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分数阶惯性时滞BAM神经网络全局Mittag-Leffler稳定和全局渐近ω-周期

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摘　要: 该文研究分数阶惯性时滞BAM神经网络的全局Mittag-Leffler稳定和全局渐近ω-周期问题. 首先, 利用Riemann-Liouville分数阶微积分性质, 通过引入适当的变量代换, 将含有两个不同分数阶导数的分数阶惯性时滞BAM神经网络模型简化为只含一个分数阶导数神经网络模型. 其次, 运用积分区间可加性和初始值条件, 当时间变量分别在小于等于时间迟滞有限区间和大于等于时间迟滞无限区间内变化时, 推导出含有时间迟滞和不含时间迟滞的状态函数分数阶积分之间的关系, 给出了判定分数阶惯性时滞BAM神经网络系统解全局Mittag-Leffler稳定和全局渐近ω-周期的充分条件. 最后, 通过数值模拟验证所得到理论结果的正确性.

关键词: 分数阶; 惯性; BAM神经网络; 全局Mittag-Leffler稳定; 全局渐近ω-周期

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§1　引　言

Kosko在1987年首次提出了双向联想记忆神经网络(BAM神经网络), BAM神经网络是由两层神经网络构成的特殊神经网络, 层与层之间是相互联系的, 在模式识别、人工智能和自动化控制等领域得到广泛的应用[1-4]. 1986年, Babcock和Westervelt将电感引入到神经网络模型中, 揭示了惯性的本质, 产生以二阶微分方程为特征的惯性神经网络[5], 对于整数阶惯性神经网络的动力学行为的研究已有不少的研究结果, 如文[6-15]. 2008年, Boroomand和Mohamma[16]在电路系统应用中, 首次使用分数阶电抗, 成功替换一般整数阶神经网络中的电容器, 由此掀开了分数阶神经网络研究的热潮, 如文[17-19]分别研究了分数阶四元数值不确定神经网络、分数阶模糊神经网络等同步稳定问题. 目前, 对于分数阶惯性神经网络的研究也有不少成果, 如文[20-24]分别研究了分数阶惯性神经网络的全局渐近稳定性、有限时间稳定性、Mittag-Leffler稳定及周期稳定性问题; 分数阶时滞惯性Cohen-Grossberg神经网络的有界性和全局Mittag-Lefler同

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步[25]; 分数阶惯性神经网络动力学分析研究[26]; 分数阶时滞模糊惯性神经网络有界性、全局Mittag-Lefler稳定性和S-渐近ω-周期性[27]等.

分数阶惯性时滞BAM神经网络的Mittag-Leffler稳定和全局渐近ω-周期是其动力特性研究的重要内容, 这对于理论上探讨以及为应用提供理论依据都具有一定的意义和价值. 从所查阅到的资料显示, 到目前为止尚未看到对这一系统的Mittag-Leffler稳定和全局渐近ω-周期的研究成果, 这将是一个新的课题. 探究的思路主要从以下几个方面展开.

1) 通过适当的变量代换, 将含有两个不同分数阶导数的分数阶惯性时滞BAM神经网络模型简化为只含一个分数阶导数的分数阶神经网络模型.

2) 当时间变量分别在小于等于时间迟滞有限区间和大于等于时间迟滞无限区间内变化时, 推导出含有时间迟滞和不含时间迟滞状态函数的分数阶积分之间的关系.

3) 给出了判定分数阶惯性时滞BAM神经网络系统解全局Mittag-Leffler稳定和全局渐近ω-周期的充分条件.

4) 通过仿真模拟检验理论推导结果与模拟结果的一致性.

§2 预备知识

定义2.1[28] 设$q>0$是任意正实数, 对于函数$f(t)$的q阶分数阶积分(Riemann-Liourille积分)定义为
$$D_t^{-q}f(t)=\frac{1}{\Gamma(q)}\int_0^t(t-r)^{q-1}f(r)\mathrm{d}r,$$
其中$\Gamma(\cdot)$是Gamma函数, 即$\Gamma(r)=\int_0^{+\infty}t^{r-1}\mathrm{e}^{-t}\mathrm{d}t, r>0$.

定义2.2[28] 对于函数$f(t)$的q阶分数阶导数(Riemann-Liourille导数)定义为
$$D_t^qf(t)=\frac{1}{\Gamma(n-q)}\frac{\mathrm{d}^n}{\mathrm{d}t^n}\int_0^t\frac{f(s)}{(t-s)^{q-n+1}}\mathrm{d}s,$$
其中$n-1\le q<n, n\in\mathbf{Z}^+$, $\Gamma(\cdot)$是Gamma函数.

定义2.3[29] 含一个参数q的Mittag-Leffler函数定义为
$$E_q(z)=\sum_{k=0}^{+\infty}\frac{z^k}{\Gamma(kq+1)},$$
其中复数q的实部$\mathrm{Re}(q)>0$, z是复数, $\Gamma(.)$是Gamma函数.

考虑分数阶惯性时滞BAM神经网络
$$\begin{cases}D_t^{2\alpha}x_i(t)=-c_iD_t^{\alpha}x_i(t)-\alpha_ix_i(t)+\sum\limits_{j=1}^na_{ij}f_j(y_j(t))+\sum\limits_{j=1}^nb_{ij}f_j(y_j(t-\tau_{ij}))+I_i(t),\\ D_t^{2\alpha}y_j(t)=-d_jD_t^{\alpha}y_j(t)-\beta_jy_j(t)+\sum\limits_{i=1}^ng_{ji}f_i(x_i(t))+\sum\limits_{i=1}^nh_{ji}f_i(x_i(t-\sigma_{ji}))+J_j(t),\end{cases}\tag{1}$$
其中$t\ge0, i,j=1,2,\cdots,n, D_t^{2\alpha}, D_t^{\alpha}$分别是$2\alpha$阶和$\alpha$阶Riemann-Liouville分数阶导数, $0<\alpha<1$, $x_i(t)$和$y_j(t)$表示第i和j个神经元在t时刻的状态. $c_i>0, d_j>0, \alpha_i>0, \beta_j>0$, $a_{ij}, b_{ij}, g_{ji}, h_{ji}$分别表示神经元之间的连接权重, $f_i(\cdot)$表示第i个神经元的激励函数, $I_i(t)$和$J_j(t)$表示第i个和第j个神经元在t时刻的外部输入, τ_{ij}和σ_{ji}表示t时刻第i个神经元和第j个神经元的信号传输时滞, 且满足$0<\tau_{ij}\le\tau$, $0<\sigma_{ji}\le\sigma$.

给定(1)的初始条件为
$$\begin{cases}x_i(s)=\varphi_{1i}(s), D_t^{\alpha}x_i(s)=\varphi_{2i}(s), -\sigma\le s\le0,\\ y_j(s)=\psi_{1j}(s), D_t^{\alpha}y_j(s)=\psi_{2j}(s), -\tau\le s\le0,\end{cases}\tag{2}$$

其中$i,j=1,2,\cdots,n,\varphi_{1i}(s),\varphi_{2i}(s),\psi_{1j}(s),\psi_{2j}(s)$在$[-\delta,0]$上连续有界, $\delta=\max\{\sigma,\tau\}$.

在本文讨论中, 作以下的假设.

(H_1) 激励函数$f_i(\cdot)$满足Lipschitz条件且有界, 即存在常数$l_i>0,\overline{f}_i>0$, 使得

$$|f_i(\xi_1)-f_i(\xi_2)|\le l_i|\xi_1-\xi_2|,|f_i(\cdot)|\le\overline{f}_i,\ \xi_1,\xi_2\in\mathbf{R},i=1,2,\cdots,n.$$

本文中所涉及到的范数, 作以下定义.

对于$x(t)=(x_1(t),x_2(t),\cdots,x_n(t))^{\mathrm{T}}\in\mathbf{R}$, 其范数定义为$\|x(t)\|=\sum\limits_{i=1}^{n}|x_i(t)|$.

定义2.4 若存在正常数$\rho_1>0$和$\rho_2>0$, 对于系统(1), 设$X(t)$, $\overline{X}(t)$分别为在初始条件

$$\begin{cases}x_i(s)=\varphi_{1i}(s),D_t^{\alpha}x_i(s)=\varphi_{2i}(s),-\sigma\le s\le 0,\\ y_j(s)=\psi_{1j}(s),D_t^{\alpha}y_j(s)=\psi_{2j}(s),-\tau\le s\le 0\end{cases}$$

和

$$\begin{cases}\overline{x}_i(s)=\overline{\varphi}_{1i}(s),D_t^{\alpha}\overline{x}_i(s)=\overline{\varphi}_{2i}(s),-\sigma\le s\le 0,\\ \overline{y}_j(s)=\overline{\psi}_{1j}(s),D_t^{\alpha}\overline{y}_j(s)=\overline{\psi}_{2j}(s),-\tau\le s\le 0\end{cases}$$

下的两个解, 如果$X(t)$, $\overline{X}(t)$满足

$$\|X(t)-\overline{X}(t)\|\le[M(|\varphi-\overline{\varphi}|,|\psi-\overline{\psi}|)E_q(-\rho_1t^q)]^{\rho_2},t\ge 0,$$

则称系统(1)是全局Mittag-Leffler稳定的, 其中

$$X(t)=(x_1(t),x_2(t),\cdots,x_n(t),y_1(t),y_2(t),\cdots,y_n(t))^{\mathrm{T}},$$
$$\overline{X}(t)=(\overline{x}_1(t),\overline{x}_2(t),\cdots,\overline{x}_n(t),\overline{y}_1(t),\overline{y}_2(t),\cdots,\overline{y}_n(t))^{\mathrm{T}},$$
$$\varphi(t)=(\varphi_{11}(t),\varphi_{12}(t),\cdots,\varphi_{1n}(t),\psi_{11}(t),\psi_{12}(t),\cdots,\psi_{1n}(t))^{\mathrm{T}},$$
$$\overline{\varphi}(t)=(\overline{\varphi}_{11}(t),\overline{\varphi}_{12}(t),\cdots,\overline{\varphi}_{1n}(t),\overline{\psi}_{11}(t),\overline{\psi}_{12}(t),\cdots,\overline{\psi}_{1n}(t))^{\mathrm{T}},$$
$$\|X(t)-\overline{X}(t)\|=\sum_{i=1}^{n}|x_i(t)-\overline{x}_i(t)|+\sum_{j=1}^{n}|y_j(t)-\overline{y}_j(t)|,$$
$$|\varphi-\overline{\varphi}|=\sup_{-\sigma\le s\le 0}\sum_{i=1}^{n}|\varphi_{1i}-\overline{\varphi}_{1i}|,|\psi-\overline{\psi}|=\sup_{-\sigma\le s\le 0}\sum_{i=1}^{n}|\psi_{1i}-\overline{\psi}_{1i}|,$$

$M(|\varphi-\overline{\varphi}|,|\psi-\overline{\psi}|)\ge 0,M(0,0)=0,E_q(\cdot)$表示一个参数的Mittag-Leffler函数.

设$C_b([0,+\infty),X)$表示从$[0,+\infty)$到Banach空间X的连续有界泛函的集合并赋予一致收敛范数$\|\cdot\|_{\infty}$, 参见[30].

定义2.5[30] 设函数$\xi(t)\in(C_b[0,+\infty),\mathbf{R})$, 如果存在常数$\omega>0$, 使得

$$\lim_{t\to+\infty}[\xi(t+\omega)-\xi(t)]=0,$$

则称ω是函数$\xi(t)$一个渐近周期, $\xi(t)$称为是S-渐近ω-周期的.

定义2.6[30] 如果存在以ω为周期的周期函数$X^*(t)$, 使得系统(1)的所有解$X(t)$都收敛到$X^*(t)$, 即$\lim\limits_{t\to+\infty}X(t)=X^*(t)$, 则称系统(1)的解是全局渐近$\omega$-周期的, 其中

$$X^*(t)=(x_1^*(t),x_2^*(t),\cdots,x_n^*(t),y_1^*(t),y_2^*(t),\cdots,y_n^*(t))^{\mathrm{T}}.$$

在Riemann-Liouville分数阶导数和积分定义下, 有下列结果.

引理2.7[24] 如果$x(t)\in\mathbf{C}^r[0,+\infty)$, $n-1\le q<n,m-1\le p<m,n,m\in\mathbf{Z}^+,r=\max\{n,m\}$, 那么

(1) $D_t^qC=\frac{Ct^{-q}}{\Gamma(1-q)}$, C是常数.

(2) $D_t^q(ax_1(t)+bx_2(t))=aD_t^qx_1(t)+bD_t^qx_2(t)$, a,b是常数.

引理2.8[24] 如果$x(t)\in\mathbf{R}$在$[0,\delta](\delta>0)$上连续可微, $0<q<1,n-1\le p<n,n\in\mathbf{Z}^+$, 那么有

(1) $D_t^p D_t^q x(t) = D_t^{p+q} x(t)$.

(2) $D_t^{-p} D_t^q x(t) = D^{-p+q} x(t)$.

引理2.9[22] 设$u(t)$是定义在$[0,+\infty)$上的连续函数, 若存在常数$d_1>0$和$d_2>0$, 使得

$$u(t) \leq -d_1 D_t^{-q} u(t) + d_2, \quad t \geq 0,$$

则$u(t) \leq d_2 E_q(-d_1 t^q)$, 其中$0<q<1$, $E_q(\cdot)$表示一个参数的Mittag-Leffler函数.

对系统(1), 引入变量替换

$$\begin{cases} u_i(t) = D_t^\alpha x_i(t) + \gamma_i x_i(t), \\ v_j(t) = D_t^\alpha y_j(t) + \gamma_j y_j(t), \end{cases} \quad \gamma_i > 0, \gamma_j > 0, i,j = 1,2,\cdots,n.$$

由引理2.8, (1)可变换为

$$\begin{cases} D_t^\alpha x_i(t) = -\gamma_i x_i(t) + u_i(t), \\ D_t^\alpha u_i(t) = -(\alpha_i + \gamma_i^2 - \gamma_i c_i) x_i(t) - (c_i - \gamma_i) u_i(t) + \\ \qquad \sum\limits_{j=1}^n a_{ij} f_j(y_j(t)) + \sum\limits_{j=1}^n b_{ij} f_j(y_j(t-\tau_{ij})) + I_i(t), \\ D_t^\alpha y_j(t) = -\gamma_j y_j(t) + v_j(t), \\ D_t^\alpha v_j(t) = -(\beta_j + \gamma_j^2 - \gamma_j d_j) y_j(t) - (d_j - \gamma_j) v_j(t) + \\ \qquad \sum\limits_{i=1}^n g_{ji} f_i(x_i(t)) + \sum\limits_{i=1}^n h_{ji} f_i(x_i(t-\sigma_{ji})) + J_j(t). \end{cases} \tag{3}$$

§3 主要结果

本节主要研究分数阶惯性时滞BAM神经网络的全局Mittag-Leffler稳定性和全局渐近ω-周期, 利用分数阶微积分性质, 通过对时间区间的有效划分和借助一些分析知识, 给出其判定的充分条件.

定理3.1 对于系统(1), 假设条件(H_1)成立, 如果

$$\begin{aligned} \eta = \min\Big\{ & \min_{1\leq i\leq n}\{\gamma_i - |\alpha_i + {\gamma_i}^2 - \gamma_i c_i| - \sum_{j=1}^n |g_{ji}| l_i - \sum_{j=1}^n |h_{ji}| l_i, c_i - \gamma_i - 1\}, \\ & \min_{1\leq j\leq n}\{\gamma_j - |\beta_j + {\gamma_j}^2 - \gamma_j d_j| - \sum_{i=1}^n |a_{ij}| l_j - \sum_{i=1}^n |b_{ij}| l_j, d_j - \gamma_j - 1\}\Big\} > 0, \end{aligned}$$

那么系统(1)的解是全局Mittag-Leffler稳定的.

证 设$X(t) = (x_1(t), x_2(t), \cdots, x_n(t), y_1(t), y_2(t), \cdots, y_n(t))^{\mathrm{T}}$和$\overline{X}(t) = (\overline{x}_1(t), \overline{x}_2(t), \cdots, \overline{x}_n(t), \overline{y}_1(t), \overline{y}_2(t), \cdots, \overline{y}_n(t))^{\mathrm{T}}$分别是系统(1)在初值$x_i(s) = \varphi_{1i}(s)$, $D_t^q x_i(s) = \varphi_{2i}(s), y_i(s) = \psi_{1i}(s)$, $D_t^q x_i(s) = \psi_{2i}(s)$和$\overline{x}_i(s) = \overline{\varphi}_{1i}(s)$, $D_t^q \overline{x}_i(s) = \overline{\varphi}_{2i}(s)$, $\overline{y}_i(s) = \overline{\psi}_{1i}(s)$, $D_t^q \overline{y}_i(s) = \overline{\psi}_{2i}(s)$下的两个解. 令

$z_i(t) = x_i(t) - \overline{x}_i(t), w_i(t) = y_i(t) - \overline{y}_i(t)$, $p_i(t) = u_i(t) - \overline{u}_i(t)$, $q_i(t) = v_i(t) - \overline{v}_i(t), i = 1,2,\cdots,n$.

由系统(3)可得

$$\begin{cases} D_t^\alpha z_i(t) = -\gamma_i z_i(t) + p_i(t), \\ D_t^\alpha p_i(t) = -(\alpha_i + {\gamma_i}^2 - \gamma_i c_i) z_i(t) - (c_i - \gamma_i) p_i(t) + \\ \qquad \sum\limits_{j=1}^n a_{ij}(f_j(y_j(t)) - f_j(\overline{y}_j(t))) + \sum\limits_{j=1}^n b_{ij}(f_j(y_j(t-\tau_{ij})) - f_j(\overline{y}_j(t-\tau_{ij}))), \\ D_t^\alpha w_j(t) = -\gamma_j w_j(t) + q_j(t), \\ D_t^\alpha q_j(t) = -(\beta_j + {\gamma_j}^2 - \gamma_j d_j) w_j(t) - (d_j - \gamma_j) q_j(t) + \\ \qquad \sum\limits_{i=1}^n g_{ji}(f_i(x_i(t)) - f_i(\overline{x}_i(t))) + \sum\limits_{i=1}^n h_{ji}(f_i(x_i(t-\sigma_{ji})) - f_i(\overline{x}_i(t-\sigma_{ji}))). \end{cases} \tag{4}$$

对于任意连续函数$g(x)$, 由定义2.2知$D_t^\alpha|g(t)| \le \mathrm{sgn}(g(t))D_t^\alpha g(t)$, 从(4)得

$$\begin{cases} D_t^\alpha|z_i(t)| \le -\gamma_i|z_i(t)| + |p_i(t)|, \\ D_t^\alpha|p_i(t)| \le |\alpha_i + {\gamma_i}^2 - \gamma_i c_i)||z_i(t)| - (c_i - \gamma_i)|p_i(t)| + \sum\limits_{j=1}^n |a_{ij}|l_j|w_j(t)| + \\ \qquad\qquad \sum\limits_{j=1}^n |b_{ij}|l_j|w_j(t-\tau_{ij})|, \\ D_t^\alpha|w_j(t)| \le -\gamma_j|w_j(t)| + |q_j(t)|, \\ D_t^\alpha|q_j(t)| \le |\beta_j + {\gamma_j}^2 - \gamma_j d_j||w_j(t)| - (d_j - \gamma_j)|q_j(t)| + \sum\limits_{i=1}^n |g_{ji}|l_i|z_i(t)| + \\ \qquad\qquad \sum\limits_{i=1}^n |h_{ji}|l_i|z_i(t-\sigma_{ji})|. \end{cases} \tag{5}$$

由引理2.8, 从(5)可得

$$\begin{cases} |z_i(t)| \le -\gamma_i D_t^{-\alpha}|z_i(t)| + D_t^{-\alpha}|p_i(t)|, \\ |p_i(t)| \le |\alpha_i + {\gamma_i}^2 - \gamma_i c_i)|D_t^{-\alpha}|z_i(t)| - (c_i - \gamma_i)D_t^{-\alpha}|p_i(t)| + \\ \qquad \sum\limits_{j=1}^n |a_{ij}|l_j D_t^{-\alpha}|w_j(t)| + \sum\limits_{j=1}^n |b_{ij}|l_j D_t^{-\alpha}|w_j(t-\tau_{ij})|, \\ |w_j(t)| \le -\gamma_j D_t^{-\alpha}|w_j(t)| + D_t^{-\alpha}|q_j(t)|, \\ |q_j(t)| \le |\beta_j + {\gamma_j}^2 - \gamma_j d_j|D_t^{-\alpha}|w_j(t)| - (d_j - \gamma_j)D_t^{-\alpha}|q_j(t)| + \\ \qquad \sum\limits_{i=1}^n |g_{ji}|l_i D_t^{-\alpha}|z_i(t)| + \sum\limits_{i=1}^n |h_{ji}|l_i D_t^{-\alpha}|z_i(t-\sigma_{ji})|. \end{cases} \tag{6}$$

当$t \in [0, \tau_{ij}]$时, 有$t - \tau_{ij} \le 0$, 则

$$\begin{aligned} D_t^{-\alpha}|w_j(t-\tau_{ij})| = & \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}|w_j(s-\tau_{ij})|\mathrm{d}s = \\ & \frac{1}{\Gamma(\alpha)}\int_{-\tau_{ij}}^{t-\tau_{ij}} (t-u-\tau_{ij})^{\alpha-1}|w_j(u)|\mathrm{d}u \le \\ & \frac{w_j^*}{\Gamma(\alpha)}\int_{-\tau_{ij}}^{t-\tau_{ij}} (t-u-\tau_{ij})^{\alpha-1}\mathrm{d}u = \\ & -\frac{w_j^*(t-u-\tau_{ij})^\alpha}{\alpha\Gamma(\alpha)} \Big|_{-\tau_{ij}}^{t-\tau_{ij}} = \frac{w_j^* t^\alpha}{\Gamma(\alpha+1)} \le \frac{w_j^*\tau^\alpha}{\Gamma(\alpha+1)}, \end{aligned}$$

其中$w_j(s) = y_j(s) - \overline{y}_j(s) = \psi_{1j}(s) - \overline{\psi}_{1j}(s)$, $w_j^* = \sup\limits_{-\tau \le s \le 0}\{|\psi_{1j}(s) - \overline{\psi}_{1j}(s)|\}$.

当$t \in [\tau_{ij}, +\infty)$时, 有$t - \tau_{ij} \ge 0$, 则

$$\begin{aligned} & D_t^{-\alpha}|w_j(t-\tau_{ij})| = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1}|w_j(s-\tau_{ij})|\mathrm{d}s = \\ & \frac{1}{\Gamma(\alpha)}\int_{-\tau_{ij}}^{t-\tau_{ij}} (t-u-\tau_{ij})^{\alpha-1}|w_j(u)|\mathrm{d}u = \\ & \frac{1}{\Gamma(\alpha)}\int_{-\tau_{ij}}^{0} (t-u-\tau_{ij})^{\alpha-1}|w_j(u)|\mathrm{d}u + \frac{1}{\Gamma(\alpha)}\int_0^{t-\tau_{ij}} (t-u-\tau_{ij})^{\alpha-1}|w_j(u)|\mathrm{d}u \le \\ & \frac{w_j^*\tau^\alpha}{\Gamma(\alpha+1)} + \frac{1}{\Gamma(\alpha)}\int_0^t (t-u)^{\alpha-1}|w_j(u)|\mathrm{d}u = \\ & \frac{w_j^*\tau^\alpha}{\Gamma(\alpha+1)} + D_t^{-\alpha}|w_j(t)|. \end{aligned}$$

从上述推导结果, 可得

$$D_t^{-\alpha}|w_j(t-\tau_{ij})| \le \frac{w_j^*\tau^\alpha}{\Gamma(\alpha+1)} + D_t^{-\alpha}|w_j(t)|. \tag{7}$$

同理可推得

$$D_t^{-\alpha}|z_i(t-\sigma_{ji})| \le \frac{z_i^*\sigma^\alpha}{\Gamma(\alpha+1)} + D_t^{-\alpha}|z_i(t)|, \tag{8}$$

其中$z_i^* = \sup\limits_{-\sigma\le s\le 0}\{|\varphi_{1i}(s)-\overline{\varphi}_{1i}(s)|\}$.

由(6)-(8)式, 可得

$$\begin{aligned}
&\sum_{i=1}^n[|z_i(t)|+|p_i(t)|]+\sum_{j=1}^n[|w_j(t)|+|q_j(t)|]\le\\
&\sum_{i=1}^n\{[-\gamma_i+|\alpha_i+\gamma_i{}^2-\gamma_i c_i|]D_t^{-\alpha}|z_i(t)|-[c_i-\gamma_i-1]D_t^{-\alpha}|p_i(t)|+\\
&\sum_{j=1}^n|a_{ij}|l_jD_t^{-\alpha}|w_j(t)|+\sum_{j=1}^n|b_{ij}|l_j[\frac{w_j^*\tau^\alpha}{\Gamma(\alpha+1)}+D_t^{-\alpha}|w_j(t)|]\}+\\
&\sum_{j=1}^n\{[-\gamma_j+|\beta_j+\gamma_j{}^2-\gamma_jd_j|]D_t^{-\alpha}|w_i(t)|-[(d_j-\gamma_j)-1]D_t^{-\alpha}|q_j(t)|+\sum_{i=1}^n|g_{ji}|l_iD_t^{-\alpha}|z_i(t)|+\\
&\sum_{i=1}^n|h_{ji}|l_i[\frac{z_i^*\sigma^\alpha}{\Gamma(\alpha+1)}+D_t^{-\alpha}|z_i(t)|]\}=\\
&\sum_{i=1}^n[-\gamma_i+|\alpha_i+\gamma_i{}^2-\gamma_ic_i|+\sum_{j=1}^n|g_{ji}|l_i+\sum_{j=1}^n|h_{ji}|l_i]D_t^{-\alpha}|z_i(t)|-\sum_{i=1}^n[c_i-\gamma_i-1]D_t^{-\alpha}|p_i(t)|+\\
&\sum_{j=1}^n[-\gamma_j+|\beta_j+\gamma_j{}^2-\gamma_jd_j|+\sum_{i=1}^n|a_{ij}|l_j+\sum_{i=1}^n|b_{ij}|l_j]D_t^{-\alpha}|w_j(t)|-\\
&\sum_{j=1}^n[(d_j-\gamma_j)-1]D_t^{-\alpha}|q_j(t)|+\sum_{i=1}^n\sum_{j=1}^n[|b_{ij}|l_j[\frac{w_j^*\tau^\alpha}{\Gamma(\alpha+1)}+|h_{ji}|l_i\frac{z_i^*\sigma^\alpha}{\Gamma(\alpha+1)}]\le\\
&-\eta\{\sum_{i=1}^n[D_t^{-\alpha}|z_i(t)|+D_t^{-\alpha}|p_i(t)|]+\sum_{j=1}^n[D_t^{-\alpha}|w_j(t)|+D_t^{-\alpha}|q_j(t)|]\}+M(|\varphi-\overline{\varphi}|,|\psi-\overline{\psi}|),
\end{aligned}$$

其中$\eta=\min\{\min\limits_{1\le i\le n}\{\gamma_i-|\alpha_i+\gamma_i{}^2-\gamma_ic_i|-\sum\limits_{j=1}^n|g_{ji}|l_i-\sum\limits_{j=1}^n|h_{ji}|l_i, c_i-\gamma_i-1\}$,

$$\min_{1\le j\le n}\{\gamma_j-|\beta_j+\gamma_j{}^2-\gamma_jd_j|-\sum_{i=1}^n|a_{ij}|l_j-\sum_{i=1}^n|b_{ij}|l_j, d_j-\gamma_j-1\}\},$$

$$\xi(|\varphi-\overline{\varphi}|,|\psi-\overline{\psi}|)=\sum_{i=1}^n\sum_{j=1}^n[|b_{ij}|l_j\frac{w_j^*\tau^\alpha}{\Gamma(\alpha+1)}+|h_{ji}|l_i\frac{z_i^*\sigma^\alpha}{\Gamma(\alpha+1)}].$$

即有

$$\begin{aligned}
&\sum_{i=1}^n[|z_i(t)|+|p_i(t)|]+\sum_{j=1}^n[|w_j(t)|+|q_j(t)|]\le\\
&-\eta D_t^{-\alpha}\{\sum_{i=1}^n[|z_i(t)|+|p_i(t)|]+\sum_{j=1}^n[|w_j(t)|+|q_j(t)|]\}+\xi(|\varphi-\overline{\varphi}|,|\psi-\overline{\psi}|).
\end{aligned}\tag{9}$$

由引理2.9, 从(9)知

$$\sum_{i=1}^{n}[|z_i(t)|+|p_i(t)|]+\sum_{j=1}^{n}[|w_j(t)|+|q_j(t)|]\le M(|\varphi-\overline{\varphi}|,|\psi-\overline{\psi}|)E_\alpha(-\eta t^\alpha).$$

从而可得

$$\|X(t)-\overline{X}(t)\|=\sum_{i=1}^{n}|x_i(t)-\overline{x}_i(t)|+\sum_{j=1}^{n}|y_j(t)-\overline{y}_j(t)|\le\xi(|\varphi-\overline{\varphi}|,|\psi-\overline{\psi}|)E_\alpha(-\eta t^\alpha).$$

显然$\xi(0,0)=0,\xi(|\varphi-\overline{\varphi}|,|\psi-\overline{\psi}|)\ge 0$. 由定义2.4知, 系统(1)的解$X(t)$是全局Mittag-Leffler稳定的.

定理3.2 对于系统(1), 假设定理3.1条件成立, 如果存在常数$\omega>0,I_i>0,J_j>0$, 使得$I_i(t+\omega)=I_i(t),J_j(t+\omega)=J_j(t),|I_i(t)|\le I_i,|J_i(t)|\le J_j,i,j=1,2\cdots n$, 那么系统(1)的解是全局渐近$\omega$-周期的.

证 由系统(3)可得

$$\begin{cases}D_t^\alpha|x_i(t)|\le-\gamma_i|x_i(t)|+|u_i(t)|,\\ D_t^\alpha|u_i(t)|\le|\alpha_i+\gamma_i^2-\gamma_ic_i||x_i(t)|-(c_i-\gamma_i)|u_i(t)|+\sum\limits_{j=1}^{n}(|a_{ij}|+|b_{ij}|)\overline{f}_j+I_i,\\ D_t^\alpha|y_j(t)|\le-\gamma_j|y_j(t)|+|v_j(t)|,\\ D_t^\alpha|v_j(t)|\le|\beta_j+\gamma_j^2-\gamma_jd_j||y_j(t)|-(d_j-\gamma_j)|v_j(t)|+\sum\limits_{i=1}^{n}(|g_{ji}|+|h_{ji}|)\overline{f}_i+J_j.\end{cases}\tag{10}$$

由(10)得

$$\begin{aligned}&D_t^\alpha[|x_i(t)|+|u_i(t)|]\le\\&-(\gamma_i-|\alpha_i+\gamma_i^2-\gamma_ic_i|)|x_i(t)|-(c_i-\gamma_i-1)|u_i(t)|+\sum_{j=1}^{n}(|a_{ij}|+|b_{ij}|)\overline{f}_j+I_i\le\\&-\eta_{1i}[|x_i(t)|+|u_i(t)|]+\sum_{j=1}^{n}(|a_{ij}|+|b_{ij}|)\overline{f}_j+I_i,\end{aligned}\tag{11}$$

其中$\eta_{1i}=\min\{\gamma_i-|\alpha_i+\gamma_i^2-\gamma_ic_i|,c_i-\gamma_i-1\}>0$.

令$z_i(t)=|x_i(t)|+|u_i(t)|-\dfrac{1}{\eta_{1i}}[\sum\limits_{j=1}^{n}(|a_{ij}|+|b_{ij}|)\overline{f}_j+I_i]$, 由引理2.7及(11)可推得

$$\begin{aligned}D_t^\alpha z_i(t)=D_t^\alpha[|x_i(t)|+|u_i(t)|]-D_t^\alpha\{\tfrac{1}{\eta_{1i}}[\sum_{j=1}^{n}(|a_{ij}|+|b_{ij}|)\overline{f}_j+I_i]\}=\\ D_t^\alpha[|x_i(t)|+|u_i(t)|]-\tfrac{1}{\eta_{1i}\Gamma(1-\alpha)}[\sum_{j=1}^{n}(|a_{ij}|+|b_{ij}|)\overline{f}_j+I_i]t^{-\alpha}\le\\ -\eta_{1i}[|x_i(t)|+|u_i(t)|]+\sum_{j=1}^{n}(|a_{ij}|+|b_{ij}|)\overline{f}_j+I_i+\tfrac{1}{\eta_{1i}\Gamma(1-\alpha)}[\sum_{j=1}^{n}(|a_{ij}|+|b_{ij}|)\overline{f}_j+I_i]=\\ -\eta_{1i}z_i(t)+\tfrac{1}{\eta_{1i}\Gamma(1-\alpha)}[\sum_{j=1}^{n}(|a_{ij}|+|b_{ij}|)\overline{f}_j+I_i].\end{aligned}$$

由引理2.9可得$z_i(t)\le\dfrac{1}{\eta_{1i}\Gamma(1-\alpha)}[\sum\limits_{j=1}^{n}(|a_{ij}|+|b_{ij}|)\overline{f}_j+I_i]E_\alpha(-\eta_{1i}t^\alpha)$, 即

$$\begin{gathered}|x_i(t)|+|u_i(t)|\le\\ \frac{1}{\eta_{1i}\Gamma(1-\alpha)}[\sum_{j=1}^{n}(|a_{ij}|+|b_{ij}|)\overline{f}_j+I_i]E_q(-\eta_{1i}t^\alpha)+\frac{1}{\eta_{1i}}[\sum_{j=1}^{n}(|a_{ij}|+|b_{ij}|)\overline{f}_j+I_i].\end{gathered}$$

于是得到$x_i(t)$和$u_i(t)$有界, $i=1,2,\cdots,n$.

同理可推得

$$|y_j(t)|+|v_j(t)|\leq$$
$$\frac{1}{\eta_{2j}\Gamma(1-\alpha)}[\sum_{i=1}^{n}(|g_{ji}|+|h_{ji}|)\overline{f}_j+J_j]E_\alpha(-\eta_{2j}t^\alpha)+\frac{1}{\eta_{2j}}[\sum_{i=1}^{n}(|g_{ji}|+|h_{ji}|)\overline{f}_j+J_j],$$

其中$\eta_{2j}=\min\{\gamma_j-|\beta_j+\gamma_j^2-\gamma_jd_j|,d_j-\gamma_j-1\}>0$. 可知$y_j(t)$和$v_j(t)$有界, $j=1,2,\cdots,n$.

由引理2.8, 从(3)可得

$$\begin{cases}x_i(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}[-\gamma_ix_i(s)+u_i(s)]\mathrm{ds},\\ u_i(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}[-(\alpha_i+\gamma_i^2-\gamma_ic_i)x_i(s)-(c_i-\gamma_i)u_i(s)+\\ \qquad\sum\limits_{j=1}^{n}a_{ij}f_j(y_j(s))+\sum\limits_{j=1}^{n}b_{ij}f_j(y_j(s-\tau_{ij}))+I_i(s)]\mathrm{ds},\\ y_j(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}[-\gamma_jy_j(s)+v_j(s)]\mathrm{ds},\\ v_j(t)=\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}[-(\beta_j+\gamma_j^2-\gamma_jd_j)y_j(s)-(d_j-\gamma_j)v_j(s)+\\ \qquad\sum\limits_{i=1}^{n}g_{ji}f_i(x_i(s))+\sum\limits_{i=1}^{n}b_{ji}f_i(x_i(s-\sigma_{ji}))+J_j(s)]\mathrm{ds}.\end{cases}\tag{12}$$

由(12)可得

$$\begin{aligned}x_i(t+\omega)=&\frac{1}{\Gamma(\alpha)}\int_0^{t+\omega}(t+\omega-s)^{\alpha-1}[-\gamma_ix_i(s)+u_i(s)]\mathrm{ds}=\\&\frac{1}{\Gamma(\alpha)}\int_{-\omega}^{t}(t-s)^{\alpha-1}[-\gamma_ix_i(s+\omega)+u_i(s+\omega)]\mathrm{ds}=\\&\frac{1}{\Gamma(\alpha)}\int_{0}^{t}(t-s)^{\alpha-1}[-\gamma_ix_i(s+\omega)+u_i(s+\omega)]\mathrm{ds}+\\&\frac{1}{\Gamma(\alpha)}\int_{-\omega}^{0}(t-s)^{\alpha-1}[-\gamma_ix_i(s+\omega)+u_i(s+\omega)]\mathrm{ds}.\end{aligned}\tag{13}$$

同理可得

$$\begin{aligned}u_i(t+\omega)=&\frac{1}{\Gamma(\alpha)}\int_0^{t}(t-s)^{\alpha-1}[-(\alpha_i+\gamma_i^2-\gamma_ic_i)x_i(s+\omega)-(c_i-\gamma_i)u_i(s+\omega)+\\&\sum_{j=1}^{n}a_{ij}f_j(y_j(s+\omega))+\sum_{j=1}^{n}b_{ij}f_j(y_j(s+\omega-\tau_{ij}))+I_i(s)]\mathrm{ds}+\\&\frac{1}{\Gamma(\alpha)}\int_{-\omega}^{0}(t-s)^{\alpha-1}[-(\alpha_i+\gamma_i^2-\gamma_ic_i)x_i(s+\omega)-(c_i-\gamma_i)u_i(s+\omega)+\\&\sum_{j=1}^{n}a_{ij}f_j(y_j(s+\omega))+\sum_{j=1}^{n}b_{ij}f_j(y_j(s+\omega-\tau_{ij}))+I_i(s)]\mathrm{ds},\end{aligned}\tag{14}$$

$$\begin{aligned}y_j(t+\omega)=&\frac{1}{\Gamma(\alpha)}\int_0^{t}(t-s)^{\alpha-1}[-\gamma_jy_j(s+\omega)+v_j(s+\omega)]\mathrm{ds}+\\&\frac{1}{\Gamma(\alpha)}\int_{-\omega}^{0}(t-s)^{\alpha-1}[-\gamma_jy_j(s+\omega)+v_j(s+\omega)]\mathrm{ds},\end{aligned}\tag{15}$$

$$
\begin{aligned}
v_j(t+\omega)=&\frac{1}{\Gamma(\alpha)}\int_0^t(t-s)^{\alpha-1}[-(\beta_j+\gamma_j^2-\gamma_j d_j)y_j(s+\omega)-(d_j-\gamma_j)v_j(s+\omega)+\\
&\sum_{i=1}^n g_{ji}f_i(x_i(s+\omega))+\sum_{i=1}^n b_{ji}f_i(x_i(s+\omega-\sigma_{ji}))+J_j(s+\omega)]\mathrm{d}s+\\
&\frac{1}{\Gamma(\alpha)}\int_{-\omega}^0(t-s)^{\alpha-1}[-(\beta_j+\gamma_j^2-\gamma_j d_j)y_j(s+\omega)-(d_j-\gamma_j)v_j(s+\omega)+\\
&\sum_{i=1}^n g_{ji}f_i(x_i(s+\omega))+\sum_{i=1}^n b_{ji}f_i(x_i(s+\omega-\sigma_{ji}))+J_j(s+\omega)]\mathrm{d}s.
\end{aligned}\tag{16}
$$

从(12)-(13)可得

$$
\begin{aligned}
x_i(t+\omega)-x_i(t)=&D_t^{-\alpha}[-\gamma_i(x_i(t+\omega)-x_i(t))+u_i(t+\omega)-u_i(t)]+\\
&\frac{1}{\Gamma(\alpha)}\int_{-\omega}^0(t-s)^{\alpha-1}[-\gamma_j x_i(s+\omega)+u_i(s+\omega)]\mathrm{d}s.
\end{aligned}\tag{17}
$$

由于$x_i(t)$和$u_i(t)$有界, $\exists M_i>0$, 使得$|x_i(t)|\le M_i, |u_i(t)|\le M_i, i=1,2,\cdots,n$. 从(17)可得

$$
\begin{aligned}
&|x_i(t+\omega)-x_i(t)|\le D_t^{-\alpha}[-\gamma_i|x_i(t+\omega)-x_i(t)|+|u_i(t+\omega)-u_i(t)|]+\\
&\frac{1}{\Gamma(\alpha)}\int_{-\omega}^0(t-s)^{\alpha-1}[\gamma_j|x_i(s+\omega)|+|u_i(s+\omega)|]\mathrm{d}s\le\\
&D_t^{-\alpha}[-\gamma_i|x_i(t+\omega)-x_i(t)|+|u_i(t+\omega)-u_i(t)|]+\frac{(\gamma_i+1)M_i}{\Gamma(\alpha)}\int_{-\omega}^0(t-s)^{\alpha-1}\mathrm{d}s\le\\
&D_t^{-\alpha}[-\gamma_i|x_i(t+\omega)-x_i(t)|+|u_i(t+\omega)-u_i(t)|]+\frac{(\gamma_i+1)M_i}{\Gamma(\alpha)}\int_{-\omega}^0(-s)^{\alpha-1}\mathrm{d}s=\\
&D_t^{-\alpha}[-\gamma_i|x_i(t+\omega)-x_i(t)|+|u_i(t+\omega)-u_i(t)|]+\frac{\omega^\alpha}{\Gamma(\alpha+1)}(\gamma_i+1)M_i.
\end{aligned}\tag{18}
$$

由于$y_j(t)$和$v_j(t)$有界, $\exists N_j>0$, 使得$|y_j(t)|\le N_j, |v_j(t)|\le N_j$, 类似于(18)结果的推导过程, 从(12), (14)-(16)可推得

$$
\begin{aligned}
&|u_i(t+\omega)-u_i(t)|\le D_t^{-\delta}[|\alpha_i+\gamma_i^2-\gamma_i c_i||x_i(t+\omega)-x_i(t)|-(c_i-\gamma_i)|u_i(t+\omega)-u_i(t)|+\\
&\sum_{j=1}^n|a_{ij}|l_j|y_j(t+\omega)-y_i(t)|+\sum_{j=1}^n|b_{ij}|l_j|y_j(t+\omega-\tau_{ij})-y_j(t-\tau_{ij})|+\\
&\frac{\omega^\alpha}{\Gamma(\alpha+1)}[(|\alpha_i+\gamma_i^2-\gamma_i c_i|+|c_i-\gamma_i|)M_i+\sum_{j=1}^n(|a_{ij}|+|b_{ij}|)\overline{f_j}+I_i],
\end{aligned}\tag{19}
$$

$$
|y_j(t+\omega)-y_j(t)|\le D_t^{-\delta}[-\gamma_j|y_j(t+\omega)-y_j(t)|+|v_j(t+\omega)-v_j(t)|]+\frac{\omega^\alpha}{\Gamma(\alpha+1)}(\gamma_j+1)N_j,\tag{20}
$$

$$
\begin{aligned}
|v_j(t+\omega)-v_j(t)|\le& D_t^{-\delta}[|\beta_j+\gamma_j^2-\gamma_j d_j||y_j(t+\omega)-y_j(t)|-(d_j-\gamma_j)|v_j(t+\omega)-v_j(t)|+\\
&\textstyle\sum_{i=1}^n|g_{ji}|l_j|x_i(t+\omega)-x_i(t)|+\sum_{i=1}^n|h_{ji}|l_i|x_i(t+\omega-\sigma_{ji})-x_i(t-\sigma_{ji})|+\\
&\textstyle\frac{\omega^\alpha}{\Gamma(\alpha+1)}[(|\beta_j+\gamma_j^2-\gamma_j c_j|+|d_j-\gamma_j|)N_j+\sum_{i=1}^n(|g_{ji}|+|h_{ji}|)\overline{f_j}+J_j].
\end{aligned}\tag{21}
$$

类似于(7)结果的推导过程, 可得

$$
\begin{cases}
D_t^{-\alpha}|y_j(t+\omega-\tau_{ij})-y_j(t-\tau_{ij})|\le\dfrac{2\psi_{1j}^*\tau^\alpha}{\Gamma(\alpha+1)}+D_t^{-\alpha}|y_j(t+\omega)-y_j(t)|,\\
D_t^{-\alpha}|x_i(t+\omega-\sigma_{ij})-x_i(t-\sigma_{ij})|\le\dfrac{2\varphi_{1j}^*\tau^\alpha}{\Gamma(\alpha+1)}+D_t^{-\alpha}|x_i(t+\omega)-x_i(t)|,
\end{cases}\tag{22}
$$

其中$\psi_{1j}^*=\sup\limits_{-\tau\le s\le 0}\{|\psi_{1j}(s)|\},\varphi_{1i}^*=\sup\limits_{-\sigma\le s\le 0}\{|\varphi_{1i}(s)|\}$.

从(18)-(22)可得

$$\begin{aligned}&\sum_{i=1}^n[|x_i(t+\omega)-x_i(t)|+|u_i(t+\omega)-u_i(t)|]+\sum_{j=1}^n[|y_j(t+\omega)-y_j(t)|+|v_j(t+\omega)-v_j(t)|]\le\\&\sum_{i=1}^n\{D_t^{-\alpha}[-\gamma_i+|\alpha_i+\gamma_i^2-\gamma_ic_i|+\\&\quad\sum_{i=1}^n(|g_{ji}|+|h_{ji}|l_i)]|x_i(t+\omega)-x_i(t)|+D_t^{-\alpha}[1-c_i+\gamma_i]|u_i(t+\omega)-u_i(t)|\}+\\&\quad\sum_{j=1}^n\{D_t^{-\alpha}[-\gamma_i+|\beta_j+\gamma_j^2-\gamma_jd_j|+\\&\quad\sum_{j=1}^n(|a_{ij}|+|b_{ij}|)]|y_j(t+\omega)-y_j(t)|+D_t^{-\alpha}[1-d_j+\gamma_j]|v_j(t+\omega)-v_j(t)|\}+\\&\quad\sum_{i=1}^n\frac{\omega^\alpha}{\Gamma(\alpha+1)}(\gamma_i+1)M_i+\sum_{j=1}^n\frac{\omega^\alpha}{\Gamma(\alpha+1)}(\gamma_j+1)N_j+\\&\quad\sum_{i=1}^n\frac{\omega^\alpha}{\Gamma(\alpha+1)}[(|\alpha_i+\gamma_i^2-\gamma_ic_i|+|c_i-\gamma_i|)M_i+\sum_{j=1}^n(|a_{ij}|+|b_{ij}|)\overline{f_j}+I_i]+\\&\quad\sum_{j=1}^n\frac{\omega^\alpha}{\Gamma(\alpha+1)}[(|\beta_j+\gamma_j^2-\gamma_jc_j|+|d_j-\gamma_j|)N_j+\sum_{i=1}^n(|g_{ji}|+|h_{ji}|)\overline{f_j}+J_j]\le\\&-\eta D_t^{-\alpha}\{\sum_{i=1}^n[|x_i(t+\omega)-x_i(t)|+|u_i(t+\omega)-u_i(t)|]+\\&\quad\sum_{j=1}^n[|y_j(t+\omega)-y_j(t)|+|v_j(t+\omega)-v_j(t)|]\}+\delta,\end{aligned}\tag{23}$$

其中η由定理2.1所给定,

$$\begin{aligned}\delta=&\sum_{i=1}^n\frac{\omega^\alpha}{\Gamma(\alpha+1)}[(\gamma_i+1)M_i+(|\alpha_i+\gamma_i^2-\gamma_ic_i|+|c_i-\gamma_i|)M_i+\sum_{j=1}^n(|a_{ij}|+|b_{ij}|)\overline{f_j}+I_i]+\\&\sum_{j=1}^n\frac{\omega^\alpha}{\Gamma(\alpha+1)}[(\gamma_j+1)N_j+(|\beta_j+\gamma_j^2-\gamma_jc_j|+|d_j-\gamma_j|)N_j+\sum_{i=1}^n(|g_{ji}|+|h_{ji}|)\overline{f_j}+J_j].\end{aligned}$$

由引理2.9, 从(23)可推得

$$\sum_{i=1}^n[|x_i(t+\omega)-x_i(t)|+|u_i(t+\omega)-u_i(t)|]+\sum_{j=1}^n[|y_j(t+\omega)-y_j(t)|+|v_j(t+\omega)-v_j(t)|]\le\delta E_\alpha(-\eta t^\alpha).\tag{24}$$

从(24)可得$\lim\limits_{t\to+\infty}\sum\limits_{i=1}^n|x_i(t)-x_i(t+\omega)|=0,\lim\limits_{t\to+\infty}\sum\limits_{j=1}^n|y_j(t)-y_j(t+\omega)|=0$. 由定义2.5知, 系统(1)的解$X(t)$在$(0,+\infty)$上是$S$-渐近$\omega$-周期解.

设$X(t)$是系统(1)的S-渐近ω-周期解, 那么对于任意$k\in\mathbf{N}$, $X(t+k\omega)$也是系统(1)的解, 由于$X(t)$有界, 则可得函数序列$\{X(t+k\omega)\}_{k\in\mathbf{N}}$是等度连续且一致有界的, 利用Arzela-Asoli定理, 可选取一个子序列$\{k\omega\}_{k\in\mathbf{N}}$使得$\{X(t+k\omega)\}_{k\in\mathbf{N}}$一致收敛于$[0,+\infty)$内紧集上的连续函数

$X^*(t)=(x_1^*(t),x_2^*(t),\cdots,x_n^*(t),y_1^*(t),y_2^*(t),\cdots,y_n^*(t))^{\mathrm{T}}$.

下证$X^*(t)$是周期函数. 事实上

$$X^*(t+\omega)=\lim_{k\to+\infty}X(t+\omega+k\omega)=\lim_{k\to+\infty}X(t+(k+1)\omega)=X^*(t). \tag{25}$$

即$X^*(t)$是ω-周期函数. 由于

$$|x_i(t)-x_i^*(t)|\le|x_i(t)-x_i(t+\omega)|+|x_i(t+\omega)-x_i(t+k\omega)|+|x_i(t+k\omega)-x_i^*(t)|. \tag{26}$$

因为$x_i(t)$是S-渐近ω-周期的, 可得

$$\lim_{t\to+\infty}\sum_{i=1}^{n}|x_i(t)-x_i(t+\omega)|=0. \tag{27}$$

又因为系统(1)是Mittag-Leffler稳定, 那么有

$$\lim_{t\to+\infty}|x_i(t+\omega)-x_i(t+k\omega)|=0. \tag{28}$$

利用$X^*(t)$的定义可得

$$\lim_{k\to+\infty}|x_i(t+k\omega)-x_i^*(t)|=0. \tag{29}$$

从(26)-(29)可得

$$\lim_{t\to+\infty}|x_i(t)-x_i^*(t)|=0, i=1,2,\cdots,n. \tag{30}$$

同理可得

$$\lim_{t\to+\infty}|y_j(t)-y_j^*(t)|=0, i=1,2,\cdots,n. \tag{31}$$

从(30)-(31)式, 由定义2.6可知, 系统(1)的解是全局渐近ω-周期的.

§4 数值例子

例4.1 考虑分数阶惯性时滞BAM神经网络

$$\begin{cases} D_t^{2\alpha}x_i(t)=-c_iD_t^{\alpha}x_i(t)-\alpha_ix_i(t)+\sum\limits_{j=1}^{2}a_{ij}f_j(y_j(t))+\sum\limits_{j=1}^{2}b_{ij}f_j(y_j(t-\tau_{ij}))+I_i(t),\\ D_t^{2\alpha}y_j(t)=-d_jD_t^{\alpha}y_j(t)-\beta_jy_j(t)+\sum\limits_{i=1}^{2}g_{ji}f_i(x_i(t))+\sum\limits_{i=1}^{2}h_{ji}f_i(x_i(t-\sigma_{ji}))+J_j(t),\end{cases} \tag{32}$$

其中$t\ge0, i,j=1,2$.

$\alpha=0.8$, $f_i(x_i)=\frac{1}{2}(|x_i+1|-|x_i-1|)$, $\tau_{ij}=\sigma_{ij}=1$, $i,j=1,2$. $I_1(t)=0.02\sin(t)$, $I_2(t)=0.05\sin(t)$, $J_1(t)=0.02\cos(t)$, $J_2(t)=0.05\cos(t)$, $\alpha_1=5$, $\alpha_2=4$, $\beta_1=4$, $\beta_2=3$, $c_1=5$, $c_2=4.5$, $d_1=3.2$, $d_2=3.5$, $a_{11}=1$, $a_{12}=0.5$, $a_{21}=0.5$, $a_{22}=0.8$, $b_{11}=-0.1$, $b_{12}=0.1$, $b_{21}=0.2$, $b_{22}=0.2$, $g_{11}=1$, $g_{12}=0.5$, $g_{21}=0.5$, $g_{22}=0.8$, $h_{11}=0.1, h_{12}=0.5, h_{21}=0.4, h_{22}=0.2$.

根据给定的$f_i(x_i)$, 取$l_i=1, i=1,2$. 选取$\gamma_i=2, i=1,2$. 通过计算得到

$$\eta=\min\Big\{\min_{1\le i\le2}\{\gamma_i-|\alpha_i+\gamma_i{}^2-\gamma_ic_i|-\sum_{j=1}^{2}|g_{ji}|l_i-\sum_{j=1}^{2}|h_{ji}|l_i, c_i-\gamma_i-1\},$$
$$\min_{1\le j\le2}\{\gamma_j-|\beta_j+\gamma_j{}^2-\gamma_jd_j|-\sum_{i=1}^{2}|a_{ij}|l_j-\sum_{i=1}^{2}|b_{ij}|l_j, d_j-\gamma_j-1\}\Big\}=0.2>0,$$

由此可知, 系统(32)满足定理3.1和定理3.2的所有条件, 故其解是Mittag-Leffler稳定的, 且是S-渐近2π-周期解.

通过计算机数值模拟, 可得到系统$x_1(t),x_2(t),y_1(t),y_2(t)$在$t$时刻的变化状态, 如图1和图2所示. 从图可知所得到数值模拟的结果与定理结果相一致, 从而验证了理论推导所得结果的正确性.

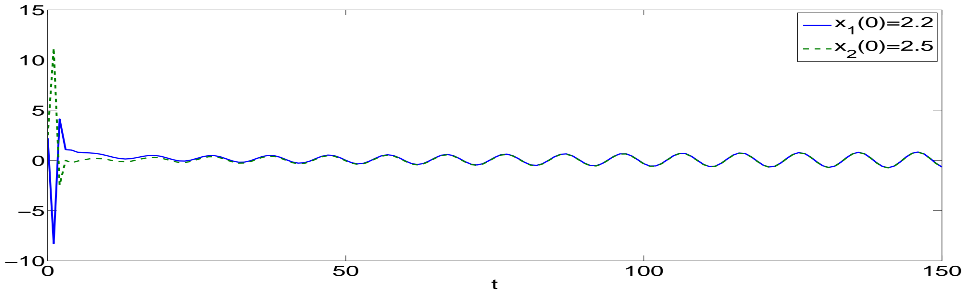

图 1 例4.1中系统(32) $x_1(t), x_2(t)$在t时刻的变化轨迹

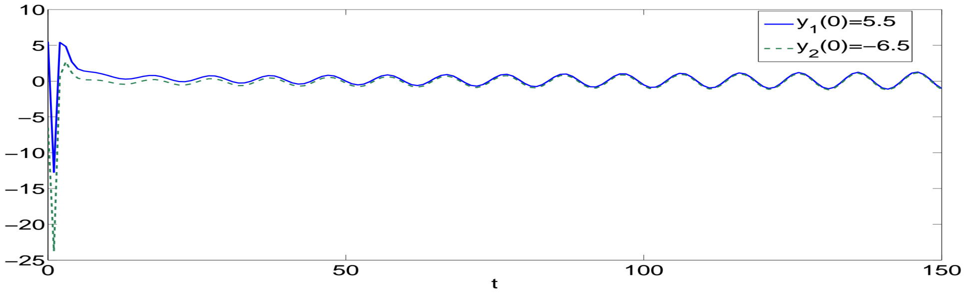

图 2 例4.1中系统(32) $y_1(t), y_2(t)$在t时刻的变化轨迹

§5 结　论

本文是对分数阶惯性时滞BAM神经网络的动力学行为进行研究. 通过引入变量代换, 对含有二个不同分数阶导数的BAM神经网络模型转化为只含一个分数阶导数神经网络的模型. 利用积分的区间可加性和系统的初始值, 推导出含有时间迟滞和不含时间迟滞的状态函数分数阶积分之间的关系. 分别给出了系统解全局Mittag-Leffler稳定和全局渐近ω-周期判定的充分条件, 即定理3.1-定理3.2, 所得到研究结果是新的. 同时通过数值模拟例子验证了本文理论推导所得结果的正确性. 这对于进一步研究分数阶惯性神经网络的动力学特性具有理论意义和应用价值. 利用本文研究的思路和方法, 可进一步研究其他类型的分数阶惯性神经网络的稳定性和周期性问题.

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Global Mittag-Leffler stability and global asymptotic ω-period for fractional inertial BAM neural network with time-delay

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Abstract: The paper investigates the global Mittag-Leffler stability and global asymptotic ω-periodicity of fractional-order inertial BAM neural networks with time delays. Firstly, by utilizing the properties of Riemann-Liouville fractional calculus and introducing appropriate variable substitutions, the fractional-order inertial BAM neural network model, which involves two different fractional-order derivatives, is simplified to a model containing only one fractional-order derivative. Next, by applying the additivity of integral intervals and initial value conditions, the relationship between the fractional-order integrals of the state functions with and without time delays is derived when the time variable varies within finite intervals less than or equal to the time delay and within infinite intervals greater than or equal to the time delay. Sufficient conditions for determining the global Mittag-Leffler stability and global asymptotic ω-periodicity of the solutions for the fractional-order inertial BAM neural network system are provided. Finally, numerical simulations are conducted to verify the effectiveness and correctness of the theoretical results obtained.

Keywords: fractional; inertial; BAM neural networks; global Mittag-Leffler stability; global asymptotic ω-period

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