu B+\kappa_\rho\theta_\rho C+(\beta\theta_r-\eta_r^2 N)D+\frac{1}{2}\eta_\rho^2\left(1-\mathrm{E}^{\mathbb{Q}^\top}[\rho^2(t)]\right)C^2+$$
$$\frac{1}{2}\eta_r^2 D^2+\eta_\rho\rho_1\mathrm{E}^{\mathbb{Q}^\top}[\sqrt{\nu(t)}]\mathrm{E}^{\mathbb{Q}^\top}[\sqrt{1-\rho^2(t)}]\mathrm{i}\phi C-d\mathrm{i}\phi.$$

(A.8)是关于B的具有常系数的Riccati方程, 通过简单计算可求出B; (A.10)是关于D的一元线性常微分方程, 因此可直接求解. 求出的B和D分别为

$$B(\phi,\tau)=\frac{\kappa_\nu-b_1(\phi)}{\eta_\nu^2}\frac{1-\mathrm{e}^{-b_1(\phi)\tau}}{1-g_1(\phi)\mathrm{e}^{-b_1(\phi)\tau}},\tag{A.11}$$

$$D(\phi,\tau)=\frac{\mathrm{i}\phi(1-\mathrm{e}^{-\beta\tau})}{\beta},\tag{A.12}$$

其中

$$b_1(\phi)=\sqrt{\kappa_\nu^2+\eta_\nu^2(\mathrm{i}\phi+\phi^2)},\quad g_1(\phi)=\frac{\kappa_\nu-b_1(\phi)}{\kappa_\nu+b_1(\phi)}.$$

求解$C(\phi,\tau)$, 可以将(A.11)和$\mathrm{E}^{\mathbb{Q}^\top}[\nu(t)]=(\nu(0)-\theta_\nu)\,\mathrm{e}^{-\kappa_\nu(T-\tau)}+\theta_\nu$ 代入(A.9)得

$$C(\phi,\tau)=\frac{C_1\eta_\nu\left(\theta_\nu-\nu(0)\right)}{\kappa_\nu+\kappa_\rho-l_1}\mathrm{e}^{(\kappa_\nu-l_1)\tau-\kappa_\nu T}+\frac{C_1\eta_\nu\left(\nu(0)-\theta_\nu\right)}{\kappa_\nu+\kappa_\rho}\mathrm{e}^{\kappa_\nu(\tau-T)}+$$
$$\frac{C_1\eta_\nu\theta_\nu}{\kappa_\rho}-\frac{C_1\eta_\nu\theta_\nu}{\kappa_\rho-l_1}\mathrm{e}^{-l_1\tau}+C_1C_2\mathrm{e}^{-\kappa_\rho\tau},\tag{A.13}$$

其中

$$\begin{aligned}C_2&=-\frac{\eta_\nu(\theta_\nu-\nu(0))}{\kappa_\nu+\kappa_\rho-l_1}\mathrm{e}^{-\kappa_\nu T}-\frac{\eta_\nu(\nu(0)-\theta_\nu)}{\kappa_\nu+\kappa_\rho}\mathrm{e}^{-\kappa_\nu T}-\frac{\eta_\nu\theta_\nu}{\kappa_\rho}+\frac{\eta_\nu\theta_\nu}{\kappa_\rho-l_1},\\ l_1&=-\ln\left(\frac{\mathrm{e}^{-b_1(\phi)}-g_1(\phi)\mathrm{e}^{-b_1(\phi)}}{1-g_1(\phi)\mathrm{e}^{-b_1(\phi)}}\right),C_1=\mathrm{i}\phi\frac{\kappa_\nu-b_1(\phi)}{\eta_\nu^2}.\end{aligned}$$

求解$A(\phi,\tau)$, 可以利用(A.4)-(A.6)近似替换$\mathrm{E}^{\mathbb{Q}^\top}[\sqrt{\nu(t)}]$, $\mathrm{E}^{\mathbb{Q}^\top}[\rho^2(t)]$和$\mathrm{E}^{\mathbb{Q}^\top}[\sqrt{1-\rho^2(t)}]$, 从而得到

$$\frac{\partial A}{\partial \tau}=\kappa_\nu\theta_\nu B+\kappa_\rho\theta_\rho C+(\beta\theta_r-\eta_r^2 L)D+\frac{1}{2}\eta_\rho^2[1-(\mathrm{e}^{-m_2t}+b_2\mathrm{e}^{-n_2t}+a_2)]C^2+$$
$$\frac{1}{2}\eta_r^2D^2+\eta_\rho\rho_1[m+n\mathrm{e}^{-lt}][\mathrm{e}^{-m_3t}+b_3\mathrm{e}^{-n_3t}+a_3]\mathrm{i}\phi C-d\mathrm{i}\phi.\tag{A.14}$$

把(A.11)-(A.13)和$L(t,T)=\frac{1}{\beta}(1-\mathrm{e}^{-\beta(T-t)})$代入(A.14), 然后直接对$A(\phi,\tau)$进行积分得到

$$A(\phi,\tau)=\kappa_\nu\theta_\nu H_0(\phi,\tau)+(\kappa_\rho\theta_\rho+\eta_\rho\rho_1ma_3\mathrm{i}\phi)H_1(\phi,\tau)+\beta\theta_rH_2(\phi,\tau)-\eta_r^2H_3(\phi,\tau)-$$
$$d\mathrm{i}\phi\tau+\frac{1}{2}\eta_r^2H_4(\phi,\tau)+\frac{1}{2}\eta_\rho^2(1-a_2)H_5(\phi,\tau)+\eta_\rho\rho_1m\mathrm{i}\phi H_6(\phi,\tau)+\eta_\rho\rho_1mb_3\mathrm{i}\phi H_7(\phi,\tau)+$$
$$\eta_\rho\rho_1n\mathrm{i}\phi H_8(\phi,\tau)+\eta_\rho\rho_1nb_3\mathrm{i}\phi H_9(\phi,\tau)+\eta_\rho\rho_1na_3\mathrm{i}\phi H_{10}(\phi,\tau)-\frac{1}{2}\eta_\rho^2H_{11}(\phi,\tau)-$$
$$\frac{1}{2}\eta_\rho^2b_2H_{12}(\phi,\tau),$$

其中

$$\begin{aligned}
H_0(\phi,\tau) &= \frac{1}{\eta_\nu^2}((\kappa_\nu - b_1(\phi))\tau - 2\ln(\frac{1-g_1(\phi)\mathrm{e}^{-b_1(\phi)\tau}}{1-g_1(\phi)})),\\
H_1(\phi,\tau) &= \frac{C_1\eta_\nu(\theta_\nu-\nu(0))\mathrm{e}^{\kappa_\nu(\tau-T)-l_1\tau}}{(\kappa_\nu+\kappa_\rho-l_1)(\kappa_\nu-l_1)} + \frac{C_1\eta_\nu(\nu(0)-\theta_\nu)\mathrm{e}^{\kappa_\nu(\tau-T)}}{(\kappa_\nu+\kappa_\rho)\kappa_\nu} + \frac{C_1\eta_\nu\theta_\nu\tau}{\kappa_\rho} +\\
&\quad \frac{C_1\eta_\nu\theta_\nu\mathrm{e}^{-l_1\tau}}{(\kappa_\rho-l_1)l_1} - \frac{C_1C_2\mathrm{e}^{-\kappa_\rho\tau}}{\kappa_\rho} + H_{2c},\\
H_2(\phi,\tau) &= \frac{\mathrm{i}\phi}{\beta^2}\mathrm{e}^{-\beta\tau} + \frac{\mathrm{i}\phi}{\beta}\tau - \frac{\mathrm{i}\phi}{\beta^2},\\
H_3(\phi,\tau) &= \frac{\mathrm{i}\phi}{2\beta^3}(1-\mathrm{e}^{-2\beta\tau}) + \frac{2\mathrm{i}\phi}{\beta^3}(\mathrm{e}^{-\beta\tau}-1) + \frac{\mathrm{i}\phi}{\beta^2}\tau,\\
H_4(\phi,\tau) &= \frac{\phi^2}{2\beta^3}(\mathrm{e}^{-2\beta\tau}-1) - \frac{2\phi^2}{\beta^3}(\mathrm{e}^{-\beta\tau}-1) - \frac{\phi^2}{\beta^2}\tau,\\
H_{10}(\phi,\tau) &= \frac{C_1\eta_\nu(\theta_\nu-\nu(0))\mathrm{e}^{\tau(\kappa_\nu+l-l_1)-T(\kappa_\nu+l)}}{(\kappa_\nu+\kappa_\rho-l_1)(\kappa_\nu+l-l_1)} + \frac{C_1\eta_\nu(\nu(0)-\theta_\nu)\mathrm{e}^{(\tau-T)(l+\kappa_\nu)}}{(l+\kappa_\nu)(\kappa_\nu+\kappa_\rho)} +\\
&\quad \frac{C_1\eta_\nu\theta_\nu\mathrm{e}^{l(\tau-T)}}{\kappa_\rho l} - \frac{C_1\eta_\nu\theta_\nu\mathrm{e}^{\tau(l-l_1)-lT}}{(\kappa_\rho-l_1)(l-l_1)} + \frac{C_1C_2\mathrm{e}^{\tau(l-\kappa_\rho)-lT}}{l-\kappa_\rho} - H_{3c},\\
H_5(\phi,\tau) &= I_1\mathrm{e}^{2\kappa_\nu(\tau-T)} + I_4\mathrm{e}^{-\tau l_1} + I_5\mathrm{e}^{(-l_1-\kappa_\rho)\tau} + I_9\mathrm{e}^{\tau(\kappa_\nu-\kappa_\rho)-\kappa_\nu T} +\\
&\quad I_3\mathrm{e}^{2\tau(\kappa_\nu-l_1)-2\kappa_\nu T} + I_2\mathrm{e}^{2\kappa_\nu(\tau-T)-l_1\tau} +\\
&\quad I_{11}\mathrm{e}^{\tau(\kappa_\nu-\kappa_\rho-l_1)-\kappa_\nu T} + I_{12}\mathrm{e}^{\tau(\kappa_\nu-l_1)-\kappa_\nu T} +\\
&\quad I_{13}\mathrm{e}^{\tau(\kappa_\nu-2l_1)-\kappa_\nu T} + I_6\mathrm{e}^{-2\kappa_\rho\tau} + I_7\mathrm{e}^{-\kappa_\rho\tau} +\\
&\quad I_8\mathrm{e}^{-2l_1\tau} + I_{10}\mathrm{e}^{(\kappa_\nu-l_1)\tau-\kappa_\nu T} + I_{14}\mathrm{e}^{\kappa_\nu(\tau-T)} + \frac{C_1^2\eta_\nu^2\theta_\nu^2\tau}{\kappa_\rho^2} - H_{4c},
\end{aligned}$$

以及

$$\begin{aligned}
H_{2c} &= \frac{C_1\eta_\nu(\nu(0)-\theta_\nu)\mathrm{e}^{-\kappa_\nu T}}{(\kappa_\nu+\kappa_\rho-l_1)(\kappa_\nu-l_1)} - \frac{C_1\eta_\nu(\nu(0)-\theta_\nu)\mathrm{e}^{-\kappa_\nu T}}{\kappa_\nu(\kappa_\nu+\kappa_\rho)} - \frac{C_1\eta_\nu\theta_\nu}{(\kappa_\rho-l_1)l_1} + \frac{C_1C_2}{\kappa_\rho},\\
H_{3c} &= \frac{C_1\eta_\nu(\theta_\nu-\nu(0))\mathrm{e}^{-T(\kappa_\nu+l)}}{(\kappa_\nu+\kappa_\rho-l_1)(\kappa_\nu+l-l_1)} + \frac{C_1\eta_\nu(\nu(0)-\theta_\nu)\mathrm{e}^{-T(l+\kappa_\nu)}}{(l+\kappa_\nu)(\kappa_\nu+\kappa_\rho)} + \frac{C_1\eta_\nu\theta_\nu\mathrm{e}^{-lT}}{\kappa_\rho l} -\\
&\quad \frac{C_1\eta_\nu\theta_\nu\mathrm{e}^{-lT}}{(\kappa_\rho-l_1)(l-l_1)} + \frac{C_1C_2\mathrm{e}^{-lT}}{l-\kappa_\rho},\\
H_{4c} &= (I_1+I_2+I_3)\,\mathrm{e}^{-2\kappa_\nu T} + I_4+I_5+I_6+I_7+I_8 +\\
&\quad (I_9+I_{10}+I_{11}+I_{12}+I_{13}+I_{14})\,\mathrm{e}^{-\kappa_\nu T},
\end{aligned}$$

$$\begin{aligned}
I_1 &= \frac{C_1^2\eta_\nu^2(\nu(0)-\theta_\nu)^2}{2\kappa_\nu(\kappa_\nu+\kappa_\rho)^2}, \quad I_2=\frac{-2C_1^2\eta_\nu^2(\nu(0)-\theta_\nu)^2}{(\kappa_\nu+\kappa_\rho-l_1)(\kappa_\nu+\kappa_\rho)(2\kappa_\nu-l_1)},\\
I_3 &= \frac{C_1^2\eta_\nu^2(\nu(0)-\theta_\nu)^2}{2(\kappa_\nu+\kappa_\rho-l_1)^2(\kappa_\nu-l_1)}, \quad I_4=\frac{2C_1^2\eta_\nu^2\theta_\nu^2}{\kappa_\rho l_1(\kappa_\rho-l_1)},\\
I_5 &= \frac{2\eta_\nu\theta_\nu C_1^2C_2}{\kappa_\rho^2-l_1^2}, \quad I_6=-\frac{1}{2}\frac{C_1^2C_2^2}{\kappa_\rho}, \quad I_7=-\frac{2\eta_\nu\theta_\nu C_1^2C_2}{\kappa_\rho^2},\\
I_{10} &= \frac{2C_1^2\eta_\nu^2(\theta_\nu^2-\nu(0)\theta_\nu)}{(\kappa_\nu+\kappa_\rho)(\kappa_\nu-l_1)(\kappa_\rho-l_1)}, \quad I_8=-\frac{1}{2}\frac{\eta_\nu^2\theta_\nu^2C_1^2}{l_1(\kappa_\rho-l_1)^2},\\
I_{11} &= \frac{2\eta_\nu(\theta_\nu-\nu(0))C_1^2C_2}{(\kappa_\nu+\kappa_\rho-l_1)(\kappa_\nu-\kappa_\rho-l_1)}, \quad I_9=\frac{2(\nu(0)-\theta_\nu)C_1^2C_2\eta_\nu}{(\kappa_\nu+\kappa_\rho)(\kappa_\nu-\kappa_\rho)},\\
I_{12} &= \frac{2C_1^2\eta_\nu^2(\theta_\nu^2-\nu(0)\theta_\nu)}{\kappa_\rho(\kappa_\nu-l_1)(\kappa_\nu+\kappa_\rho-l_1)}, \quad I_{14}=\frac{2C_1^2\eta_\nu^2(\nu(0)\theta_\nu-\theta_\nu^2)}{\kappa_\nu\kappa_\rho(\kappa_\nu+\kappa_\rho)},\\
I_{13} &= \frac{2C_1^2\eta_\nu^2(\nu(0)\theta_\nu-\theta_\nu^2)}{(\kappa_\rho-l_1)(\kappa_\nu-2l_1)(\kappa_\nu+\kappa_\rho-l_1)}.
\end{aligned}$$

令$H_{10}(\phi,\tau)=H_{10}(\phi,\tau,l)$, 则$H_6(\phi,\tau)=H_{10}(\phi,\tau,m_3)$, $H_7(\phi,\tau)=H_{10}(\phi,\tau,n_3)$, $H_8(\phi,\tau)=H_{10}(\phi,\tau,l+m_3)$, $H_9(\phi,\tau)=H_{10}(\phi,\tau,l+n_3)$. 令$H(\phi,\tau,y)=\int_0^\tau \mathrm{e}^{-y(T-\tau)}C^2(\phi,\tau)\mathrm{d}\tau$, 可得$H_{11}(\phi,\tau)=H(\phi,\tau,m_2)$, $H_{12}(\phi,\tau)=H(\phi,\tau,n_2)$.

对$H(\phi,\tau,y)$求积分求得

$$\begin{aligned}
H(\phi,\tau,y) = {} & I_1\mathrm{e}^{(2\kappa_\nu+y)(\tau-T)}+I_4\mathrm{e}^{(-l_1+y)\tau-yT}+I_5\mathrm{e}^{-(\kappa_\rho+l_1+y)\tau-yT}+I_2\mathrm{e}^{(2\kappa_\nu+y)(\tau-T)-l_1\tau}+\\
& I_3\mathrm{e}^{(2(\kappa_\nu-l_1)+y)\tau-(2\kappa_\nu+y)T}+I_9\mathrm{e}^{(\kappa_\nu-\kappa_\rho+y)\tau-(\kappa_\nu+y)T}+\\
& I_{11}\mathrm{e}^{(\kappa_\nu-\kappa_\rho-l_1+y)\tau-(\kappa_\nu+y)T}+I_{12}\mathrm{e}^{(\kappa_\nu-l_1+y)\tau-(\kappa_\nu+y)T}+\\
& I_{13}\mathrm{e}^{(\kappa_\nu-2l_1+y)\tau-(\kappa_\nu+y)T}+I_6\mathrm{e}^{(-2\kappa_\rho+y)\tau-yT}+I_7\mathrm{e}^{(-\kappa_\rho+y)\tau-yT}+\\
& I_8\mathrm{e}^{(-2l_1+y)\tau-yT}+I_{10}\mathrm{e}^{(\kappa_\nu-l_1+y)\tau-(\kappa_\nu+y)T}+I_{14}\mathrm{e}^{(\kappa_\nu+y)(\tau-T)}+\\
& I_{15}\mathrm{e}^{y(\tau-T)}-H_{4c},
\end{aligned}$$

其中

$$\begin{aligned}
H_{4c} &= (I_1+I_2+I_3)\,\mathrm{e}^{-(2\kappa_\nu+y)T}+(I_4+I_5+I_6+I_7+I_8+I_{15})\mathrm{e}^{-yT}+\\
&\quad (I_9+I_{10}+I_{11}+I_{12}+I_{13}+I_{14})\,\mathrm{e}^{-(\kappa_\nu+y)T},\\
I_1 &= \frac{C_1^2\eta_\nu^2(\nu(0)-\theta_\nu)^2}{(2\kappa_\nu+y)(\kappa_\nu+\kappa_\rho)^2}, \quad I_2=\frac{-2C_1^2\eta_\nu^2(\nu(0)-\theta_\nu)^2}{(\kappa_\nu+\kappa_\rho-l_1)(\kappa_\nu+\kappa_\rho)(2\kappa_\nu+y-l_1)},\\
I_3 &= \frac{C_1^2\eta_\nu^2(\nu(0)-\theta_\nu)^2}{(\kappa_\nu+\kappa_\rho-l_1)^2(2(\kappa_\nu-l_1)+y)}, \quad I_4=-\frac{2C_1^2\eta_\nu^2\theta_\nu^2}{\kappa_\rho(-l_1+y)(\kappa_\rho-l_1)},\\
I_5 &= \frac{2\eta_\nu\theta_\nu C_1^2C_2}{(\kappa_\rho-l_1)(\kappa_\rho+l_1+y)}, \quad I_6=\frac{C_1^2C_2^2}{-2\kappa_\rho+y}, \quad I_7=\frac{2\eta_\nu\theta_\nu C_1^2C_2}{\kappa_\rho(-\kappa_\rho+y)},\\
I_{10} &= \frac{2C_1^2\eta_\nu^2(\theta_\nu^2-\nu(0)\theta_\nu)}{(\kappa_\nu+\kappa_\rho)(\kappa_\nu-l_1+y)(\kappa_\rho-l_1)}, \quad I_8=\frac{\eta_\nu^2\theta_\nu^2C_1^2}{(\kappa_\rho-l_1)^2(-2l_1+y)},\\
I_{11} &= \frac{2\eta_\nu(\theta_\nu-\nu(0))C_1^2C_2}{(\kappa_\nu+\kappa_\rho-l_1)(\kappa_\nu-\kappa_\rho-l_1+y)}, \quad I_9=\frac{2(\nu(0)-\theta_\nu)C_1^2C_2\eta_\nu}{(\kappa_\nu+\kappa_\rho)(\kappa_\nu-\kappa_\rho+y)},
\end{aligned}$$

$$
\begin{aligned}
I_{12} &= \frac{2C_1^2\eta_\nu^2(\theta_\nu^2-\nu(0)\theta_\nu)}{\kappa_\rho(\kappa_\nu-l_1+y)(\kappa_\nu+\kappa_\rho-l_1)}, \quad I_{14}=\frac{2C_1^2\eta_\nu^2(\nu(0)\theta_\nu-\theta_\nu^2)}{(\kappa_\nu+y)\kappa_\rho(\kappa_\nu+\kappa_\rho)},\\
I_{13} &= \frac{2C_1^2\eta_\nu^2(\nu(0)\theta_\nu-\theta_\nu^2)}{(\kappa_\rho-l_1)(\kappa_\nu-2l_1+y)(\kappa_\nu+\kappa_\rho-l_1)}, \quad I_{15}=\frac{C_1^2\eta_\nu^2\theta_\nu^2}{\kappa_\rho^2 y},
\end{aligned}
$$

从而求出$A(\phi,\tau)$, $B(\phi,\tau)$, $C(\phi,\tau)$和$D(\phi,\tau)$, 进而求得$f_C(\phi;\tau,X,\nu,\rho,\lambda,r)$. 通过(A.7)和(13)可得$X(t)$的特征函数

$$
f(\phi;\tau,X,\nu,\rho,\lambda,r)=f_C(\phi;\tau,X,\nu,\rho,\lambda,r)f_J(\phi;\tau,X,\nu,\rho,\lambda,r).
$$

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Option valuation with stochastic correlation under the Hawkes jump-diffusion model

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Abstract: Based on the stochastic volatility model, assuming that the underlying asset price's jumps are driven by the self-exciting Hawkes jump processes, the risk-free interest rate is stochastic, and the underlying price is randomly correlated with its volatility. An analytical pricing expression for the standard European options pricing has been obtained under the proposed model. In the numerical analysis, the option value of the proposed model is higher than the constant interest rate model and the constant jump intensity model, but when the expiration time is short, the random interest rate and random jump intensity have little effect on the option value. In addition, the proposed model can produce flexible pattern of the implied volatility surface, and therefore it is capable of fitting both the implied volatility smile and the implied volatility skew which are empirically observed in option markets.

Keywords: option pricing; Hawkes processes; stochastic volatility; stochastic interest rate; stochastic correlation

MR Subject Classification: 60E10; 91G20