高校应用数学学报
2025, 40(1): 89-97

具有非局部时滞的Lotka-Volterra竞争系统的行波解

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摘　要: 该文考虑了具有非局部时滞的两种群竞争系统的行波解, 借助于具有非局部时滞反应扩散方程行波解的存在性理论, 证明了当选取特定核函数且波速c大于某临界值时, 连接系统两个半平凡平衡点行波解的存在性.

关键词: 反应扩散方程; 非局部时滞; 行波解; 存在性

中图分类号: O175.26

文献标识码: A　　**文章编号**: 1000-4424(2025)01-0089-09

§1　引　言

自从Lotka[1]和Volterra[2]提出著名的Lotka-Volterra种群模型后, 种群动力学的研究已经取得了长足发展. 基于Lotka-Volterra竞争系统, 许多学者提出了各种不同含有扩散项的系统并研究了其动力学行为, 获得了十分丰富的结论, 参看文献[3-6]. Al-Omari和Gourley[7]考虑了含有扩散项的Lotka-Volterra竞争模型

$$\begin{cases} \frac{\partial U_1(x,t)}{\partial t} = d_1 \Delta U_1(x,t) + \alpha_1 \mathrm{e}^{-\gamma_1\tau_1} U_1(x,t-\tau_1) - \beta_1 U_1^2(x,t) - c_1 U_1(x,t)U_2(x,t), \\ \frac{\partial U_2(x,t)}{\partial t} = d_2 \Delta U_2(x,t) + \alpha_2 \mathrm{e}^{-\gamma_2\tau_2} U_2(x,t-\tau_2) - \beta_2 U_2^2(x,t) - c_2 U_1(x,t)U_2(x,t), \end{cases} \tag{1.1}$$

其中$(x,t)\in \Omega\times[0,+\infty)$, $\Omega=\mathbf{R}^n$, Δ为关于x的Laplace算子, $U(x,t)$和$V(x,t)$分别表示两个竞争成年种群的密度, 正常数d_1和d_2分别表示两个种群成年种群的扩散系数, 正常数α_1和α_2分别表示两个种群的成年种群的出生率, 正常数γ_1和γ_2分别表示两种群在成熟过程中的死亡率, 正常数β_1和β_2分别表示两个种群的成熟种群的死亡率, 正常数c_1和c_2分别表示两种群间的竞争效应, $\tau_1,\tau_2>0$是常数.

文献[7]首先得到了系统(1.1)三个常数边界平衡解

$$E_0=(0,0),\ \hat{E}_1=\left(\frac{\alpha_1}{\beta_1}\mathrm{e}^{-\gamma_1\tau_1},0\right),\ \hat{E}_2=\left(0,\frac{\alpha_2}{\beta_2}\mathrm{e}^{-\gamma_2\tau_2}\right)$$

的全局渐近稳定性, 并在$\hat{E}_2$不稳定且$\hat{E}_1$线性稳定, 即系统(1.1)不存在共存平衡点时, 借助于文献[8]关于具有离散时滞反应扩散方程行波解的存在性理论得到了连接系统(1.1)两个半平凡平衡点$\hat{E}_2$和$\hat{E}_1$行波解的存在性.

收稿日期: 2024-09-18　　修回日期: 2025-02-23
基金项目: 国家自然科学基金(12261054)

在提出系统(1.1)时, 假设两种群个体仅在自己当前所处的栖息位置上存在竞争、同一种群的成熟时间是相同的且忽略了种群个体在栖息地中的走动. 为了考虑这些因素, 文献[9-10]提出了非局部时滞(时间-空间的非局部)的概念, 它是通过一个加权函数(在假设种群个体是随机走动的条件下利用概率分析得到的)表示的. 基于此并结合系统(1.1), 本文主要考虑以下具有非局部时滞的Lotka-Volterra竞争系统

$$\begin{cases} \frac{\partial U(x,t)}{\partial t} = d_1 \triangle U + \alpha_u \int_\Omega \int_{-\infty}^t g(x-y,t-s)U(y,s)\mathrm{d}y\mathrm{d}s - \\ \qquad \beta_u U^2(x,t) - c_1 U(x,t)V(x,t), \\ \frac{\partial V(x,t)}{\partial t} = d_2 \triangle V + \alpha_v \int_\Omega \int_{-\infty}^t h(x-y,t-s)V(y,s)\mathrm{d}y\mathrm{d}s - \\ \qquad \beta_v V^2(x,t) - c_2 U(x,t)V(x,t), \end{cases} \tag{1.2}$$

其中核函数$g(y,s)$和$h(y,s)$在其变量$s \in (0,+\infty)$, $y \in \Omega$情况下是可积的非负函数; 卷积项$\int_\Omega \int_0^\infty g(y,s)U(y,s)\mathrm{d}s$和$\int_\Omega \int_0^\infty h(y,s)V(y,s)\mathrm{d}s$分别描述了两种群内部对资源的竞争; 此外规定

$$\int_\Omega \int_0^\infty g(y,s)\mathrm{d}s = \int_\Omega \int_0^\infty h(y,s)\mathrm{d}s = 1.$$

可以得到, 对任意可行参数值, 系统(1.2)总是有平凡平衡点$E_0 = (0,0)$, 半平凡平衡点$\hat{E_u} = \left(\frac{\alpha_u}{\beta_u}, 0\right)$和$\hat{E_v} = \left(0, \frac{\alpha_v}{\beta_v}\right)$. 此后借助于线性化方法分析容易得到, 系统(1.2)的平衡点E_0总是不稳定的; 当条件

(H1) $c_2\alpha_u > \beta_u\alpha_v$

成立时, 系统(1.2)的平衡点$\hat{E_u}$是线性稳定的; 当条件

(H2) $c_1\alpha_v < \beta_v\alpha_u$

成立时, 系统(1.2)的平衡点$\hat{E_v}$是不稳定的; 当系统(1.2)两个半平凡平衡点$\hat{E_u}$和$\hat{E_v}$都线性稳定或者都不稳定时, 共存平衡点$\hat{E}(\hat{u},\hat{v})$是存在的, 反之共存平衡点$\hat{E}(\hat{u},\hat{v})$不存在, 其中

$$\hat{u} = \frac{\beta_v\alpha_u - c_1\alpha_v}{\beta_u\beta_v - c_1c_2}, \hat{v} = \frac{\beta_u\alpha_v - c_2\alpha_u}{\beta_u\beta_v - c_1c_2}.$$

§2主要阐述文献[11]建立的具有非局部时滞反应扩散方程行波解的存在性的相关理论; §3主要借助于§2的理论考虑当条件(H1)和(H2)成立且选取特定核函数$g(y,s) = h(y,s) = \frac{1}{\tau_0}\mathrm{e}^{-\frac{s}{\tau_0}}\delta(y), \tau_0 > 0$, $\Omega = (-\infty,+\infty)$时, 连接系统(1.2)两个半平凡平衡点$\hat{E_v}$和$\hat{E_u}$之间行波解的存在性问题.

§2 预备知识

本节主要阐述文献[11]中建立的具有非局部时滞反应扩散方程行波解的存在性的相关理论.

考虑具有非局部时滞的反应扩散方程

$$\frac{\partial u(x,t)}{\partial t} = D\frac{\partial^2 u(x,t)}{\partial x^2} + f(u(x,t),(g_1*u)(x,t),\cdots,(g_m*u)(x,t)), \tag{2.1}$$

其中$(x,t) \in \mathbf{R}\times[0,+\infty)$, $D = \mathrm{diag}(d_1,\cdots,d_n)$, $d_i > 0, i = 1,\cdots,n, n \in \mathbf{N}$, $u \in \mathbf{R}^n, f \in C(\mathbf{R}^{(m+1)n},\mathbf{R}^n)$, 且

$$(g_j*u)(x,t) = \int_{-\infty}^t \int_{-\infty}^\infty g_j(x-y,t-s)u(y,s)\mathrm{d}y\mathrm{d}s,$$

核函数$g_j(x,t)$是可积的非负函数, 满足$g_j(-x,t)=g_j(x,t)$, $\int_0^{+\infty}\int_{-\infty}^{\infty}g_j(y,s)\mathrm{d}y\mathrm{d}s=1, j=1,\cdots,m, m\in\mathbf{N}$; 此外假设核函数对于$t\in[0,a], a>0$有$\int_{-\infty}^{+\infty}g_j(x,t)\mathrm{d}x$是一致收敛的, 即存在$\varepsilon>0$, 对于任意$t\in[0,a]$, 有$\int_M^{+\infty}g_j(x,t)\mathrm{d}x<\varepsilon$.

首先, 将$u(x,t)=\Phi(z), z=x+ct$, $c(c>0)$为波速, 代入系统(2.1)后, 变为行波方程

$$D\Phi''(z)-c\Phi'(z)+f(\Phi(z),(g_1*\Phi)(z),\cdots,(g_m*\Phi)(z))=0, z\in\mathbf{R}, \tag{2.2}$$

其中$(g_j*\Phi)(z)=\int_0^{+\infty}\int_{-\infty}^{+\infty}g_j(y,s)\Phi(z-y-cs), j=1,\cdots,m$, 系统(2.1)波速为$c(c>0)$的行波解$\Phi\in C^2(\mathbf{R},\mathbf{R}^n)$, 且满足边界条件

$$\Phi(-\infty)=\mathbf{0}, \Phi(+\infty)=\boldsymbol{K}=(K_1,\cdots,K_n)^{\mathrm{T}}, \text{ 其中}K_i>0, i=1,2,\cdots,n.$$

然后, 为了证明行波解的存在性, 假设以下条件(i)-(iii)成立.

(i) 存在对角矩阵$\gamma=\mathrm{diag}(\gamma_1,\cdots,\gamma_n), \gamma_i>0, i=1,\cdots,n$使得

$$f(\Phi_2(z),(g_1*\Phi_2)(z),\cdots,(g_m*\Phi_2)(z))-f(\Phi_1(z),(g_1*\Phi_1)(z),\cdots,(g_m*\Phi_1)(z))\geq -\gamma(\Phi_2(z)-\Phi_1(z)),$$

其中$\Phi_1,\Phi_2\in C(\mathbf{R},\mathbf{R}^n)$, 当$z\in\mathbf{R}$ 时, 有$\mathbf{0}\leq\Phi_1(z)\leq\Phi_2(z)\leq\boldsymbol{K}$;

(ii) 当$\mathbf{0}<\mu<\boldsymbol{K}$时, 有$f(\mu,\cdots,\mu)\neq\mathbf{0}$;

(iii) 当$\mu=\mathbf{0}$或$\boldsymbol{K}$时, 有$f(\mu,\cdots,\mu)=\mathbf{0}$.

此外定义$\Gamma=\left\{\Phi\in C(\mathbf{R},\mathbf{R}^n):\Phi\text{是单调递增的}; \Phi(-\infty)=\mathbf{0}, \Phi(\infty)=\boldsymbol{K}\right\}$.

接下来给出系统(2.2)上、下解的定义.

定义 2.1 函数$\bar{\Phi}\in C(\mathbf{R},\mathbf{R}^n)$称为系统(2.2)的上解(或下解), 如果$\bar{\Phi}', \bar{\Phi}''$在$\mathbf{R}$的边界处及其它任意处是几乎处处存在的, 且$\bar{\Phi}$满足

$$D\bar{\Phi}''(z)-c\bar{\Phi}'(z)+f(\bar{\Phi}(z),(g_1*\bar{\Phi})(z),\cdots,(g_m*\bar{\Phi})(z))\leq 0(\text{或}\geq 0), z\in\mathbf{R}. \tag{2.3}$$

§3 行波解的存在性

本节主要考虑当条件(H1)和(H2)成立且选取特定核函数$g(y,s)=h(y,s)=\frac{1}{\tau_0}\mathrm{e}^{-\frac{s}{\tau_0}}\delta(y), \tau_0>0$, $\Omega=(-\infty,+\infty)$时, 连接系统(1.2)两个半平凡平衡点$\hat{E_v}$和$\hat{E_u}$之间的行波解.

将$U(x,t)=\Phi_1(z), V(x,t)=\Phi_2(z)$, $z=x+ct$, $c(c>0)$为波速, 代入系统(1.2)后, 则对应的行波方程为

$$\begin{cases} d_1\Phi_1''(z)-c\Phi_1'(z)+\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\Phi_1(z-y-cs)\mathrm{d}y\mathrm{d}s- \\ \qquad\beta_u\Phi_1^2(z)-c_1\Phi_1(z)\Phi_2(z)=0, \\ d_2\Phi_2''(z)-c\Phi_2'(z)+\alpha_v\int_{-\infty}^{+\infty}\int_0^{+\infty}h(y,s)\Phi_2(z-y-cs)\mathrm{d}y\mathrm{d}s- \\ \qquad\beta_v\Phi_2^2(z)-c_2\Phi_1(z)\Phi_2(z)=0, \end{cases} \tag{3.1}$$

其中$\Phi_1(-\infty)=0, \Phi_2(-\infty)=\frac{\alpha_v}{\beta_v}, \Phi_1(+\infty)=\frac{\alpha_u}{\beta_u}, \Phi_2(+\infty)=0$.

容易得到如下结论成立.

引理 3.1 当条件(H2)成立时, 存在$c^*>0, \lambda^*>0$使得

(i) $\Delta_{1c^*}(\lambda^*)=0, \frac{\partial}{\partial\lambda}\Delta_{1c^*}(\lambda)\mid_{\lambda=\lambda^*}=0$;

(ii) 当$0<c<c^*$且$\lambda>0$时, 有$\Delta_{1c}(\lambda)>0$;

(iii) 当$c>c^*$时, 方程$\Delta_{1c}(\lambda)=0$有两个正实根$\lambda_{11},\lambda_{12}$, 满足$0<\lambda_{11}<\lambda_{12}$, 且

$$\Delta_{1c}(\lambda)=\begin{cases}>0, & \lambda<\lambda_{11},\\ <0, & \lambda\in(\lambda_{11},\lambda_{12}),\\ >0, & \lambda>\lambda_{12}.\end{cases}$$

证 当$z\to-\infty$时, 线性化系统(3.1)第一式得

$$d_1\Phi_1''(z)-c\Phi_1'(z)+\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\Phi_1(z-y-cs)\mathrm{d}y\mathrm{d}s-\frac{c_1\alpha_v}{\beta_v}\Phi_1(z)=0.$$

令上式中$\Phi_1(z)=b_1\mathrm{e}^{\lambda z}$, b_1是非零常数, 则得到$\Delta_{1c}(\lambda)=0$, 其中

$$\Delta_{1c}(\lambda)=d_1\lambda^2-c\lambda+\frac{\alpha_u}{1+\lambda c\tau_0}-\frac{c_1\alpha_v}{\beta_v},\lambda\in\mathbf{R}. \tag{3.2}$$

当$c=0$时, 由条件(H2)得$\Delta_{10}(\lambda)=d_1\lambda^2+\frac{\beta_v\alpha_u-c_1\alpha_v}{\beta_v}>0$; 当$c(c>0)$增大时, 易证此引理.

引理 3.2 当条件(H1)成立时, 无论$c(c>0)$取任何值, 方程$\Delta_{2c}(\lambda)=0$有一个负实根λ_{21}和一个正实根λ_{22}, 且

$$\Delta_{2c}(\lambda)=\begin{cases}>0, & \lambda<\lambda_{21},\\ <0, & \lambda\in(\lambda_{21},\lambda_{22}),\\ >0, & \lambda>\lambda_{22}.\end{cases}$$

证 当$z\to+\infty$时, 线性化系统(3.1)第二式得

$$d_2\Phi_2''(z)-c\Phi_2'(z)+\alpha_v\int_{-\infty}^{+\infty}\int_0^{+\infty}h(y,s)\Phi_2(z-y-cs)\mathrm{d}y\mathrm{d}s-\frac{c_2\alpha_u}{\beta_u}\Phi_2(z)=0,$$

令上式中$\Phi_2(z)=b_2\mathrm{e}^{\lambda z}$, b_2是非零常数, 则得到$\Delta_{2c}(\lambda)=0$, 其中

$$\Delta_{2c}(\lambda)=d_2\lambda^2-c\lambda+\frac{\alpha_v}{1+\lambda c\tau_0}-\frac{c_2\alpha_u}{\beta_u},\lambda\in\mathbf{R}, \tag{3.3}$$

从而证得此引理.

接下来证明连接系统(1.2)两个半平凡平衡点$\left(0,\frac{\alpha_v}{\beta_v}\right)$和$\left(\frac{\alpha_u}{\beta_u},0\right)$之间行波解的存在性.

首先令$\widetilde{U}=U,\widetilde{V}=\frac{\alpha_v}{\beta_v}-V$, 则系统(1.2)变成如下系统

$$\begin{cases}\frac{\partial\widetilde{U}(x,t)}{\partial t}=d_1\triangle\widetilde{U}+\alpha_u\int_{-\infty}^{+\infty}\int_{-\infty}^{t}g(x-y,t-s)\widetilde{U}(y,s)\mathrm{d}y\mathrm{d}s-\beta_u\widetilde{U}^2(x,t)-\\ \qquad\frac{c_1\alpha_v}{\beta_v}\widetilde{U}(x,t)+c_1\widetilde{U}(x,t)\widetilde{V}(x,t),\\ \frac{\partial\widetilde{V}(x,t)}{\partial t}=d_2\triangle\widetilde{V}+\alpha_v\int_{-\infty}^{+\infty}\int_{-\infty}^{t}h(x-y,t-s)\widetilde{V}(y,s)\mathrm{d}y\mathrm{d}s-2\alpha_v\widetilde{V}(x,t)+\\ \qquad\beta_v\widetilde{V}^2(x,t)+\frac{c_2\alpha_v}{\beta_v}\widetilde{U}(x,t)-c_2\widetilde{U}(x,t)\widetilde{V}(x,t),\end{cases} \tag{3.4}$$

其中系统平衡点是$\mathbf{0}=(0,0)$和$\boldsymbol{K}=(\frac{\alpha_u}{\beta_u},\frac{\alpha_v}{\beta_v})$, 且$\mathbf{0}$是不稳定的, $\boldsymbol{K}$是线性稳定的.

然后将$\widetilde{U}(x,t)=\Phi_1(z),\widetilde{V}(x,t)=\Phi_2(z),z=x+ct$代入系统(3.4)后, 变为如下行波方程

$$\begin{cases}d_1\Phi_1''(z)-c\Phi_1'(z)+\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\Phi_1(z-y-cs)\mathrm{d}y\mathrm{d}s-\beta_u\Phi_1^2(z)-\\ \qquad\frac{c_1\alpha_v}{\beta_v}\Phi_1(z)+c_1\Phi_1(z)\Phi_2(z)=0,\\ d_2\Phi_2''(z)-c\Phi_2'(z)+\alpha_v\int_{-\infty}^{+\infty}\int_0^{+\infty}h(y,s)\Phi_2(z-y-cs)\mathrm{d}y\mathrm{d}s+\beta_v\Phi_2^2(z)-\\ \qquad 2\alpha_v\Phi_2(z)+\frac{c_2\alpha_v}{\beta_v}\Phi_1(z)-c_2\Phi_1(z)\Phi_2(z)=0,\end{cases} \tag{3.5}$$

其中$\Phi_1(-\infty)=0$, $\Phi_2(-\infty)=0$, $\Phi_1(+\infty)=\frac{\alpha_u}{\beta_u}$, $\Phi_2(+\infty)=\frac{\alpha_v}{\beta_v}$.

注意到证明连接系统(1.2)两个半平凡平衡点$\hat{E}_v$和$\hat{E}_u$之间行波解的存在性, 相当于证明连接行波方程(3.5)平衡点$\mathbf{0}$与$\boldsymbol{K}$之间行波解的存在性. 因此接下来首先给出定理3.3, 然后借助于行波方程(3.5)证明此定理.

定理 3.3 当条件(H1)和(H2)成立时, 存在$c^*>0$, 对任意$c>c^*$, 假设

(H3) $\varDelta_{2c}(\lambda_{11})\le 2(\alpha_v-\frac{c_2\alpha_u}{\beta_u})$

成立, 则连接系统(1.2)半平凡平衡点$\left(0,\frac{\alpha_v}{\beta_v}\right)$和$\left(\frac{\alpha_u}{\beta_u},0\right)$之间存在波速为$c$的行波解.

证 令$\varPhi=(\varPhi_1,\varPhi_2), f_c(\varPhi)=(f_{c_1}(\varPhi),f_{c_2}(\varPhi))$, 则由系统(3.5)得

$$f_{c_1}(\varPhi)=\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\varPhi_1(z-y-cs)\mathrm{d}y\mathrm{d}s-\beta_u\varPhi_1^2(z)-\frac{c_1\alpha_v}{\beta_v}\varPhi_1(z)+ c_1\varPhi_1(z)\varPhi_2(z),$$

$$f_{c_2}(\varPhi)=\alpha_v\int_{-\infty}^{+\infty}\int_0^{+\infty}h(y,s)\varPhi_2(z-y-cs)\mathrm{d}y\mathrm{d}s-2\alpha_v\varPhi_2(z)+\beta_v\varPhi_2^2(z)+ \frac{c_2\alpha_v}{\beta_v}\varPhi_1(z)-c_2\varPhi_1(z)\varPhi_2(z).$$

首先证f_c满足拟单调条件. 设$\varPhi=(\varPhi_1,\varPhi_2)$和$\varPsi=(\varPsi_1,\varPsi_2)\in C(\mathbf{R},\mathbf{R}^2)$, 且$\mathbf{0}\le\varPsi\le\varPhi\le\boldsymbol{K}$, 则

$$\begin{aligned}&f_{c_1}(\varPhi)-f_{c_1}(\varPsi)=\\&\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\varPhi_1(-y-cs)\mathrm{d}y\mathrm{d}s-\beta_u\varPhi_1^2(0)-\frac{c_1\alpha_v}{\beta_v}\varPhi_1(0)+c_1\varPhi_1(0)\varPhi_2(0)-\\&\quad\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\varPsi_1(-y-cs)\mathrm{d}y\mathrm{d}s+\beta_u\varPsi_1^2(0)+\frac{c_1\alpha_v}{\beta_v}\varPsi_1(0)-c_1\varPsi_1(0)\varPsi_2(0)=\\&\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\left[\varPhi_1(-y-cs)-\varPsi_1(-y-cs)\right]\mathrm{d}y\mathrm{d}s-\beta_u(\varPhi_1^2(0)-\varPsi_1^2(0))-\\&\quad\frac{c_1\alpha_v}{\beta_v}(\varPhi_1(0)-\varPsi_1(0))-c_1(\varPsi_1(0)\varPsi_2(0)-\varPhi_1(0)\varPhi_2(0))\ge\\&-\beta_u(\varPhi_1(0)-\varPsi_1(0))(\varPhi_1(0)+\varPsi_1(0))-\frac{c_1\alpha_v}{\beta_v}(\varPhi_1(0)-\varPsi_1(0))\ge\\&\left[-2\alpha_u-\frac{c_1\alpha_v}{\beta_v}\right](\varPhi_1(0)-\varPsi_1(0)),\end{aligned}$$

取$\delta_1\ge 2\alpha_u+\frac{c_1\alpha_v}{\beta_v}$, 则

$$f_{c_1}(\varPhi)-f_{c_1}(\varPsi)+\delta_1\left[\varPhi_1(0)-\varPsi_1(0)\right]\ge[-2\alpha_u-\frac{c_1\alpha_v}{\beta_v}+\delta_1](\varPhi_1(0)-\varPsi_1(0))\ge 0.$$

类似地

$$f_{c_2}(\varPhi)-f_{c_2}(\varPsi)=\alpha_v\int_{-\infty}^{+\infty}\int_0^{+\infty}h(y,s)\varPhi_2(-y-cs)\mathrm{d}y\mathrm{d}s-2\alpha_v\varPhi_2(0)+\beta_v\varPhi_2^2(0)+$$

$$\frac{c_2\alpha_v}{\beta_v}\Phi_1(0)-c_2\Phi_1(0)\Phi_2(0)-\frac{c_2\alpha_v}{\beta_v}\Psi_1(0)+c_2\Phi_1(0)\Phi_2(0)-$$

$$\alpha_v\int_{-\infty}^{+\infty}\int_0^{+\infty}h(y,s)\Psi_2(-y-cs)\mathrm{d}y\mathrm{d}s+2\alpha_v\Psi_2(0)-\beta_v\Psi_2^2(0)=$$

$$\alpha_v\int_{-\infty}^{+\infty}\int_0^{+\infty}h(y,s)[\Phi_2(-y-cs)-\Psi_2(-y-cs)]\mathrm{d}y\mathrm{d}s+$$

$$\beta_v(\Phi_2^2(0)-\Psi_2^2(0))+\frac{c_2\alpha_v}{\beta_v}(\Phi_1(0)-\Psi_1(0))-$$

$$2\alpha_v(\Phi_2(0)-\Psi_2(0))-c_2(\Phi_1(0)\Phi_2(0)-\Psi_1(0)\Psi_2(0))\geq$$

$$-2\alpha_v(\Phi_2(0)-\Psi_2(0))+\frac{c_2\alpha_v}{\beta_v}(\Phi_1(0)-\Psi_1(0))-$$

$$c_2\Phi_2(0)(\Phi_1(0)-\Psi_1(0))-c_2\Psi_1(0)(\Phi_2(0)-\Psi_2(0))\geq$$

$$-2\alpha_v(\Phi_2(0)-\Psi_2(0))+\frac{c_2\alpha_v}{\beta_v}(\Phi_1(0)-\Psi_1(0))-$$

$$\frac{c_2\alpha_v}{\beta_v}(\Phi_1(0)-\Psi_1(0))-\frac{c_2\alpha_u}{\beta_u}(\Phi_2(0)-\Psi_2(0))=$$

$$\left[-2\alpha_v-\frac{c_2\alpha_u}{\beta_u}\right](\Phi_2(0)-\Psi_2(0)),$$

取$\delta_2\geq 2\alpha_v+\frac{c_2\alpha_u}{\beta_u}$, 则

$$f_{c_2}(\Phi)-f_{c_2}(\Psi)+\delta_2[\Phi_2(0)-\Psi_2(0)]\geq[-2\alpha_v-\frac{c_2\alpha_u}{\beta_u}+\delta_2](\Phi_2(0)-\Psi_2(0))\geq 0,$$

从而可得$f_c(\Phi)-f_c(\Psi)+\delta[\Phi(0)-\Psi(0)]\geq 0$, 其中$\delta=\mathrm{diag}(\delta_1,\delta_2)$, 因此$f_c$满足拟单调条件.

定义$\Gamma=\left\{\Phi\in C(\mathbf{R},\mathbf{R}^n):\Phi\text{是单调递增的};\ \Phi(-\infty)=\mathbf{0},\ \Phi(\infty)=\boldsymbol{K}\right\}$, 其中$\boldsymbol{K}=\left(\frac{\alpha_u}{\beta_u},\frac{\alpha_v}{\beta_v}\right)$.

然后寻找系统(3.5)的上解. 设$\lambda_{11},\lambda_{12}$满足引理3.1, 定义

$$\bar\Phi_1(z)=\min\left\{\frac{\alpha_u}{\beta_u}\mathrm{e}^{\lambda_{11}z},\frac{\alpha_u}{\beta_u}\right\},\ \bar\Phi_2(z)=\min\left\{\frac{\alpha_v}{\beta_v}\mathrm{e}^{\lambda_{11}z},\frac{\alpha_v}{\beta_v}\right\},$$

其中$\bar\Phi(z)=(\bar\Phi_1(z),\bar\Phi_2(z))^{\mathrm{T}}$是系统(3.5)的上解, 且$\bar\Phi(z)\in\Gamma$.

对于系统(3.5)第一式有$z\geq 0$和$z<0$两种情况.

(i) 当$z\geq 0$时, $\bar\Phi_1(z)=\frac{\alpha_u}{\beta_u}$, $\bar\Phi_1(z-y-cs)\leq\frac{\alpha_u}{\beta_u}$, $\bar\Phi_2(z)=\frac{\alpha_v}{\beta_v}$, 则

$$d_1\bar\Phi_1''(z)-c\bar\Phi_1'(z)+\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\bar\Phi_1(z-y-cs)\mathrm{d}y\mathrm{d}s-$$

$$\beta_u\bar\Phi_1^2(z)-\frac{c_1\alpha_v}{\beta_v}\bar\Phi_1(z)+c_1\bar\Phi_1(z)\bar\Phi_2(z)\leq$$

$$\frac{\alpha_u^2}{\beta_u}-\frac{\alpha_u^2}{\beta_u}-\frac{c_1\alpha_u\alpha_v}{\beta_u\beta_v}+\frac{c_1\alpha_u\alpha_v}{\beta_u\beta_v}=0;$$

(ii) 当$z<0$时, $\bar\Phi_1(z)=\frac{\alpha_u}{\beta_u}\mathrm{e}^{\lambda_{11}z}$, $\bar\Phi_1(z-y-cs)=\frac{\alpha_u}{\beta_u}\mathrm{e}^{\lambda_{11}(z-y-cs)}$, $\bar\Phi_2(z)=\frac{\alpha_v}{\beta_v}\mathrm{e}^{\lambda_{11}z}$; $\Delta_{1c}(\lambda_{11})=0$, 且条件(H2)成立, 则

$$d_1\bar\Phi_1''(z)-c\bar\Phi_1'(z)+\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\bar\Phi_1(z-y-cs)\mathrm{d}y\mathrm{d}s-\beta_u\bar\Phi_1^2(z)-$$

$$\frac{c_1\alpha_v}{\beta_v}\bar\Phi_1(z)+c_1\bar\Phi_1(z)\bar\Phi_2(z)=$$

$$\frac{\alpha_u}{\beta_u}\mathrm{e}^{\lambda_{11}z}\underbrace{\left(d_1\lambda_{11}^2-c\lambda_{11}+\frac{\alpha_u}{1+\lambda_{11}\tau_0}-\frac{c_1\alpha_v}{\beta_v}\right)}_{=\Delta_{1c}(\lambda_{11})=0}+\frac{\alpha_u}{\beta_u}\mathrm{e}^{2\lambda_{11}z}\underbrace{\left(\frac{c_1\alpha_v-\alpha_u\beta_v}{\beta_v}\right)}_{<0}<$$

$$\frac{\alpha_u}{\beta_u}\mathrm{e}^{\lambda_{11}z}\Delta_{1c}(\lambda_{11})=0,$$

因此容易得到

$$d_1\bar{\Phi}_1''(z)-c\bar{\Phi}_1'(z)+\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\bar{\Phi}_1(z-y-cs)\mathrm{d}y\mathrm{d}s-\beta_u\bar{\Phi}_1^2(z)-$$
$$\frac{c_1\alpha_v}{\beta_v}\bar{\Phi}_1(z)+c_1\bar{\Phi}_1(z)\bar{\Phi}_2(z)<0, z\in\mathbf{R}.$$

对于系统(3.5)第二式有$z\geq 0$和$z<0$两种情况.

(i) 当$z\geq 0$时, 有

$$d_2\bar{\Phi}_2''(z)-c\bar{\Phi}_2'(z)+\alpha_v\int_{-\infty}^{+\infty}\int_0^{+\infty}h(y,s)\bar{\Phi}_2(z-y-cs)\mathrm{d}y\mathrm{d}s-$$
$$2\alpha_v\bar{\Phi}_2(z)+\beta_v\bar{\Phi}_2^2(z)+\frac{c_2\alpha_v}{\beta_v}\bar{\Phi}_1(z)-c_2\bar{\Phi}_1(z)\bar{\Phi}_2(z)\leq$$
$$\frac{\alpha_v^2}{\beta_v}-\frac{2\alpha_v^2}{\beta_v}+\frac{c_2\alpha_u\alpha_v}{\beta_u\beta_v}-\frac{c_2\alpha_u\alpha_v}{\beta_u\beta_v}+\frac{\alpha_v^2}{\beta_v}=0;$$

(ii) 当$z<0$且条件(H1)和(H3)成立时, 有

$$d_2\bar{\Phi}_2''(z)-c\bar{\Phi}_2'(z)+\alpha_v\int_{-\infty}^{+\infty}\int_0^{+\infty}h(y,s)\bar{\Phi}_2(z-y-cs)\mathrm{d}y\mathrm{d}s-2\alpha_v\bar{\Phi}_2(z)+$$
$$\beta_v\bar{\Phi}_2^2(z)+\frac{c_2\alpha_v}{\beta_v}\bar{\Phi}_1(z)-c_2\bar{\Phi}_1(z)\bar{\Phi}_2(z)=$$
$$\frac{\alpha_v}{\beta_v}\mathrm{e}^{\lambda_{11}z}\left\{d_2\lambda_{11}^2-c\lambda_{11}+\frac{\alpha_v}{1+\lambda_{11}c\tau_0}-2\alpha_v+\frac{c_2\alpha_u}{\beta_u}\right\}+\frac{\alpha_v}{\beta_v}\mathrm{e}^{2\lambda_{11}z}\underbrace{\left(\frac{\alpha_v\beta_u-c_2\alpha_u}{\beta_u}\right)}_{<0}<$$
$$\frac{\alpha_v}{\beta_v}\mathrm{e}^{\lambda_{11}z}\left\{\Delta_{2c}(\lambda_{11})+2\left(\frac{c_2\alpha_u}{\beta_u}-\alpha_v\right)\right\}\leq 0,$$

因此可得

$$d_2\bar{\Phi}_2''(z)-c\bar{\Phi}_2'(z)+\alpha_v\int_{-\infty}^{+\infty}\int_0^{+\infty}h(y,s)\bar{\Phi}_2(z-y-cs)\mathrm{d}y\mathrm{d}s-$$
$$2\alpha_v\bar{\Phi}_2(z)+\beta_v\bar{\Phi}_2^2(z)+\frac{c_2\alpha_v}{\beta_v}\bar{\Phi}_1(z)-c_2\bar{\Phi}_1(z)\bar{\Phi}_2(z)\leq 0, z\in\mathbf{R}.$$

于是可说明$\bar{\Phi}=(\bar{\Phi}_1,\bar{\Phi}_2)^{\mathrm{T}}$是系统(3.5)的上解.

最后构造下解. 设$\lambda_{11},\lambda_{12}$是$\Delta_{1c}(\lambda)=0$的两个正实根. 由引理3.1知, 存在$\varepsilon>0$, 对于$\lambda_{11}<\lambda_{11}+\varepsilon<\lambda_{12}$有$\Delta_{1c}(\lambda_{11}+\varepsilon)<0$, 此外$\lambda_{11}+\varepsilon\leq 2\lambda_{11}$. 假设$M>1$, 且定义下解为

$$\underline{\Phi}_2(z)=0, \underline{\Phi}_1(z)=\begin{cases}(1-M\mathrm{e}^{\varepsilon z})\mathrm{e}^{\lambda_{11}z}, & z<z_1,\\ 0, & z\geq z_1,\end{cases}$$

其中$z_1=-\frac{1}{\varepsilon}\ln M<0$. 则对任意$z$, $\underline{\Phi}_1(z)\geq 0$.

对于系统(3.5)第一式有$z\geq z_1$和$z<z_1$两种情况.

(i) 当$z \geq z_1$时, $\underline{\Phi}_1(z)=0,\ \underline{\Phi}_1(z-y-cs)\geq 0,\ y\in\mathbf{R}$, 则

$$d_1\underline{\Phi}_1''(z)-c\underline{\Phi}_1'(z)+\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\underline{\Phi}_1(z-y-cs)\mathrm{d}y\mathrm{d}s-$$
$$\beta_u\underline{\Phi}_1^2(z)-\frac{c_1\alpha_v}{\beta_v}\underline{\Phi}_1(z)+c_1\underline{\Phi}_1(z)\underline{\Phi}_2(z)\geq 0,z\in\mathbf{R};$$

(ii) 当$z<z_1$时, $\underline{\Phi}_1(z)=(1-M\mathrm{e}^{\varepsilon z})\mathrm{e}^{\lambda_{11}z}$, $\underline{\Phi}_1'(z)=\lambda_{11}\mathrm{e}^{\lambda_{11}z}-M(\lambda_{11}+\varepsilon)\mathrm{e}^{(\lambda_{11}+\varepsilon)z}$; $\underline{\Phi}_1''(z)=\lambda_{11}^2\mathrm{e}^{\lambda_{11}z}-M(\lambda_{11}+\varepsilon)^2\mathrm{e}^{(\lambda_{11}+\varepsilon)z}$, 则

$$d_1\underline{\Phi}_1''(z)-c\underline{\Phi}_1'(z)+\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\underline{\Phi}_1(z-y-cs)\mathrm{d}y\mathrm{d}s-$$
$$\beta_u\underline{\Phi}_1^2(z)-\frac{c_1\alpha_v}{\beta_v}\underline{\Phi}_1(z)+c_1\underline{\Phi}_1(z)\underline{\Phi}_2(z)=$$
$$-\beta_u(1-M\mathrm{e}^{\varepsilon z})^2\mathrm{e}^{2\lambda_{11}z}+\mathrm{e}^{\lambda_{11}z}\underbrace{\left(d_1\lambda_{11}^2-c\lambda_{11}+\frac{\alpha_u}{1+\lambda_{11}c\tau_0}-\frac{c_1\alpha_v}{\beta_v}\right)}_{=\Delta_{1c}(\lambda_{11})=0}-$$
$$M\mathrm{e}^{(\lambda_{11}+\varepsilon)z}\underbrace{\left(d_1(\lambda_{11}+\varepsilon)^2-c(\lambda_{11}+\varepsilon)+\frac{\alpha_u}{1+(\lambda_{11}+\varepsilon)c\tau_0}-\frac{c_1\alpha_v}{\beta_v}\right)}_{=\Delta_{1c}(\lambda_{11}+\varepsilon)}=$$
$$-M\mathrm{e}^{(\lambda_{11}+\varepsilon)z}\Delta_{1c}(\lambda_{11}+\varepsilon)-\beta_u(1-M\mathrm{e}^{\varepsilon z})^2\mathrm{e}^{2\lambda_{11}z},$$

此外, 因为$z<z_1<0$, 所以$0\leq 1-M\mathrm{e}^{\varepsilon z}<1$, 且$2\lambda_{11}>\lambda_{11}+\varepsilon, z<0$, 则$\mathrm{e}^{2\lambda_{11}z}\leq\mathrm{e}^{(\lambda_{11}+\varepsilon)z}$, 故

$$d_1\underline{\Phi}_1''(z)-c\underline{\Phi}_1'(z)+\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\underline{\Phi}_1(z-y-cs)\mathrm{d}y\mathrm{d}s-$$
$$\beta_u\underline{\Phi}_1^2(z)-\frac{c_1\alpha_v}{\beta_v}\underline{\Phi}_1(z)+c_1\underline{\Phi}_1(z)\underline{\Phi}_2(z)\geq$$
$$-M\mathrm{e}^{(\lambda_{11}+\varepsilon)z}\Delta_{1c}(\lambda_{11}+\varepsilon)-\beta_u\mathrm{e}^{(\lambda_{11}+\varepsilon)z}=M\mathrm{e}^{(\lambda_{11}+\varepsilon)z}\left\{\underbrace{-\Delta_{1c}(\lambda_{11}+\varepsilon)}_{>0}-\frac{\beta_u}{M}\right\},$$

注意到现在为确保上式右边是大于零的, 必须选择$M>1$足够大, 且$\sup\limits_{z\in\mathbf{R}}\underline{\Phi}_1(z)<\frac{\alpha_u}{\beta_u}$. 因此可得

$$d_1\underline{\Phi}_1''(z)-c\underline{\Phi}_1'(z)+\alpha_u\int_{-\infty}^{+\infty}\int_0^{+\infty}g(y,s)\underline{\Phi}_1(z-y-cs)\mathrm{d}y\mathrm{d}s-$$
$$\beta_u\underline{\Phi}_1^2(z)-\frac{c_1\alpha_v}{\beta_v}\underline{\Phi}_1(z)+c_1\underline{\Phi}_1(z)\underline{\Phi}_2(z)\geq 0,z\in\mathbf{R}.$$

对于系统(3.5)第二式有

$$d_2\underline{\Phi}_2''(z)-c\underline{\Phi}_2'(z)+\alpha_v\int_{-\infty}^{+\infty}\int_0^{+\infty}h(y,s)\underline{\Phi}_2(z-y-cs)\mathrm{d}y\mathrm{d}s-2\alpha_v\Phi_2(z)+$$
$$\beta_v\underline{\Phi}_2^2(z)+\frac{c_2\alpha_v}{\beta_v}\underline{\Phi}_1(z)-c_2\underline{\Phi}_1(z)\underline{\Phi}_2(z)\geq 0,z\in\mathbf{R}.$$

于是可说明$\underline{\Phi}=(\underline{\Phi}_1,\underline{\Phi}_2)^{\mathrm{T}}$是系统(3.5)的下解.

注意到$\mathbf{0}\leq\underline{\Phi}(z)\leq\bar{\Phi}(z)\leq\boldsymbol{K}, z\in\mathbf{R}$, 且$\bar{\Phi}(z)\not\equiv 0$, 因此综上所述, 可证定理3.3.

例 3.4 取核函数$g(y,s)=h(y,s)=\mathrm{e}^{-s}\delta(y)$, 参数$d_1=1$, $d_2=0.5$, $\alpha_u=\alpha_v=0.5$, $\beta_u=\beta_v=0.25$, $c_1=0.2$, $c_2=0.3$, 可以验证条件(H1)及(H2)成立. 此时半平凡平衡点为$\hat{E}_u=(2,0)$和$\hat{E}_v=(0,2)$. 通过计算$\Delta_{1c^*}(\lambda^*)=0$, $\frac{\partial}{\partial\lambda}\Delta_{1c^*}(\lambda)\mid_{\lambda=\lambda^*}=0$得到$(\lambda^*,c^*)\approx(0.5074,0.8113)$.

令波速$c = 1 > c^*$, 通过计算方程$\varDelta_{1c}(\lambda) = 0$可以得到$\lambda_{11} \approx 0.0717$, $\lambda_{12} \approx 1.1457$. 可以验证条件(H3) $\varDelta_{2c}(\lambda_{11}) \approx -0.2026 < 2(\alpha_v - \frac{c_2\alpha_u}{\beta_u}) = -0.2$ 成立. 由定理3.3连接系统(1.2)半平凡平衡点$\hat{E}_u$和$\hat{E}_v$之间存在波速为1的行波解.

参考文献:

[1] Lotka A J. Elements of Physical Biology[M]. New York: Williams and Wilkins, 1925.

[2] Volterra V. Variazionie fluttuazioni del numero d'individui in specie animali conviventi[J]. Mem Acad Licei, 1926, 2: 31-113.

[3] Lin Guo, Li Wantong. Bistable wavefronts in a diffusive and competitive Lotka-Volterra type system with nonlocal delays[J]. J Differ Equation, 2008, 244(3): 487-513.

[4] Vicen Méndez, Fort J, Farjas J. Speed of wave-front solutions to hyperbolic reaction-diffusion equations[J]. Phys Rev E, Statistical Physics, Plasmas, Fluids, and Related Interdisciplinary Topics, 1999, 60(5 Pt A): 5231-5243.

[5] Lv Guangying, Wang Mingxin. Traveling wave front in diffusive and competitive Lotka-Volterra system with delays[J]. Nonlinear Anal Real World Appl, 2010, 11(3): 1323-1329.

[6] Huang Aimei, Weng Peixuan. Traveling wavefronts for a Lotka-Volterra system of type-K with delay[J]. Nonlinear Anal Real World Appl, 2013, 14(2): 1114-1129.

[7] Al-Omari J F M, Gourley S A. Stability and traveling fronts in Lotka-Volterra competition models with stage structure[J]. SIAM J Appl Math, 2003, 63(6): 2063-2086.

[8] Wu Jianhong, Zou Xingfu. Traveling wave fronts of reaction-diffusion systems with delay[J]. J Dyn Differ Equation, 2001, 13(3): 651-687.

[9] Britton N F. Aggregation and the competitive exclusion principle[J]. J Theoret Biol, 1989, 136: 57-66.

[10] Britton N F. Spatial structures and periodic travelling waves in an integro-differential reaction-diffusion population model[J]. SIAM J Appl Math, 1990, 50: 1663-1688.

[11] Li Wantong, Ruan Shigui, Wang Zhicheng. On the diffusive Nicholson's blowflies equation with nonlocal delay[J]. J Nonlinear Science, 2007, 17(6): 505-525.

Traveling wave solutions for Lotka-Volterra competitive systems with nonlocal delay

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Abstract: This article considers the traveling wave solutions of a two-species competition system with nonlocal delay. By means of the theory of existence for the traveling wave solutions of reaction-diffusion equations with nonlocal delay, the existence of the traveling wave solutions connecting two semi-trivial equilibria of the systems is proved when a particular kernel function is selected and the wave velocity c is greater than a critical value.

Keywords: reaction-diffusion equation; nonlocal delay; travelling wave solutions; existence

MR Subject Classification: 35C07; 35R10; 92D25